

Marko Zlokarnik

**Scale-Up in Chemical
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Marko Zlokarnik

Scale-Up in Chemical Engineering

Second, Completely Revised and Extended Edition



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This book is dedicated to my friend and teacher

Dr. phil. Dr.-Ing. h.c. Juri Pawlowski

Contents

Preface to the 1st Edition XIII

Preface to the 2nd Edition XV

Symbols XVII

1 Introduction 1

2 Dimensional Analysis 3

2.1 The Fundamental Principle 3

2.2 What is a Dimension? 3

2.3 What is a Physical Quantity? 3

2.4 Base and Derived Quantities, Dimensional Constants 4

2.5 Dimensional Systems 5

2.6 Dimensional Homogeneity of a Physical Content 7

Example 1: What determines the period of oscillation of a pendulum? 7

Example 2: What determines the duration of fall θ of a body in a homogeneous gravitational field (Law of Free Fall)? What determines the speed v of a liquid discharge out of a vessel with an opening? (Torricelli's formula) 9

Example 3: Correlation between meat size and roasting time 12

2.7 The Pi Theorem 14

3 Generation of Pi-sets by Matrix Transformation 17

Example 4: The pressure drop of a homogeneous fluid in a straight, smooth pipe (ignoring the inlet effects) 17

4 Scale Invariance of the Pi-space – the Foundation of the Scale-up 25

Example 5: Heat transfer from a heated wire to an air stream 27

5	Important Tips Concerning the Compilation of the Problem Relevance List	31
5.1	Treatment of Universal Physical Constants	31
5.2	Introduction of Intermediate Quantities	31
Example 6:	Homogenization of liquid mixtures with different densities and viscosities	33
Example 7:	Dissolved air flotation process	34
6	Important Aspects Concerning the Scale-up	39
6.1	Scale-up Procedure for Unavailability of Model Material Systems	39
Example 8:	Scale-up of mechanical foam breakers	39
6.2	Scale-up Under Conditions of Partial Similarity	42
Example 9:	Drag resistance of a ship's hull	43
Example 10:	Rules of thumb for scaling up chemical reactors: Volume-related mixing power and the superficial velocity as design criteria for mixing vessels and bubble columns	47
7	Preliminary Summary of the Scale-up Essentials	51
7.1	The Advantages of Using Dimensional Analysis	51
7.2	Scope of Applicability of Dimensional Analysis	52
7.3	Experimental Techniques for Scale-up	53
7.4	Carrying out Experiments Under Changes of Scale	54
8	Treatment of Physical Properties by Dimensional Analysis	57
8.1	Why is this Consideration Important?	57
8.2	Dimensionless Representation of a Material Function	59
Example 11:	Standard representation of the temperature dependence of the viscosity	59
Example 12:	Standard representation of the temperature dependence of density	63
Example 13:	Standard representation of the particle strength for different materials in dependence on the particle diameter	64
Example 14:	Drying a wet polymeric mass. Reference-invariant representation of the material function $D(T, F)$	66
8.3	Reference-invariant Representation of a Material Function	68
8.4	Pi-space for Variable Physical Properties	69
Example 15:	Consideration of the dependence $\mu(T)$ using the μ_w/μ term	70
Example 16:	Consideration of the dependence $\rho(T)$ by the Grashof number Gr	72
8.5	Rheological Standardization Functions and Process Equations in Non-Newtonian Fluids	72
8.5.1	Rheological Standardization Functions	73
8.5.1.1	Flow Behavior of Non-Newtonian Pseudoplastic Fluids	73
8.5.1.2	Flow Behavior of Non-Newtonian Viscoelastic Fluids	76
8.5.1.3	Dimensional-analytical Discussion of Viscoelastic fluids	78
8.5.1.4	Elaboration of Rheological Standardization Functions	80

Example 17: Dimensional-analytical treatment of <i>Weissenberg's</i> phenomenon – Instructions for a PhD thesis	81
8.5.2 Process Equations for Non-Newtonian Fluids	85
8.5.2.1 Concept of the Effective Viscosity μ_{eff} According to <i>Metzner–Otto</i>	86
8.5.2.2 Process Equations for Mechanical Processes with Non-Newtonian Fluids	87
Example 18: Power characteristics of a stirrer	87
Example 19: Homogenization characteristics of a stirrer	90
8.5.2.3 Process Equations for Thermal Processes in Association with Non-Newtonian Fluids	91
8.4.2.4 Scale-up in Processes with Non-Newtonian Fluids	91
9 Reduction of the Pi-space	93
9.1 The Rayleigh – Riabouchinsky Controversy	93
Example 20: Dimensional-analytical treatment of <i>Boussinesq's</i> problem	95
Example 21: Heat transfer characteristic of a stirring vessel	97
10 Typical Problems and Mistakes in the Use of Dimensional Analysis	101
10.1 Model Scale and Flow Conditions – Scale-up and Miniplants	101
10.1.1 The Size of the Laboratory Device and Fluid Dynamics	102
10.1.2 The Size of the Laboratory Device and the Pi-space	103
10.1.3 Micro and Macro Mixing	104
10.1.4 Micro Mixing and the Selectivity of Complex Chemical Reactions	105
10.1.5 Mini and Micro Plants from the Viewpoint of Scale-up	105
10.2 Unsatisfactory Sensitivity of the Target Quantity	106
10.2.1 Mixing Time θ	106
10.2.2 Complete Suspension of Solids According to the 1-s Criterion	106
10.3 Model Scale and the Accuracy of Measurement	107
10.3.1 Determination of the Stirrer Power	108
10.3.2 Mass Transfer in Surface Aeration	108
10.4 Complete Recording of the Pi-set by Experiment	109
10.5 Correct Procedure in the Application of Dimensional Analysis	111
10.5.1 Preparation of Model Experiments	111
10.5.2 Execution of Model Experiments	111
10.5.3 Evaluation of Test Experiments	111
11 Optimization of Process Conditions by Combining Process Characteristics	113
Example 22: Determination of stirring conditions in order to carry out a homogenization process with minimum mixing work	113
Example 23: Process characteristics of a self-aspirating hollow stirrer and the determination of its optimum process conditions	118
Example 24: Optimization of stirrers for the maximum removal of reaction heat	121

12 Selected Examples of the Dimensional-analytical Treatment of Processes in the Field of Mechanical Unit Operations 125

Introductory Remark 125

Example 25: Power consumption in a gassed liquid. Design data for stirrers and model experiments for scaling up 125

Example 26: Scale-up of mixers for mixing of solids 131

Example 27: Conveying characteristics of single-screw machines 135

Example 28: Dimensional-analytical treatment of liquid atomization 140

Example 29: The hanging film phenomenon 143

Example 30: The production of liquid/liquid emulsions 146

Example 31: Fine grinding of solids in stirred media mills 150

Example 32: Scale-up of flotation cells for waste water purification 156

Example 33: Description of the temporal course of spin drying in centrifugal filters 163

Example 34: Description of particle separation by means of inertial forces 166

Example 35: Gas hold-up in bubble columns 170

Example 36: Dimensional analysis of the tableting process 174

13 Selected Examples of the Dimensional-analytical Treatment of Processes in the Field of Thermal Unit Operations 181

13.1 Introductory Remarks 181

Example 37: Steady-state heat transfer in mixing vessels 182

Example 38: Steady-state heat transfer in pipes 184

Example 39 Steady-state heat transfer in bubble columns 185

13.2 Foundations of the Mass Transfer in a Gas/Liquid (G/L) System 189

A short introduction to Examples 40, 41 and 42 189

Example 40: Mass transfer in surface aeration 191

Example 41: Mass transfer in volume aeration in mixing vessels 193

Example 42: Mass transfer in the G/L system in bubble columns with injectors as gas distributors. Optimization of the process conditions with respect to the efficiency of the oxygen uptake $E \equiv G/\Sigma P$ 196

13.3 Coalescence in the Gas/Liquid System 203

Example 43: Scaling up of dryers 205

14 Selected Examples for the Dimensional-analytical Treatment of Processes in the Field of Chemical Unit Operations 211

Introductory Remark 211

Example 44: Continuous chemical reaction process in a tubular reactor 212

Example 45: Description of the mass and heat transfer in solid-catalyzed gas reactions by dimensional analysis 218

Example 46: Scale-up of reactors for catalytic processes in the petrochemical industry 226

Example 47: Dimensioning of a tubular reactor, equipped with a mixing nozzle, designed for carrying out competitive-consecutive reactions 229

Example 48: Mass transfer limitation of the reaction rate of fast chemical reactions in the heterogeneous material gas/liquid system 233

15 Selected Examples for the Dimensional-analytical Treatment of Processes within the Living World 237

Introductory Remark 237

Example 49: The consideration of rowing from the viewpoint of dimensional analysis 238

Example 50: Why most animals swim beneath the water surface 240

Example 51: Walking on the Moon 241

Example 52: Walking and jumping on water 244

Example 53: What makes sap ascend up a tree? 245

16 Brief Historic Survey on Dimensional Analysis and Scale-up 247

16.1 Historic Development of Dimensional Analysis 247

16.2 Historic Development of Scale-up 250

17 Exercises on Scale-up and Solutions 253

17.1 Exercises 253

17.2 Solutions 256

18 List of important, named pi-numbers 259

19 References 261

Index 269

Preface to the 1st Edition

In this day and age, chemical engineers are faced with many research and design problems which are so complicated that they cannot be solved by numerical mathematics. In this context, one only has to think of processes involving fluids with temperature-dependent physical properties or non-Newtonian flow behavior. Fluid mechanics in heterogeneous physical systems exhibiting coalescence phenomena or foaming, also demonstrate this problem. The scaling up of equipment needed for dealing with such physical systems often presents serious hurdles which can frequently be overcome only with the aid of partial similarity.

In general, the university graduate has not been adequately trained to deal with such problems. On the one hand, treatises on dimensional analysis, the theory of similarity and scale-up methods included in common, “run of the mill” textbooks on chemical engineering are out of date. In addition, they are seldom written in a manner that would popularize these methods. On the other hand, there is no motivation for this type of research at universities since, as a rule, they are not confronted with scale-up tasks and are therefore not equipped with the necessary apparatus on the bench-scale.

All of these points give the totally wrong impression that the methods referred to are – at most – of only marginal importance in practical chemical engineering, because otherwise they would have been dealt with in greater depth at university level!

The aim of this book is to remedy this deficiency. It presents dimensional analysis – this being the only secure foundation for scale-up – in such a way that it can be immediately and easily understood, even without a mathematical background.

Due to the increasing importance of biotechnology, which employs non-Newtonian fluids far more frequently than the chemical industry does, variable physical properties (e.g., temperature dependence, shear-dependence of viscosity) are treated in detail. It must be kept in mind that in scaling up such processes, apart from the geometrical and process-related similarity, the physical similarity also has to be considered.

The theoretical foundations of dimensional analysis and of scale-up are presented and discussed in the first half of this book. This theoretical framework is demonstrated by twenty examples, all of which deal with interesting engineering problems taken from current practice.

The second half of this book deals with the integral dimensional-analytical treatment of problems taken from the areas of mechanical, thermal and chemical process engineering. In this respect, the term “integral” is used to indicate that, in the treatment of each problem, dimensional analysis was applied from the very beginning and that, as a consequence, the performance and evaluation of tests were always in accordance with its predictions.

A thorough consideration of this approach not only provides the reader with a practical guideline for their own use; it also shows the unexpectedly large advantage offered by these methods.

The interested reader, who is intending to solve a concrete problem but is not familiar with dimensional-analytical methodology, does not need to read this book from cover to cover in order to solve the problem in this way. It is sufficient to read the first seven chapters (ca. 50 pages), dealing with dimensional analysis and the generation of dimensionless numbers. Subsequently, the reader can scrutinize the examples given in the second part of this book and choose that example which helps to find a solution to the problem under consideration. In doing so, the task in hand can be solved in the dimensional-analytical way. Only the practical treatment of such problems facilitates understanding for the benefit and efficiency of these methods.

In the course of the past 35 years during which I have been investigating dimensional-analytical working methods from the practical point of view, my friend and colleague, Dr. *Juri Pawlowski*, has been an invaluable teacher and adviser. I am indebted to him for innumerable suggestions and tips as well as for his comments on this manuscript. I would like to express my gratitude to him at this point.

In closing, my sincere thanks also go to my former employer, the company BAYER AG, Leverkusen/Germany. In the “Engineering Department Applied Physics” I could devote my whole professional life to process engineering research and development. This company always permitted me to spend a considerable amount of time on basic research in the field of chemical engineering in addition to my company duties and corporate research.

Marko Zlokarnik

Preface to the 2nd Edition

The first English edition of this book (2002) received a surprisingly good reception and was sold out during the course of the year 2005. My suggestion to prepare a new edition instead of a further reprint was willingly accepted by the J.Wiley-VCH publishing house.

I would like to express my sincere thanks to the editors, Ms. Dr. Barbara Böck und Ms. Karin Sora.

Over the last five years I have held almost thirty seminars on this topic in the “Haus der Technik” in Essen, Berlin and Munich, in “Dechema” in Frankfurt and also in various university institutes and companies in the German speaking countries (Germany – Austria – Switzerland). Meeting young colleagues I was thus able to detect any difficulties in understanding the topic and to find out how these hurdles could be overcome. I was anxious to use this experience in the new edition.

The following topics have been added to the new edition:

1. The chapter on “Variable physical properties”, particularly non-Newtonian liquids, has been completely reworked. The following new examples have been added: Particle strength of solids in dependence on particle diameter, *Weissenberg's* phenomenon in viscoelastic fluids, and coalescence phenomena in gas/liquid (G/L) systems.
2. The problems of scale-up from miniplants in the laboratory, was examined more closely.
3. Two further interesting examples deal with the dimensional analysis of the tableting process and of walking on the moon's surface.
4. The examples concerning steady-state heat transfer include that in pipelines and in mixing vessels in addition to bubble columns.
5. Mass transfer in G/L systems has been restructured in order to present the differences in the dimensional-analytical treatment of the surface and volume aeration more clearly.
6. A brief historic survey of the development of the dimensional analysis and of scale-up is included.
7. There are 25 exercises and their solutions.

In order not to overextend the size of the book, some examples from the first edition, in which a few less important topics were treated, have been omitted.

I would like to thank my friend and teacher, Dr. Juri Pawlowski, for his advice in restructuring various chapters, especially the section dealing with rheology.

Graz, December 2005

Marko Zlokarnik

Symbols

Latin symbols

a	volume-related phase boundary surface $a \equiv A/V$
α	thermal diffusivity; $\alpha \equiv k/(\rho C_p)$
A	area, surface
$c, \Delta c$	concentration, concentration difference
c	velocity of sound in a vacuum
C_p	heat capacity, mass-related
c_s	saturation concentration
d	characteristic diameter
d_b	bubble diameter, usually formulated as “ <i>Sauter</i> mean diameter” d_{32}
d_{32}	<i>Sauter</i> mean diameter of gas bubbles and drops, respectively
d_p	particle diameter
D	vessel diameter, pipe diameter
D	diffusivity
D_{eff}	effective axial dispersion coefficient
E	energy
	enhancement factor in chemisorption
	activation energy in chemical reactions
	efficiency factor of the absorption process
f	functional dependence
F	force
F	degree of humidity
g	acceleration due to gravity
G	mass flow
G	gravitational constant
h	heat transfer coefficient
H	height
	base dimension of the amount of heat
J	<i>Joule's</i> mechanical heat equivalent
k	reaction rate constant
	thermal conductivity
	proportionality constant (Section 8.5)

k	<i>Boltzmann</i> constant
k_G	gas-side mass transfer coefficient
k_L	liquid-side mass transfer coefficient
k_{La}	volume-related liquid-side mass transfer coefficient
k_F	flotation rate constant
K	consistency index (Section 8.5)
l	characteristic length
L	base dimension of length
m	mass
m	flow index (Section 8.5)
mol	amount of substance
M	base dimension of mass
n	stirrer speed
N	base dimension of amount of substance
	number of stages
N_x	normal stress ($x = 1$ or 2); (Section 8.5)
p, Δp	pressure, pressure drop
P	power, power of stirrer
q	volume throughput
Q	heat flow
r	rank of the dimensional matrix
	reaction rate
R	heat of reaction
R	universal gas constant
S	cross-sectional area ($\propto D^2$)
S_i	coalescence parameters (in i numbers)
t	running time
T	base dimension of time
T	temperature
T	absolute temperature
u	tip speed ($u = \pi nd$)
U	overall heat transfer coefficient (Example 23)
v	velocity, superficial velocity
V	volume
z	number

Greek symbols

α	angle
β	specific breakage energy (Example 31)
β_0	temperature coefficient of density,
γ	deformation
γ_0	temperature coefficient of viscosity
$\dot{\gamma}$	shear rate

Δ	difference
δ	thickness of film, layer, wall
ε	gas hold-up in the liquid
ε	mass-related power, $\varepsilon \equiv P/\rho V$
ζ	friction factor in pipe flow
Θ	base dimension of temperature
	contact angle
	time constant (Chapter 8)
θ	duration of time
Λ	macro-scale of turbulence
λ	relaxation time (Section 8.5)
	<i>Kolmogorov's</i> micro-scale of turbulence
μ	dynamic viscosity
μ	scale factor, $\mu \equiv l_T/l_M$
ν	kinematic viscosity
ρ	density
ρC_p	heat capacity, volume-related
σ	surface tension, phase boundary tension
	tensile strength
τ	mean residence time, $\tau = V/q$
	shear stress
τ_0	yield stress
φ	portion (volume, mass)
ϕ	degree of filling

Indices

c	continuous phase
d	dispersed phase
e	end value
F	flock
G	gas (gaseous)
L	liquid
min	minimum
M	model-scale
0	start condition
p	particle
s	saturation value
	height of the layer
S	solid, foam
t	condition at time t
T	technological-scale, full-scale
w	wall

1

Introduction

A chemical engineer is generally concerned with the industrial implementation of processes in which the chemical or microbiological conversion of material takes place in conjunction with the transfer of mass, heat, and momentum. These processes are *scale-dependent*, i.e., they behave differently on a small scale (in laboratories or pilot plants) than they do on a large scale (in production). Also included are heterogeneous chemical reactions and most unit operations. Understandably, chemical engineers have always wanted to find ways of simulating these processes *in models* in order to gain knowledge which will then assist them in designing new industrial plants. Occasionally, they are faced with the same problem for another reason: an industrial facility already exists but does not function properly, if at all, and suitable measurements have to be carried out in order to discover the cause of these difficulties as well as to provide a solution.

Irrespective of whether the model involved represents a “scale-up” or a “scale-down”, certain important questions will always apply:

- How small can the model be? Is one model sufficient or should tests be carried out with models of different sizes?
- When must or when can physical properties differ? When must the measurements be carried out on the model with the original system of materials?
- Which rules govern the adaptation of the process parameters in the model measurements to those of the full-scale plant?
- Is it possible to achieve complete similarity between the processes in the model and those in its full-scale counterpart? If not: how should one proceed?

These questions touch on the theoretical fundamentals of models, these being based on dimensional analysis. Although they have been used in the field of fluid dynamics and heat transfer for more than a century – cars, aircraft, vessels and heat exchangers were scaled up according to these principles – these methods have gained only a modest acceptance in chemical engineering. The reasons for this have already been explained in the preface.

The importance of dimensional-analytical methodology for current applications in this field can be best exemplified by practical examples. Therefore, the main

emphasis of this book lies in the integral treatment of chemical engineering problems by dimensional analysis.

From the area of *mechanical* process engineering, stirring in homogeneous and in gassed fluids, as well as the mixing of particulate matter, are treated. Furthermore, atomization of liquids with nozzles, production of liquid/liquid dispersions (emulsions) in emulsifiers and the grinding of solids in stirred ball mills is dealt with. As peculiarities, scale-up procedures are presented for the flotation cells for waste water purification, for the separation of aerosols in dust separators by means of inertial forces and also for the temporal course of spin drying in centrifugal filters.

From the area of *thermal* process engineering, the mass and heat transfer in stirred vessels and in bubble columns is treated. In the case of mass transfer in the gas/liquid system, coalescence phenomena are also dealt with in detail. The problem of simultaneous mass and heat transfer is discussed in association with film drying.

In dealing with *chemical* process engineering, the conduction of chemical reactions in a tubular reactor and in a packed bed reactor (solid-catalyzed reactions) is discussed. In consecutive-competitive reactions between two liquid partners, a maximum possible selectivity is only achievable in a tubular reactor under the condition that back-mixing of educts and products is completely prevented. The scale-up for such a process is presented. Finally, the dimensional-analytical framework is presented for the reaction rate of a fast chemical reaction in the gas/liquid system, which is to a certain degree, limited by mass transfer.

Last but not least, in the final chapter it is demonstrated by a few examples that different types of motion in the *living world* can also be described by dimensional analysis. In this manner the validity range of the pertinent dimensionless numbers can be given. The processes of motion in Nature are subject to the same physical framework conditions (restrictions) as the technological world.

2

Dimensional Analysis

2.1

The Fundamental Principle

Dimensional analysis is based upon the recognition that a mathematical formulation of a chemical or physical technological problem can be of general validity only if it is *dimensionally homogenous*, i.e., if it is valid in any system of dimensions.

2.2

What is a Dimension?

A dimension is a purely *qualitative* description of a sensory perception of a physical entity or natural appearance. Length can be experienced as height, depth and breadth. Mass presents itself as a light or heavy body and time as a short moment or a long period. The dimension of length is Length (L), the dimension of a mass is Mass (M), and so on.

Each physical concept can be associated with a type of quantity and this, in turn, can be assigned to a dimension. It can happen that different quantities display the same dimension. *Example:* Diffusivity (D), thermal diffusivity (α) and kinematic viscosity (ν) all have the same dimension $[L^2 T^{-1}]$.

2.3

What is a Physical Quantity?

In contrast to a dimension, a physical quantity represents a *quantitative* description of a physical quality.

physical quantity = numerical value $\times$ measuring unit

Example: A mass of 5 kg: $m = 5$ kg. The measuring unit of length can be a meter, a foot, a cubit, a yardstick, a nautical mile, a light year, and so on. Measuring units

of energy are, for example, Joule, cal, eV. (It is therefore necessary to establish the measuring units in an appropriate measuring system.)

2.4

Base and Derived Quantities, Dimensional Constants

A distinction is made between base or primary quantities and the secondary quantities which are derived from them (derived quantities). Base quantities are based on standards and are quantified by comparison with them. According to the *Système International d' Unités* (SI), mass is classified as a primary quantity instead of force, which was used some forty years ago.

Secondary quantities are derived from primary ones according to physical laws, e.g., velocity = length/time: $[v] = L/T$. Its coherent measuring unit is m/s. Coherence of the measuring units means that the secondary quantities have to have only those measuring units which correspond with *per definitionem* fixed primary ones and therefore present themselves as power products of themselves. Giving the velocity in mph (miles per hour) would contradict this.

If a secondary quantity has been established by a physical law, it can happen that it contradicts another one.

Example a:

According to Newton's 2nd law of motion, the force, F , is expressed as a product of mass, m , and acceleration, a : $F = m a$, having the dimension of $[M L T^{-2}]$. According to Newton's law of gravitation, force is defined by $F \propto m_1 m_2/r^2$, thus leading to another dimension $[M^2 L^{-2}]$. To remedy this, the gravitational constant G – a dimensional constant – had to be introduced to ensure the dimensional homogeneity of the latter equation:

$$F = G m_1 m_2/r^2.$$

$$G [M^{-1} L^3 T^{-2}] = 6.673 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$$

Example b:

A similar example is the introduction of the universal gas constant, R , which ensures that in the perfect gas equation of state $p V = n R T$ the already fixed secondary unit for work

$W = p V [M L^2 T^{-2}]$ is not affected.

$$R [M L^2 T^{-2} N^{-1} \Theta^{-1}] = 8.313 \text{ J}/(\text{mol K})$$

Another class of derived quantities is represented by the coefficients in the transfer equations for momentum, mass and heat. They were also established by the

respective physical laws – therefore they are often called “defined quantities” – and can only be determined via measurement of their constituents.

In chemical process engineering it frequently happens that new secondary quantities have to be introduced. The dimensions and the coherent measuring units for these can easily be fixed. *Example:* volume-related liquid-side mass transfer coefficient $k_{L,a}$ [T^{-1}].

2.5 Dimensional Systems

A dimensional system consists of all the primary and secondary dimensions and corresponding measuring units. The currently used “Système International d’unités (SI) is based on seven base dimensions. They are presented in **Table 1** together with their corresponding base units.

Table 2 refers to some very frequently used secondary measuring units which have been named after famous researchers.

Table 1 Base quantities, their dimensions and units according to SI

Base quantity	Base dimension	SI base unit	
length	L	m	meter
mass	M	kg	kilogram
time	T	s	second
thermodynamic temperature	Θ	K	kelvin
amount of substance	N	mol	mole
electric current	I	A	ampere
luminous intensity	J	cd	candela

Table 2 Important secondary measuring units in mechanical engineering, named after famous researchers

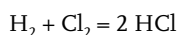
Sec. quantity	Dimension	Measuring unit	Name
force	$M L T^{-2}$	$kg m s^{-2} \equiv N$	Newton
pressure	$M L^{-1} T^{-2}$	$kg m^{-1} s^{-2} \equiv Pa$	Pascal
energy	$M L^2 T^{-2}$	$kg m^2 s^{-2} \equiv J$	Joule
power	$M L^2 T^{-3}$	$kg m^2 s^{-3} \equiv W$	Watt

Table 3 Commonly used secondary quantities and their respective dimensions according to the currently used SI units in mechanical and thermal problems

Physical entity	Dimension
area	L^2
volume	L^3
angular velocity, shear rate, frequency	T^{-1}
velocity	$L T^{-1}$
acceleration	$L T^{-2}$
kinematic viscosity, diffusivity, thermal diffusivity	$L^2 T^{-1}$
density	$M L^{-3}$
surface tension	$M T^{-2}$
dynamic viscosity	$M L^{-1} T^{-1}$
momentum	$M L T^{-1}$
force	$M L T^{-2}$
pressure, tension	$M L^{-1} T^{-2}$
angular momentum	$M L^2 T^{-1}$
mechanical energy, work, torque	$M L^2 T^{-2}$
power	$M L^2 T^{-3}$
specific heat	$L^2 T^{-2} \theta^{-1}$
heat conductivity	$M L T^{-3} \theta^{-1}$
heat transfer coefficient	$M T^{-3} \theta^{-1}$

If, in the formulation of a problem, only the base dimensions [M, L, T] occur in the dimensions of the involved quantities, then it is a mechanical problem. If [θ] occurs, then it is a thermal problem and if [N] occurs it is a chemical problem.

In a chemical reaction the molecules (or the atoms) of the reaction partners react with each other and not their masses. Their number (amount) results from the mass of the respective substance according to its molecular mass. One mole (SI unit: mole) of a chemically pure compound consists of $N_A = 6.022 \times 10^{23}$ entities (molecules, atoms). The information about the amount of substance is obtained by dividing the mass of the chemically pure substance by its molecular mass. To put it even more precisely: In the reaction between gaseous hydrogen and chlorine, one mole of hydrogen reacts with one mole of chlorine according to the equation



and two moles of hydrochloric acid are produced. Consequently, it is completely insignificant that, with respect to the masses, 2 g of H_2 reacted with 71 g of Cl_2 to produce 73 g of HCl .

2.6

Dimensional Homogeneity of a Physical Content

It has already been emphasized that a physical relationship can be of general validity only if it is formulated *dimensionally homogeneously*, i.e. if it is valid with any system of dimensions (section 2.1). The aim of dimensional analysis is to check whether the physical content under consideration can be formulated in a dimensionally homogeneous manner or not. The procedure necessary to accomplish this consists of two parts:

- a) Initially, all physical parameters necessary to describe the problem are listed. This so-called “relevance list” of the problem consists of the quantity in question (as a rule only one) and all the parameters which influence it. The target quantity is the only dependent variable and the influencing parameters should be primarily independent of each other. Example: From the material properties μ , v and ρ only two should be chosen, because they are linked together by the definition equation: $v \equiv \mu/\rho$.
- b) In the second step, the dimensional homogeneity of the physical content is checked by transferring it into a dimensionless form. This is the foundation of the so-called pi theorem (see section 2.7). The dimensional homogeneity comes to the dimensionless formulation of the physical content in question.

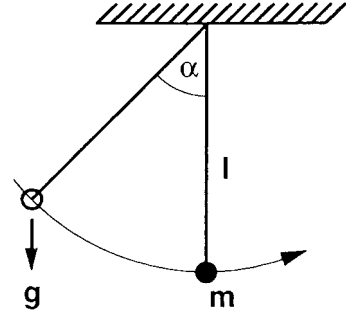
■ Note: ***A dimensionless expression is dimensionally homogeneous!***

This point will be made clear by three examples.

Example 1: What determines the period of oscillation of a pendulum?

We first draw a sketch depicting a pendulum and write down all the quantities which could be involved in this question [15]. It may be assumed that the period of oscillation of a pendulum depends on the length and mass of the pendulum, the gravitational acceleration and the amplitude of the swing:

Physical quantity	Symbol	Dimension
Period of oscillation	θ	T
Length of pendulum	l	L
Mass of pendulum	m	M
Gravitational acceleration	g	LT^{-2}
Amplitude (angle)	α	–



Our aim is to establish θ as a function of l , m , g and α :

$$\theta = f(l, m, g, \alpha) \quad (2.1)$$

Can this dependency be dimensionally homogeneous? **No!** The first thing that now becomes clear is that the base dimension of mass [M] only occurs in the mass m itself. Changing its measuring unit, e.g., from kilograms to pounds, would change the numerical value of the function. This is unacceptable. Either our list should include a further variable containing M, or mass is not a relevant variable. If we assume – by simplification^{*)} – the latter, the above relationship is reduced to:

$$\theta = f(l, g, \alpha) \quad (2.2)$$

Both l and g contain the base dimension of length. When combined as a ratio l/g they become dimensionless with respect to L and are therefore independent of changes in the base dimension of length:

$$\theta = f(l/g, \alpha) \quad (2.3)$$

On the left-hand side of the equation we have the dimension T and on the right T^2 . To remedy this, we will have to write $\sqrt{l/g}$. This expression will keep its dimension [T] only if it remains unchanged, therefore we have to put it as a constant in front of the function f . Because α is dimensionless anyway, the final result of the dimensional analysis reads:

$$\theta = \sqrt{l/g} f(\alpha) \rightarrow \theta \sqrt{g/l} = f(\alpha) \quad (2.4)$$

The dependency between four dimensional quantities, containing two base dimensions (L and T) in their dimensions, is reduced to a $4 - 2 = 2$ parametric relationship between dimensionless expressions (“numbers”)!

*) In the case of a real pendulum the density and viscosity of air should also be introduced into the relevance list. Both contain mass in their dimensions. However, this would

unnecessarily complicate the problem at this stage. Therefore we will consider a physical pendulum with a point mass in a vacuum.

This equation is the only statement that dimensional analysis can offer in this case. It is not capable of producing information on the form of f . The integration of Newton's equation of motion for small amplitudes leads to $f(a) = 2\pi$; the period of oscillation is then independent of a . The relationship can now be expressed as:

$$\theta \sqrt{g/l} = 2\pi \quad (2.5)$$

The left-hand side of the expression is a *dimensionless* number, its numerical value being 2π .

The elegant solution of this example should not tempt the reader to believe that dimensional analysis can solve every problem solely by a theoretical consideration. To treat this example by dimensional analysis, the acceleration due to gravity ($g = 9.81 \text{ m s}^{-2}$) had to be known, this being calculated from the gravitational law. *Sir Isaac Newton* derived it from *Kepler's* laws of planetary movement. *Bridgman's* ([15], p.12) comment on this situation is particularly appropriate:

"The problem cannot be solved by the philosopher in his armchair, but the knowledge involved was gathered only by someone at some time soiling his hands with direct contact".

Many a reader may have doubted the conclusion that the period of swing of a pendulum does not depend on its mass. It should be remarked that by the year 1602 *G. Galilei* had already verified this by a simple experiment [7]. He built gallows on which four pendulums of the same length, but different mass, were hung. In this way all four pendulums could be moved to the same extent and then released at the same time. It could be clearly seen that the period really does not depend on the mass. (The model of this arrangement can be found in the department of physics at Padova University in Italy.)

Example 2: What determines the duration of fall θ of a body in a homogeneous gravitational field (Law of Free Fall)? What determines the speed v of a liquid discharge out of a vessel with an opening? (*Torricelli's* formula)

This example concerns two further well-known physical laws and shows – in a similar way to the first one – that important information can often be obtained solely by dimensional-analytical treatment, without the need for any experiments.

Let us first consider Free Fall. (A drawing is unnecessary here!) The duration of fall θ depends on the length h and on the gravitational acceleration g . From the viewpoint of dimensional analysis the mass must be irrelevant, because the base dimension M is contained only in the mass itself and therefore cannot be eliminated. (In a similar way to example 1, we assume that friction of the air – caused by its density and viscosity – can be neglected; we work in a vacuum.)

In the case of Free Fall, *Galilei* had already discovered the irrelevance of mass from the consideration that he referred to in his script "De motu" in the year

1590. He referred to the Italian physicist *G. B. Benedetti* [70]. His deliberations were as follows: If the speed of fall were proportional to the mass, a body of double the mass would fall twice as fast as the original one. If both bodies were attached by a thread, the lighter one would slow down the heavier one, the combination of the two would thus fall more slowly. If the distance between both bodies were shortened so much that one single body of three times the mass were formed, would it fall faster than the one of twice the mass? This contradiction shows that in Free Fall the body mass must be irrelevant.

The relevance list of this problem is as follows:

$$\{\theta; h; g\} \quad (2.6)$$

These three quantities contain only two base dimensions [L, T], from them only one dimensionless product can be produced.

$$\frac{\theta^2 g}{h} = \text{const} \quad (2.7)$$

From this, for the duration of fall θ and the final velocity $v_{\text{end}} = g \theta$, resp., the following expressions can be deduced:

$$\theta = \sqrt{\text{const}_1 h/g} \quad \text{or} \quad v_{\text{end}} = \sqrt{\text{const}_2 g h} \quad (2.8)$$

Dimensional analysis can indicate nothing about the numerical value of the constants. These must normally be evaluated by experiment. The present case can be treated mathematically; the numerical value of both constants is obtained by integration of the Newtonian equation of motion

$$d^2y/dt^2 = -g$$

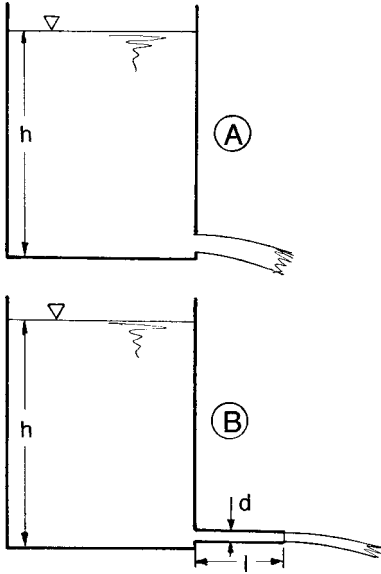
with the integration conditions

$$v = dy/dt; \quad y(t=0) = h; \quad y(t=\theta) = 0; \quad v(t=0) = 0; \quad v_{\text{end}} = v(t=\theta).$$

They are found to be equal and lead to the final result

$$\theta = \sqrt{2 h/g} \quad \text{or} \quad v_{\text{end}} = \sqrt{2 g h} \quad (2.9)$$

Now let us consider the discharge of a fluid from an opening of a vessel. Here we distinguish between friction-free and non friction-free discharge and draw two sketches.



In the case of a **friction-free discharge** (sketch A) we obtain the same relationship as in the case of Free Fall by equalizing the potential energy of the fluid (its hydrostatic pressure)

$$\rho g h = \Delta p_{\text{hydr}} \quad (2.10)$$

with its dynamic energy (pressure drop Δp in the opening)

$$v^2 \rho / 2 = \Delta p_{\text{dyn}} \quad (2.11)$$

We obtain

$$\frac{\Delta p_{\text{hydr}}}{\Delta p_{\text{dyn}}} = \frac{\rho g h}{v^2 \rho / 2} = \frac{2 g h}{v^2} \rightarrow v = \sqrt{2 g h} \quad (2.12)$$

This relationship is called the *Torricelli* discharge formula. The dimensionless expression

$$\frac{\Delta p_{\text{hydr}}}{\Delta p_{\text{dyn}}} = \frac{\Delta p}{v^2 \rho / 2} \equiv \text{Eu} \quad (2.13)$$

is called the Euler number Eu.

In the case of a **non friction-free** discharge (sketch B, the opening is combined with a pipe of diameter d and length l) the kinematic viscosity ν of a fluid becomes an important physical parameter. (This quantity cannot effect the discharge of a fluid through an opening in a thin wall.)

The initial relevance list $\{v, g, h\}$, which led to relationships (2.12) and (2.13) is extended by both geometric parameters d and l as well as by the physical parameter ν to

$$\{v, g, h, d, l; \nu\}.$$

We anticipate the geometric number l/d and note that ν can be made dimensionless in combination with v and d . Thus we obtain the following set of numbers:

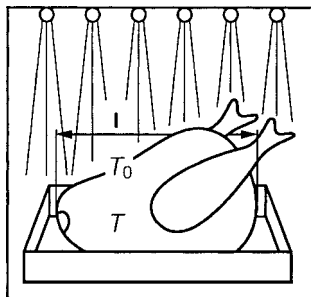
$$\left\{ \frac{v^2}{g h}; \frac{v d}{\nu}; \frac{l}{d} \right\} \rightarrow \{Eu, Re, l/d\} \quad (2.14)$$

in which Re denotes the Reynolds number. From this it follows

$$Eu = f(Re, l/d) \quad (2.15)$$

Example 3: Correlation between meat size and roasting time

First, we have to recall the physical situation and, in order to facilitate our understanding, we should draw a sketch. At high oven temperatures heat is transferred from the heating elements to the meat surface by both radiation and heat convection. From there it is transferred solely by unsteady-state heat conduction which represents the rate-limiting step of the heating process.



The higher the thermal conductivity, k , of the body, the faster the heat spreads out. The higher its volume-related heat capacity, ρC_p , the slower the heat transfer. Therefore, the unsteady-state heat conduction is characterized by only one material property, the thermal diffusivity, $a \equiv k/\rho C_p$, of the body.

Roasting is an endothermic process. The meat is ready when a certain temperature distribution (T) is reached within it. The target quantity is the time duration, θ , necessary to achieve this temperature field.

After these considerations we are able to precisely draw up the relevance list:

Physical quantity	Symbol	Dimension
roasting time	θ	T
meat surface	A	L^2
thermal diffusivity	a	$L^2 T^{-1}$
surface temperature	T_0	θ
temperature distribution	T	θ

The base dimension of temperature, Θ , appears only in two quantities. They can therefore be contained in only one dimensionless quantity:

$$\Pi_1 \equiv T/T_0 \quad \text{or} \quad (T_0 - T)/T_0 \quad (2.16)$$

The residual three quantities form one additional dimensionless number:

$$\Pi_2 \equiv a\theta/A \equiv \text{Fo} \quad (2.17)$$

In the theory of heat transfer, Π_2 is known as the Fourier number, Fo. Now, the roasting process is presented in a two-dimensional framework:

$$T/T_0 = f(\text{Fo}) \quad (2.18)$$

In this example, five dimensional quantities produce two dimensionless numbers. This was to be expected because their dimensions contain three base dimensions: $5 - 3 = 2$.

The question concerning the correlation between the roasting time and the meat size can now be easily answered, even without explicitly knowing the function f . To reach the same temperature distribution T/T_0 or $(T_0 - T)/T_0$ in differently sized bodies, the dimensionless number $\text{Fo} \equiv a\theta/A$ must display the same (= idem, identical) numerical value. Because thermal diffusivity a remains unaltered in the meat of the same species ($a = \text{idem}$), this leads to

$$T/T_0 = \text{idem} \rightarrow \text{Fo} \equiv a\theta/A = \text{idem} \rightarrow q/A = \text{idem} \rightarrow \theta \propto A \quad (2.19)$$

This statement is obviously useless as a scale-up rule because meat is bought according to weight and not to surface. One can easily remedy this. In geometrically similar bodies, the following correlation between mass, m , surface, A , and volume, V , exists:

$$m = \rho V \propto \rho L^3 \propto \rho A^{3/2} \quad (A \propto L^2)$$

Therefore, from $\rho = \text{idem}$ it follows

$$A \propto m^{2/3} \quad \text{and thus} \quad \theta \propto A \propto m^{2/3}$$

From this correlation we obtain the scale-up rule

$$\theta_2/\theta_1 \propto (m_2/m_1)^{2/3} \quad (2.20)$$

This is the scale-up criterion for the roasting time of meat of the same kind ($a, \rho = \text{idem}$). It states that in doubling the mass of meat, the cooking time will increase by $2^{2/3} = 1.58$.

G.B. West [158] refers to “inferior” cookbooks which simply say something like “20 minutes per pound”, implying a linear relationship with weight. However, there exist “superior” cookbooks, such as the “Better Homes and Gardens Cookbook” (Des Moines Meredith Corp. 1962), which recognize the nonlinear nature of this relationship. The graphical representation of measurements given in this book confirms the relationship

$$\theta \propto m^{0.6}$$

which is very close to the theoretical evaluation giving $\theta \propto m^{2/3} = m^{0.67}$.

These three transparent and easy to understand examples clearly show how dimensional analysis deals with specific problems and the conclusions it arrives at. Now, it should be easier to understand Lord Rayleigh’s sarcastic comment with which he began his short essay on “The Principle of Similitude” [118]:

“I have often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of “laws” are put forward as novelties on the basis of elaborate experiments, which might have been predicted a priori after a few minutes’ consideration”.

From the above examples we can also learn that the transformation of a physical content from a dimensional into a dimensionless form is automatically accompanied by an essential *compression* of the statement: The set of the dimensionless numbers is smaller than the set of the quantities contained in them, but it describes the problem equally comprehensively. (In the third example the dependency between 5 dimensional parameters is reduced to a dependency between only 2 dimensionless numbers!) This is the proof of the so-called pi theorem (pi after Π , the sign used for products), which is formulated in the next Section:

2.7

The Pi Theorem

- Every physical relationship between n physical quantities can be reduced to a relationship between $m = n - r$ mutually independent dimensionless groups, where r stands for the rank of the dimensional matrix, made up of the physical quantities in question and generally equal to (or in some few cases smaller than) the number of the base quantities contained in their secondary dimensions.

(Concerning the keyword “Reduction of the rank of the matrix” see Example 5.) For mathematical proofs see e.g. Pawlowski [103] und Görtler [36].

The pi theorem is often associated with the name of *E. Buckingham*, because he introduced this term in 1914. However, the proof of the theorem had already been accomplished in the course of a mathematical analysis of partial differential equations by *A. Federmann* in 1911. (See Section 16.1: Historical Development of Dimensional Analysis.)

The first important advantage of using dimensional analysis exists in the essential **compression of the statement**. The second important advantage of its use is related to the **safeguarding of a secure scale-up**. This will be convincingly shown in the next two examples.

3

Generation of Pi-sets by Matrix Transformation

As a rule, more than two dimensionless numbers are necessary to describe a physico-technological problem; they cannot be produced as shown in the first three examples. The classical method of approaching this problem involved the solution of a system of linear algebraic equations. They were formed separately for each of the base dimensions by exponents with which they appeared in the physical quantities. *J. Pawlowski* [103] replaced this relatively awkward and involved method by a simple and transparent **matrix transformation** ("equivalence transformation") which will be presented in detail in the next example.

Example 4: The pressure drop of a homogeneous fluid in a straight, smooth pipe (ignoring the inlet effects)

Here, the relevance list consists of the following elements:

target quantity:	pressure drop, Δp
geometric parameters:	diameter, d , and length, l , of the pipe
material parameters:	density, ρ , and kinematic viscosity, ν , of the fluid
process related parameter:	volume-related throughput, q

$$\{\Delta p; d, l; \rho, \nu; q\} \quad (3.1)$$

With the dimensions of these quantities a dimensional matrix is formed. Their columns are assigned to the individual physical quantities and the rows to the exponents with which the base dimensions appear in the respective dimensions of these quantities (example: $\Delta p [M^1 L^{-1} T^{-2}]$). This dimensional matrix is subdivided into a quadratic core matrix and a residual matrix, where the rank r of the matrix (here $r = 3$) in most cases corresponds to the number of base dimensions appearing in the dimensions of the physical quantities.

	Δp	q	d	l	ρ	v
mass M	1	0	0	0	1	0
length L	-1	3	1	1	-3	2
time T	-2	-1	0	0	0	-1
	core matrix			residual matrix		

The nature of the steps which have to be carried out now makes this dimensional matrix less than ideal because it is necessary to know that each of the individual elements of the residual matrix will appear in only one of the dimensionless numbers, while the elements of the core matrix may appear as “fillers” in the denominators of all of them. The residual matrix should therefore be loaded with essential variables such as the target quantity and the most important physical properties and process-related parameters. Variables with an, as yet, uncertain influence on the process must also be included in this group. If, later, these variables are found to be irrelevant, only the dimensionless number concerned will have to be deleted while leaving the others unaltered.

Since the core matrix has to be transformed into a unity matrix (zero-free main diagonal, otherwise zeros) the “fillers” should be arranged in such a way as to facilitate a minimum of linear transformations.

The following reorganization of the above dimensional matrix fulfils both of these aims:

	ρ	d	v	Δp	q	l
mass M	1	0	0	1	0	0
length L	-3	1	2	-1	3	1
time T	0	0	-1	-2	-1	0
	core matrix			residual matrix		

Now, with the first linear transformation of the rows the so-called Gaussian algorithm is carried out (zero-free main diagonal, beneath it zeros). It determines the rank of the matrix. The rank $r = 3$ is confirmed.

	ρ	d	ν	Δp	q	l
$Z_1 = M$	1	0	0	1	0	0
$Z_2 = 3M + L$	0	1	2	2	3	1
$Z_3 = -T$	0	0	1	2	1	0
	core matrix			residual matrix		

However, in this procedure, it could happen that a zero-free diagonal does not arise. Before concluding that the rank of the matrix is in fact $r < 3$, one should examine whether another arrangement of the core matrix makes a zero-free main diagonal possible.

Now, only one further linear transformation of the rows is necessary to transform the core matrix into a unity matrix, see the matrix below.

When generating dimensionless numbers, each element of the residual matrix forms the numerator of a fraction while its denominator consists of the fillers from the unity matrix with the exponents indicated in the residual matrix, see eq. (3.2).

	ρ	d	ν	Δp	q	l
$Z'_1 = Z_1$	1	0	0	1	0	0
$Z'_2 = Z_2 - 2 Z_3$	0	1	0	-2	1	1
$Z'_3 = Z_3$	0	0	1	2	1	0
	unity matrix			residual matrix		

$$\Pi_1 \equiv \frac{\Delta p}{\rho^1 d^{-2} \nu^2} = \frac{\Delta p d^2}{\rho \nu^2} \quad \Pi_2 \equiv \frac{q}{\rho^0 d^1 \nu^1} = \frac{q}{d \nu} \quad \Pi_3 \equiv \frac{1}{\rho^0 d^1 \nu^0} = \frac{1}{d} \quad (3.2)$$

The dimensionless number Π_1 does not usually occur as a target number for Δp . It has the disadvantage that it contains the essential physical property, kinematic viscosity ν , which is already contained in the process number (which is where it belongs). This disadvantage can easily be overcome by appropriately combining the dimensionless numbers Π_1 and Π_2 . This results in the well-known Euler number

$$Eu \equiv \Pi_1 \Pi_2^{-2} \equiv \frac{\Delta p d^4}{\rho q^2} \quad (3.3)$$

which is often combined with $\Pi_3 \equiv L/d$ to obtain an *intensively formulated target number*. This takes into account that in sufficiently long pipes the effects of the

inlet can certainly be ignored and the proportionality $\Delta p \propto l$ exists. This combination of dimensionless numbers is called the friction factor in pipe flow ζ :

$$\zeta \equiv \text{Eu } d/l \equiv \Pi_1 \Pi_2^{-2} \Pi_3^{-1} \equiv \frac{\Delta p}{\rho} \frac{d^4}{q^2 l} \quad (3.4)$$

In the scientific literature ζ is normally called the “friction factor” or sometimes even the “friction coefficient”. These terms are misleading because $\zeta \equiv \text{Eu } d/l$ is a dimensionless number, whereas coefficients are dimensional constants!

■ Note: *The pi-theorem only stipulates the number of dimensionless numbers and not their form. Their form is laid down by the user, because it must suit the physics of the process and be suitable for the evaluation and presentation of the experimental data.*

The structure of the dimensionless numbers depends on the variables contained in the core matrix. The Euler number, obtained by combining Π_1 and Π_2 in the example above, would have been obtained automatically if v and q had been exchanged in the core matrix.

■ Note: *All Π sets obtained from one and the same relevance list are equivalent to each other and can be mutually transformed at leisure!*

The dimensionless number Π_2 is, in fact, the well-known Reynolds number, Re , even though it appears in another form here. Now, we will explain the structure that a dimensionless number must have in order to be called a Reynolds number. (This example is equally valid for all other named dimensionless groups.) The Reynolds number is defined as being any dimensionless number combining a *characteristic* velocity, v , and a *characteristic* measurement of length, l , with the kinematic viscosity of the fluid, $\nu \equiv \mu/\rho$. The following dimensionless numbers are equally capable of meeting these requirements:

$$\text{Re} \equiv \frac{vl\rho}{\eta} = \frac{vl}{\nu} \triangleq \frac{q}{D\nu} \triangleq n d \frac{d}{\nu} = \frac{n d^2}{\nu} \quad (3.5)$$

Here, v is interpreted as being either a superficial velocity, $v = q/S \propto q/D^2$, or the tip speed of the stirrer, $u \propto n d$; n is the stirrer speed, d is the stirrer diameter.

This method of compiling a complete set of dimensionless numbers makes it clear that the numbers formed in this way cannot contain numerical values or any other constant. These appear in dimensionless groups only when they are established and interpreted as ratios on the basis of known physical interrelations. Examples:

$$\begin{aligned} \text{Re} &\equiv \pi n d^2 / \nu & \text{where } \pi n d \text{ is the tip speed and} \\ \text{Eu} &\equiv \Delta p / (\rho v^2 / 2) & \text{where } v^2 \rho / 2 \text{ is the kinetic energy.} \end{aligned} \quad (3.6)$$

Since such expressions are of the same value as the analytically derived ones, it is always necessary to present the definition!

In the above-mentioned case of the pressure drop of the volume flow in a straight pipe, this method of compiling a complete set of dimensionless numbers produces the relationship

$$f(\text{Eu } d/l, \text{Re}) = 0 \rightarrow f(\zeta, \text{Re}) = 0 \quad \text{with} \quad \zeta \equiv \text{Eu } d/l \quad (3.7)$$

The information contained in this relationship is the maximum that dimensional analysis can offer on the basis of a relevance list, which we assumed to be complete. Dimensional analysis cannot provide any information about the form of the function f , i.e., the sort of pi-relationship involved. This information can only be obtained experimentally or by some additional theoretical consideration.

In their famous study, *Stanton and Pannell* [135] evaluated the process equation of this problem $f(\text{Eu } d/l, \text{Re}) = 0$ by measurements. **Fig. 1** shows the result of their work which impressively demonstrates the significance of the Reynolds number for pipe flow. The remark of *B. Eck* [24] hits the nail on the head:

"If one represented – as it was once usual – ζ as a function of the velocity v , one would obtain not a curve, but a galaxy. Here, the Reynold's law must strike even a beginner with an elemental force".

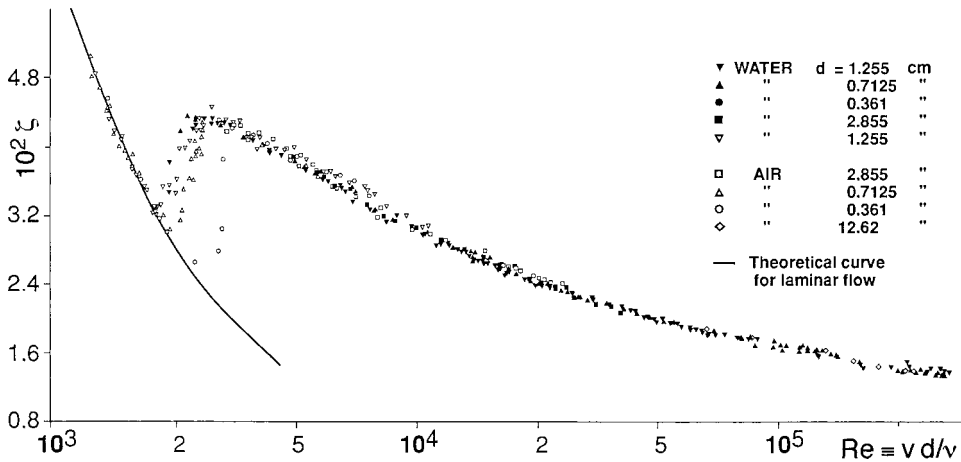


Fig. 1 Pressure drop characteristic of a straight, smooth pipe; after [135].

ζ is the “friction factor”, which is defined here (according to physical interrelation) as follows:

$$\zeta \equiv \frac{\Delta p}{(\rho/2)v^2} \frac{d}{l} = 2(\pi/4)^2 \frac{\Delta p d^4}{\rho q^2 l} = 1,24 \text{ Eu } d/l \quad (3.8)$$

The drawn-in curve is valid for the laminar flow range ($\text{Re} < 2300$) and corresponds to the analytical expression (“process equation”)

$$\zeta = 64 \text{ Re}^{-1} \quad \text{resp.} \quad \zeta \text{ Re} = 64 \quad (3.9)$$

This connection could have been clearly demonstrated had the authors chosen to present their test results in a double-logarithmic plot; this would have produced a straight line with a slope of -1 in this range of Re ! $\zeta \text{ Re}$ can be viewed as a new dimensionless number that does not include the fluid density, ρ :

$$\zeta \text{ Re} \equiv \frac{2 \Delta p}{\rho v^2} \frac{d}{l} \frac{v d \rho}{\mu} = \frac{2 \Delta p d^2}{v \mu l} = 64 \quad (3.10)$$

It is only with the pi-relationship that the relevance to the problem and the operational range of individual variables becomes clear. Only now are we able to clearly distinguish between the laminar and the turbulent range. (The scattering of the measuring points around $\text{Re} \approx 2300$ makes it clear that in smooth pipes the turbulence is often delayed.)

This example also shows that the pi-set compiled on the basis of the relevance list does no more than define the maximum pi-space, which may well shrink, from the insight gained by measurements.

In the transition range ($\text{Re} = 2.3 \times 10^3 - 1.0 \times 10^6$) the following process equations are valid:

$$\begin{aligned} \zeta &= 0.3164 \text{ Re}^{-0.25} & \text{Re} \leq 8 \times 10^4 & \quad (\text{Blasius}) \\ \zeta &= 0.0054 + 0.396 \text{ Re}^{-0.3} & \text{Re} \leq 1.5 \times 10^6 & \quad (\text{Hermann}) \end{aligned} \quad (3.11)$$

In the turbulent flow range, which appears in industrially rough ($\approx$ smooth) pipes at $\text{Re} > 10^6$, the following applies:

$$\zeta = \text{const} \quad (3.12)$$

With respect to the dependence $\Delta p(v)$ the following holds:

laminar range	$(\zeta \propto \text{Re}^{-1})$	$\Delta p \propto v$
transition range	$(\zeta \propto \text{Re}^{-0.3})$	$\Delta p \propto v^{1.7}$
turbulent range	$(\zeta = \text{const})$	$\Delta p \propto v^2$

Later on, *Nikuradze* [94] examined these correlations for artificially roughened pipes (by sticking sand particles onto the inner wall surface) and represented them in a pi-space extended by the geometrical number d_p/d . He and later researchers, for example [19], were primarily interested in the transition range of the Re number, where the wall roughness is of the same order of magnitude as the wall boundary layer.

Before the experimental data of Fig. 1 is discussed in detail with respect to the scale-up, two important conclusions can be derived from the facts presented so far:

1. The fact that the analytical presentations of the pi-relationships encountered in engineering literature often take the shape of power products does not stem from certain laws inherent in dimensional analysis. It can be simply explained by the engineer's preference for depicting test results in double-logarithmic plots. Curve sections which can be approximated as straight lines are then analytically expressed as power products. Where this proves less than easy, the engineer will often be satisfied with the curves alone, cf. Fig 1.
2. The "benefits" of dimensional analysis are often discussed. The above example provides a welcome opportunity to make the following comments. The five-parametrical dimensional relationship

$$\{\Delta p/l; d; \rho, \nu, q\}$$

can be represented by means of dimensional analysis as $\zeta(\text{Re})$ and plotted as a *single* curve (Fig. 1). If we wanted to represent this relationship in a dimensional way and avoid creating a "galaxy" at the same time [24], we would need 25 diagrams with 5 curves in each! If we had assumed that only 5 measurements per curve were sufficient, the graphic representation of this problem would still have required 625 measurements. The enormous savings in time and energy made possible by the application of dimensional analysis are consequently easy to appreciate. These significant advantages have already been pointed out by *Langhaar* [74].

Finally a critical remark must be made concerning a frequent, but completely wrong denotation of diagrams which can often be found in physical and chemical publications. Instead of denoting an axis of a diagram by, e.g.,

$$d \text{ [m]} \quad \text{or} \quad d \text{ in } m$$

d/m is used. This suggests that a dimensionless expression (a numerical value) is obtained by dividing a physical quantity by its unit of measurement.

This is real nonsense. Such a “pseudo-dimensionless” representation only means that, e.g., at a value of $d/m = 0.35$ the diameter d has this value only for the chosen measuring unit (here m). In the case of a genuine dimensionless number, however, the numerical value is independent of the measuring unit, the dimensionless number is scale-invariant!

By the way: dividing a quantity by its measuring unit does not take into consideration the fact that the product

$$\text{Physical property} = \text{numerical value} \times \text{unit of measurement}$$

does not represent a genuine multiplication, but only a procedure similar to it. (There is no way of multiplying a number by a measuring unit!) Therefore a division cannot be accepted here either. For details see [103], Section 1.2.

4

Scale Invariance of the Pi-space – the Foundation of the Scale-up

It has already been pointed out that, using dimensional analysis to handle a physical problem and, consequently, to present it in the framework of a complete set of dimensionless numbers, is the only sure way of producing a simple and reliable scale-up from the small-scale model to the full-scale technological plant. The theory of models states that:

- Two processes may be considered completely similar if they take place in a similar geometrical space and if all the dimensionless numbers necessary to describe them have the same numerical value ($\Pi_i = \text{idem}$).

This statement is supported by the results shown in Fig. 1. The researchers carried out their measurements in smooth pipes with diameters $d = 0.36 - 12.63$ cm, thereby changing the scale in the range of $1 : 35$. Furthermore, the physical properties of the fluid tested (water or air) varied widely. Nevertheless, the relationship $\zeta(\text{Re})$ did not display this change. Every numerical value of Re still corresponded to a specific numerical value of ζ ! **The pi-space is scale independent, it is scale invariant!** The pi-relationship presented is therefore valid not only for the examined laboratory devices but also for any other geometrically similar arrangement:

- Every point in a pi-framework, determined by the pi-relationship, corresponds to an infinite number of possible implementations.

This aspect is of special importance with respect to change in the linear dimension, because it is the foundation of a reliable scale-up.

From these facts the distinction between the pi-space and the original x-space is particularly clear. In an x-space $f(x_i) = 0$, which is constituted of dimensional quantities in the representation

$$x_1 = f(x_2) \quad x_j = \text{const},$$

each point of the (x_1, x_2) -space corresponds to only one realization.

This characteristic of pi-representation represents the basis of the concept of similarity based on dimensional analysis. Processes which are described by the same pi-relationship are considered similar to each other if they correspond to the

same point in the pi-framework. From the standpoint of dimensional analysis it is therefore completely insignificant whether the dimensionless numbers in question can be interpreted as ratios of forces, flows and so on. The question concerning the equality of these proportions is completely irrelevant.

Two realizations of the same physical interrelation are considered similar (*complete similarity*), when $m - 1$ dimensionless numbers of the m -dimensional pi-space have the same numerical value ($\text{pi} = \text{idem}$), because the m -th pi-number will then automatically also have the same numerical value.

To exercise the scale-up procedure on the simple example of Fig. 1, let us imagine a pipeline in which a certain fluid (natural gas or crude oil) with a certain volume-related throughput, q , must be conveyed. Our aim is to find the pressure drop, Δp , of the fluid flow in the pipeline in order to design pumps and compressors.

We start by building a geometrically similar small-scale model of the technical pipeline. We already know the physical properties of the fluid, its throughput as well as the dimensions of the industrial plant (index $_T$) and therefore we also have the numerical value for Re_T in operation. The same numerical value can be kept constant in the test apparatus (index $_M$) by the correct choice of conveying device and model fluid:

$$\text{Re} = \text{idem} \rightarrow \text{Re}_T = \text{Re}_M \rightarrow \left(\frac{v l}{\nu} \right)_T = \left(\frac{v l}{\nu} \right)_M \quad (4.1)$$

According to our knowledge of the pertinent pi-space (Fig. 1), $\text{Re} = \text{idem}$ implies $\text{Eu} = \text{idem}$. The numerical value of the Euler number Eu_M , measured in the model-scale at the given Re_M value, therefore corresponds to that of the full-scale plant. This then allows us to determine the numerical value of Δp_T in the industrial plant from the numerical value of Eu_M in the model and the given operational parameters:

$$\text{Eu} = \text{idem} \rightarrow \text{Eu}_T = \text{Eu}_M \rightarrow \left(\frac{\Delta p}{\rho v^2} \right)_T = \left(\frac{\Delta p}{\rho v^2} \right)_M \quad (4.2)$$

The procedure presented hitherto concerns the **scale-up** in the so-called **process point**. It is distinguished by the fact that the process data of the technical plant (here the volume throughput q_T and the kinematic viscosity ν_T under process conditions) are known. For the “scale-down” the process conditions (here Δp_T) will be evaluated on the laboratory scale. To do this, the knowledge of the appropriate process characteristics, Fig. 1, is unnecessary. We take it as unknown. The only assumption which has to be taken for granted is the knowledge of the correct pi-space – here $\{\zeta, \text{Re}\}$ – for the process.

This procedure is very important for scale-up, and so Exercises 7 and 8 are devoted to it.

Of course, the concept of complete similarity does not guarantee that a process will be the same in the model as in the full-scale version in *every* respect; it is only

the same with respect to the particular aspect under examination, which has been described by the appropriate pi-relationship. In order to demonstrate this fact with the help of the above example, it should be remembered that the flow conditions in two smooth pipes of different scales should be considered similar when $Re = idem$ as according to the pressure drop characteristics, it will therefore have the same numerical value of $\zeta \equiv Eu \, d/l$. However, this does not mean that heat transfer conditions prevalent in the two pipes are the same. For that to be the case, the relevant pi-relationship, $Nu = f(Re, Pr)$, requires that both the Reynolds number and the Prandtl number have the same numerical value (temperature-independent physical properties of the medium being assumed).

The more comprehensive the similarity demanded between the model ($_M$) and the full-scale device ($_T$) and the greater the

$$\text{scale-up factor } \mu \equiv l_T/l_M \quad (l - \text{any characteristic length}) \quad (4.3)$$

the harder it is to perform the scale-up. This undertaking could even fail completely if a model material system with the appropriate physical properties cannot be obtained. A further difficulty is that a scale-up involving large changes of scale may cause changes to the pi-space. An example here is the case of forced non-isothermal flow, in which progression in scale results in free convection and consequently the Grashof number, $Gr \equiv \beta \Delta T l^3 g / \nu^2$, becomes relevant to the problem. As an opposite example, it should be mentioned that with progression of the scaling down the pipe diameter, surface phenomena will sometimes come into play (e.g., the capillarity effect). In this case, a further pi-number appears which contains the surface tension σ (Weber number, We ; Bond number, Bd).

The following example has been chosen because it impressively demonstrates the scale-invariance of the pi-space. Besides this, in the matrix transformation we will encounter a reduction of the rank r of the matrix. This will enable us to understand why, in the definition of the pi-theorem (Section 2.7), it was pointed out that the rank of the matrix does *not always* equal the number of base dimensions contained in the dimensions of the respective physical quantities.

Example 5: Heat transfer from a heated wire to an air stream

This example, taken from [51], belongs to the field of heat transfer by convection. Here the heat transfer coefficient, h , represents the target quantity. This quantity can be determined only via the general heat transfer equation

$$Q = h A \Delta T \quad (4.4)$$

Electrically heated wires and pipes of diameter, d , are mounted horizontally and are cooled by an air stream. The relevance list reads:

target quantity: heat transfer coefficient, h
 geometric parameters: diameter, d , of the wire or pipe
 material parameter: density, ρ , and kin. viscosity, ν ; heat conductivity, k ;
 volume-related heat capacity, ρC_p , of the gas
 process-related parameters: flow velocity, v , of the gas

$$\{h; d; \rho, \nu, k, \rho C_p; v\} \quad (4.5)$$

A conveniently constructed dimensional matrix (for dimensions see Table 3) contains a core matrix which is transformed in three steps into the unity matrix. It just so happens that in the last linear transformation the 1st row and the ρ -columns eliminate each other. This represents a **reduction of the rank, r , of the matrix.**

	ρ	d	ν	k	h	ρC_p	v
mass M	1	0	0	1	1	1	0
length L	-3	1	1	1	0	-1	2
time T	0	0	-1	-3	-3	-2	-1
temperature Θ	0	0	0	-1	-1	-1	0
M	1	0	0	1	1	1	0
$3 M + L$	0	1	1	4	3	2	2
T	0	0	-1	-3	-3	-2	-1
Θ	0	0	0	-1	-1	-1	0
M	1	0	0	1	1	1	0
$3 M + L + T$	0	1	0	1	0	0	1
$-T$	0	0	1	3	3	2	1
Θ	0	0	0	-1	-1	-1	0
$M + \Theta$							
$3 M + L - T + \Theta$		1	0	0	-1	-1	1
$-T + 3 \Theta$		0	1	0	0	-1	1
$-\Theta$		0	0	1	1	1	0

This does not mean that the gas density, ρ , is irrelevant, but that it has just been fully taken into account by the kinematic viscosity $\nu = \mu/\rho$. If we had taken the dynamic viscosity, μ , instead of the kinematic one, ν , this elimination would not have occurred, because μ [$M L^{-1} T^{-1}$] contains mass in its dimensions. However,

the result would have been the same. In both cases $7 - 4 = 6 - 3 = 3$ pi-numbers result. These are

$$\Pi_1 \equiv \frac{h d}{k} \equiv \text{Nu} - \text{Nusselt number} \quad (4.6)$$

$$\Pi_2 \equiv \frac{\rho C_p d v}{k} \rightarrow \Pi_2 \Pi_3 \equiv \frac{\rho C_p v}{k} = \frac{\mu C_p}{k} \equiv \text{Pr} - \text{Prandtl number} \quad (4.7)$$

$$\Pi_3 \equiv \frac{v}{d v} \equiv \text{Re}^{-1} - \text{Reynolds number} \quad (4.8)$$

The functional dependence

$$\text{Nu} = f(\text{Pr}, \text{Re}) \quad (4.9)$$

is reduced when working with air to

$$\text{Nu} = f(\text{Re}). \quad (4.10)$$

This is because the Pr number here is largely temperature independent and therefore has a constant numerical value: $\text{Pr} \approx 0.70 = \text{const}$.

Functional dependence $\text{Nu} = f(\text{Re})$ is represented in **Fig. 2**. These measurements were performed with wires of $d = 0.0189\text{--}1.00$ mm and pipes of $d = 2.99\text{--}150.0$ mm. This corresponded to a scale-range of $\mu = 1 : 8.000$. Therefore, with the result in Fig. 2, the scale-invariance is proved particularly impressively.

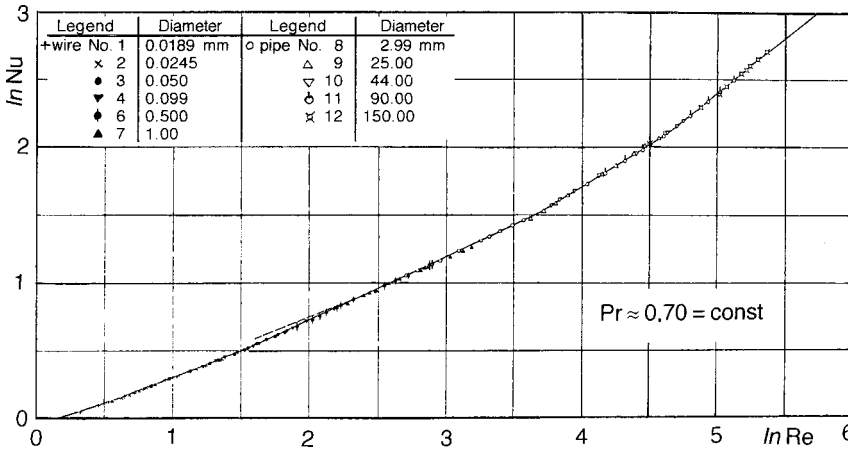


Fig. 2 Heat transfer characteristic of wires and pipes streamed crossways by the air flow; from [51].

The process equations in this pi-space read [25]:

$$\text{Nu} = 0.43 + 0.48 \text{ Re}^{0.5} \quad 1 < \text{Re} < 4 \times 10^3 \quad (4.11)$$

$$\text{Nu} \propto \text{Re}^{0.8} \quad \text{Re} > 2 \times 10^5 \quad (4.12)$$

U. Grigull [40] pointed out that the correspondence $h \propto v^{0.5}$ indicates the predominance of the laminar boundary layer, whereas $h \propto v^{0.8}$ refers to the fact that the turbulent boundary layer prevails.

5

Important Tips Concerning the Compilation of the Problem Relevance List

5.1

Treatment of Universal Physical Constants

The relevance list must also include universal physical constants such as the universal gas constant, R , the speed of light in a vacuum, c , or even the acceleration of a gravitational field (on Earth the acceleration due to gravity, g), if these constants influence the process concerned. The fact that a relevant physical quantity is a constant can never be a reason not to include it in the relevance list! By failing to consider the relevance of gravitational acceleration, the chemical engineer may find he has made a serious mistake!

Failing to consider gravitational acceleration when dealing with problems of process engineering is clearly not new. *Lord Rayleigh* [118] complained bitterly saying:

"I refer to the manner in which gravity is treated. When the question under consideration depends essentially upon gravity, the symbol of gravity (g) makes no appearance, but when gravity does not enter the question at all, g obtrudes itself conspicuously".

This is all the more surprising in view of the fact that the relevance of this quantity is easy enough to recognize if one asks the following question:

- **Would the process function differently if it took place on the Moon instead of on Earth?**

If the answer to this question is affirmative, g is a relevant variable.

In fluid dynamics g is often effective in connection with ρ or with $\Delta\rho$ as the weight or as a weight difference.

5.2

Introduction of Intermediate Quantities

Relevance lists of some problems contain a whole host of parameters. This makes the elaboration of the process characteristic a difficult endeavor. In some cases a closer look at a problem (or previous experience) facilitates a reduction in the

number of physical quantities in the relevance list. This is the case when some relevant variables affect the process by way of a so-called *intermediate quantity*. Assuming that this intermediate quantity can be measured experimentally, it should be included in the problem relevance list, if this facilitates the removal of more than one quantity from the list. If, e.g., the target quantity, y , depends on five parameters

$$y = f_1(x_1, x_2, x_3, x_4, x_5),$$

three of which can be replaced by an intermediate quantity z

$$z = f_2(x_2, x_3, x_4),$$

then the introduction of z reduces the functional dependence by two parameters to

$$y = f_3(x_1, z, x_5).$$

In the following, a few examples regarding this are mentioned.

In this context one often makes the mistake that the so-called superficial velocity $v \equiv q/S \propto q/D^2$ is also treated as an intermediate quantity. It is true that it connects two quantities (q and D) with each other but it does not in any way replace them. It only replaces the fluid throughput q . The proof follows:

For the pipe flow (see, e.g., Fig. 1) it is proved that the Reynolds number is the determining process number

$$\text{Re} \equiv v D / \nu,$$

i.e., the pipe diameter D is in fact included in the superficial velocity, but does not in any way replace it!

In bubble columns (see Example 33, Fig. 64) the extended Froude number

$$\text{Fr}' \equiv \frac{v^2 \rho}{D g \Delta \rho}$$

is without doubt the determining number for the state of flow. Here, too, the superficial velocity does not replace the column diameter.

From this point of view it is very doubtful to assume the superficial velocity as a scale-up criterion for bubble columns, as has been done for decades and which is still an accepted practice today (see Example 10 and the final remarks there). The heat transfer characteristics of bubble columns, obtained in laboratory measurements in columns of constant diameter [61, 68] were correlated by the superficial velocity, but this finding necessarily needs verification under a change of scale.

A settling process (sedimentation) depends on the parameters of the disperse phase. They can be substituted by the sedimentation velocity v_s :

$$v_s = f(d_p, g\Delta\rho, \varphi_p, \mu)$$

(d_p – mean particle diameter, $g\Delta\rho$ – weight difference, φ_p – volume portion of the solid phase, μ – dynamic density of the liquid.)

In some cases, e.g., in the flotation process, Example 7, it is possible to replace a great many influencing parameters by only one or two intermediate quantities. Then, it is legitimate to speak of intermediate quantities as “lumped parameters”.

A tableting process depends on the physical parameters, such as, e.g., mean particle diameter d_p , bulk density (porosity), flow behaviour (lubricating ability) of the powder, which depends also on humidity. These physical properties can be replaced by the lumped parameter powder compressibility κ : see Example 36.

The importance of introducing intermediate quantities will now be demonstrated by two elegant examples.

Example 6: Homogenization of liquid mixtures with different densities and viscosities

The mixing time, θ , needed for a complete homogenization of two Newtonian liquids, one resting at the start in a layer on top of the other, is normally determined by a decolorization method. For a given stirrer, installed in a vessel of known geometry, and for a material system *without* density and viscosity differences, the mixing time depends on the stirrer diameter, d , (as the characteristic geometric parameter in stirring); density, ρ , dynamic viscosity, μ , stirrer speed, n :

$$\{\theta; d; \rho, \mu; n\} \quad (5.1)$$

This 5-parametric dimensional space leads to the 2-parametric mixing time characteristic:

$$n\theta = f(\text{Re}) \quad \text{where} \quad \text{Re} \equiv n d^2 \rho / \mu = n d^2 / \nu \quad (5.2)$$

For a material system *with* density and viscosity differences, the 5-parametric relevance list, eq. (5.1), has to be extended by the physical properties of the second mixing component, by the volume ratio of both phases, $\varphi = V_2/V_1$, and – inevitably – by the weight difference, $g\Delta\rho$, due to the prevailing density differences, to a 9-parametric dimensional space [167]:

$$\{\theta; d; \rho_1, \mu_1, \rho_2, \mu_2, \varphi; g\Delta\rho, n\} \quad (5.3)$$

This leads to a mixing time characteristic consisting of six pi-numbers:

$$n\theta = f(\text{Re}, \text{Ar}, \rho_2/\rho_1, \mu_2/\mu_1, \varphi) \quad (5.4)$$

Here $\text{Re} \equiv n d^2 / \nu_1$ – Reynolds number and $\text{Ar} \equiv g\Delta\rho d^3 / (\rho_1 \nu_1^2)$ – Archimedes number.

In a meticulous visual observation of this mixing process (the slow disappearance of the *schlieren* as a result of the balancing of the density differences) one notes that the coarse macro-mixing is quickly completed as compared with the micro-mixing which takes very long time. Certainly, this long-lasting process takes place in a material system, the physical properties of which already correspond to the homogeneous mixture:

$$\mu^* = f(\mu_1, \mu_2, \varphi) \quad \text{and} \quad \rho^* = f(\rho_1, \rho_2, \varphi) \quad (5.5)$$

By the introduction of the intermediate quantities μ^* and ρ^* , the relevance list (5.3) is reduced by three parameters:

$$\{\theta; d; \rho^*, \mu^*; g\Delta\rho, n\} \quad (5.6)$$

Now, the mixing time characteristic consists of only three pi-numbers:

$$n\theta = f(\text{Re}^*, \text{Ar}^*) \quad (5.7)$$

(The pi-numbers Re^* and Ar^* have to be formed by ρ^* and μ^* .)

In this pi-space, the process equation has been evaluated for the cross-beam stirrer, whereby the scale was altered by $\mu = 1 : 2$ and the process parameters were widely varied [167]. The process equation thus found reads

$$\sqrt{n\theta} = 51.6 \text{Re}^{*-1} (\text{Ar}^{*1/3} + 3) \quad 10^1 < \text{Re}^* < 10^5; \quad 10^2 < \text{Ar}^* < 10^{11} \quad (5.8)$$

This means that the mixing time increases in this flow range by $(\Delta\rho/\rho^*)^{2/3}$. An increase of $\Delta\rho/\rho^*$ by a decade, e.g., from 10^{-2} to 10^{-1} , would lengthen the mixing time by a factor of 4.6.

This example clearly shows the advantage of introducing intermediate quantities in complicated problems. These findings will also be confirmed by the next example.

Example 7: Dissolved air flotation process

In a flotation process, tiny gas bubbles adhere to the naturally hydrophobic or artificially hydrophobized surface of solid particles making them float to the liquid surface, from where they can be removed as foam. For the removal and concentration of activated sludge in the biological waste water purification process, both dissolved air and degasifying flotation [178] can be used. In both cases, a gas mixture (air or $\text{CO}_2 + \text{N}_2$) is first dissolved in the respective liquid under higher pressure and is released, after decompression, in the form of tiny gas bubbles. The devices for this procedure consist either of longitudinally streamed through basins, or of flotation cells, **Fig. 3 a**. To enable a vertical flow through the flotation cell, the cell is spatially separated into a flotation chamber and an annular space. In the latter,

the liquid flows slowly downwards and the residual flocks rise upwards. In this way a complete removal of the particles is achieved.

Flotation is a depletion process which obeys a time law of the 1st order and is described by the flotation rate constant, k_F . The target quantity, “solids discharge A”, corresponds to the chemical conversion, X , in 1st order reactions:

$$A \equiv X \equiv (c_0 - c)/c_0 = 1 - c/c_0 = 1 - \exp(-k_F t) \quad (5.9)$$

Now we will examine the flotation process taking place in the flotation cell (Fig. 3 a) of a given geometry (the characteristic linear measurement of length being the cell diameter D).

The problem here consists of listing the material parameters because, in activated sludge, they fluctuate strongly (in contrast to ore flotation). The degree of hydrophobicity (wettability) of the particle surface (contact angle, Θ) will definitely be of importance. Furthermore, the pH value, the concentration of the flocculant (polyelectrolyte), c_F , the portion of solids, φ , in the system, the weight difference, $g\Delta\rho$, between the bacteria and the liquid – just to mention a few – will play an important role.

In contrast, it is easy to list all of the process parameters involved: liquid input, q_{in} , which leaves the cell divided into the flotage discharge, q_{F1} , and the processed liquid output, q_{out} ; releasable gas content of the liquid feed, $q_{gas}/q_{in} = Hy \Delta p/\rho_G$; gravitational acceleration g . (Hy is Henry's law constant)

The relevance list now reads:

$$\{A; D; \Theta, pH, c_F, \varphi; q_{in}, q_{aut}, q_G/q_{in}, g\Delta\rho\} \quad (5.10)$$

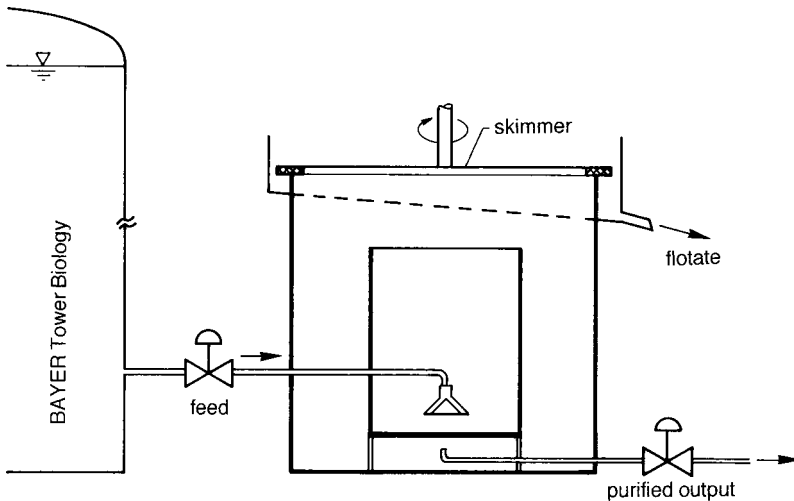


Fig. 3a Sketch of a flotation cell with spacially separated aeration and separation zones for the dissolved air flotation.

This relevance list, which is certainly incomplete with respect to physical properties, can be streamlined significantly by introducing two intermediate quantities:

1. the *superficial velocity*, v , in the annulus of the flotation cell (Fig. 3 a):

$$v = f_1(q_{in}, D) \quad \text{and} \quad (5.11)$$

2. the *rising velocity*, w , of the flocks:

$$w = f_2(\Theta, pH, c_F, \varphi, q_G/q_{in}, g\Delta\rho) \quad (5.12)$$

(The rising velocity of the flocks has to be determined by an appropriate measuring device.)

The 10-parametric relevance list in eq. (5.10), which has been assumed to be fairly complete, now reduces to only a 5-parametric one:

$$\{A, v, w, q_{in}, q_{out}\} \quad (5.13)$$

This relevance list delivers the following simple 3-parametric pi-set:

$$\{A, w/v, q_{in}/q_{out}\} \quad (5.14)$$

In this pi-space, measurements were evaluated which were performed in a bench-scale flotation cell (Fig. 3 a) of $D = 0.6$ m. The flotation cell input consisted of biologically purified waste water, containing ≈ 3 g TS/l activated sludge (TS – total solids), which was processed in the 30 m high bubble columns, the so-called “Tower Biology” of BAYER AG/Leverkusen, Germany.

The result in **Fig. 3 b** cannot be called satisfying. This is certainly not surprising, because the processed waste waters of this chemical giant originated from ca. 150 production sites and constantly changed their composition. Nevertheless, at least it can be seen that the process parameters are correctly taken into account by the ratio v/w . (The rising velocity, w , was varied each day to a small extent by the addition of different amounts of polyelectrolytic flocculants.)

The results of this examination show that biologically purified waste water can be completely freed from activated sludge under the precondition that the superficial velocity of the input amounted to the half value of the rising velocity of the flocks. (The fact that $A = 100\%$ was never achieved is understandable, because in the determination of the solids content the dissolved inorganic salts were also measured, which of course cannot be removed by flotation.)

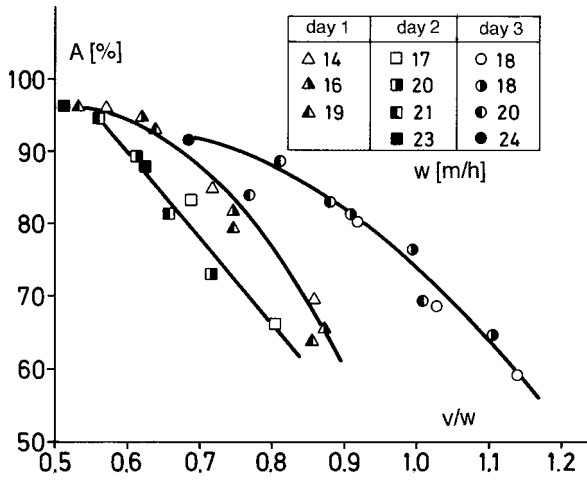


Fig. 3b Correlation between the activated sludge discharge A and the ratio v/w of the superficial velocity, v , to the rising velocity of flocks, w ; from [178].

6

Important Aspects Concerning the Scale-up

6.1

Scale-up Procedure for Unavailability of Model Material Systems

To be able to adjust the process point of the pertinent pi-space in the model experiment, one has to choose an appropriate model material system.

In order to conduct model measurements in only one model device, the numerical values of the pi-numbers in question (i.e., the fixation of the operational process point of the system) must be adjusted by the appropriate selection of the numerical values of the process parameters and/or of the model material system. If this is not feasible, then the process characteristic has to be evaluated from model experiments in devices of different scales or the operational process point has to be extrapolated from measurements in differently sized model devices.

When model material systems are not available (e.g., with non-Newtonian fluids) or the relevant material parameters are unknown (e.g., with foams, slimes and sludges), model measurements must be carried out in models of various sizes.

The unavailability of the model material systems can sometimes limit the application of the dimensional analysis. In such cases it is, of course, completely wrong to speak of “*limits of the dimensional analysis*”.

The following example will show how design and scale-up data can be obtained by model measurements with the same material system in differently sized laboratory devices.

Example 8: Scale-up of mechanical foam breakers

In certain chemical and biological as well as in some unit operations, foaming occurs to such an extent that the process is severely impaired or even comes to a complete standstill. For example, chemical reaction systems tend to foam if a gas is formed in the nascent state, because such minute gas bubbles do not coalesce to form larger ones and therefore remain in the system. Expulsion of residual monomers after emulsion polymerization often involves serious foaming problems because, in this case, very fine gas bubbles are formed in a material system which contains emulsifiers, i.e., foam-producing surfactants.

In microbiological processes, foaming occurs more often than in chemical ones since many metabolic processes produce surfactants. The process frequently takes place in 3- or 4-phase systems (G/L/(S)/micro-organisms) and intensive aeration or gas evolution (fermentation processes) are often involved. This often leads to the situation that foam evolution can no longer be controlled.

Finally, unit operations (gas absorption, distillation) may cause foaming or are even based on foaming (flotation, foam fractionation).

Mechanical foam breaking is the method most frequently used to cope with these problems. It is largely based on subjecting the foam lamellae to shear stress and to rapidly alternating pressure fields generated within the foam breaker. In addition, compressed secondary foam, which is ejected from the foam breaker, strikes against primary foam in the gas head or against the vessel wall. The task of the foam breaker is to compress the voluminous primary foam ($d_b > 1$ mm) to an easy flowing secondary foam ($d_b > 0.05$ – 0.1 mm). This can then be returned back into the gas space of the vessel.

This aim fully describes the target quantity of a foam centrifuge, s. sketch in **Fig. 4**. One has to determine the minimum rotational speed, $n_{\min}$, required to set the foam flowing. (According to [177], a resulting foam density of ca. $\rho = 0.50$ kg/l should suffice.)

The characteristic geometric parameter is undoubtedly the diameter, d , of the foam centrifuge (here the diameter of the cone base).

Density, viscosity and elasticity of the *foam lamellae*, as essential physical properties of the foam, cannot be measured. They all depend on the physical properties of the investigated multiphase material system, including the type and concentration of surfactants. These properties cannot usually be measured, particularly if they are produced *in situ* by the bacterial culture. We will therefore introduce into the relevance list the sum of all physical properties, S_i , which affect the destructibility of the foam as well as the concentration, c_T (e.g., in [ppm]), of a known tenside used in the tests.

The process parameters are first represented by the minimum rotational speed, $n_{\min}$ (as a target number), and then by the temporal foam evolution, q_f , which equals the gas throughput: $q_f \triangleq q$. In the original publication [177], the acceleration due to gravity, g , was not introduced into the relevance list on the grounds that it is the centrifugal acceleration which causes foam breaking and this exceeds the gravitational acceleration by a multiple. This is indeed true, but it also has to be taken into consideration that the state of the foam upon entry into the foam centrifuge depends very much on the force field in which it has been produced.

This reasoning leads to the following relevance list of the problem:

$$\{n_{\min}; d; S_i; c_T; q; g\} \quad (6.1)$$

and delivers the following pi-space:

$$Q^{-1} = f(\text{Fr}, S_i^*, c_T) \quad \text{with} \quad (6.2)$$

$$Q^{-1} \equiv \frac{n_{\min} d^3}{q} \quad \text{and} \quad \text{Fr} \equiv \frac{q^2}{d^5 g}$$

Q^{-1} represents the reciprocal value of the well known gas throughput number Q . Fr is the Froude number, here formed with the gas throughput and c_T is the dimensionless concentration (in ppm) of the foaming agent in the liquid. S_i^* are the physical properties which affect foam stability. Because they are neither known by number (i) nor by kind, instead of S_i^* the type of the foaming agent (name and chemical structure) must be given.

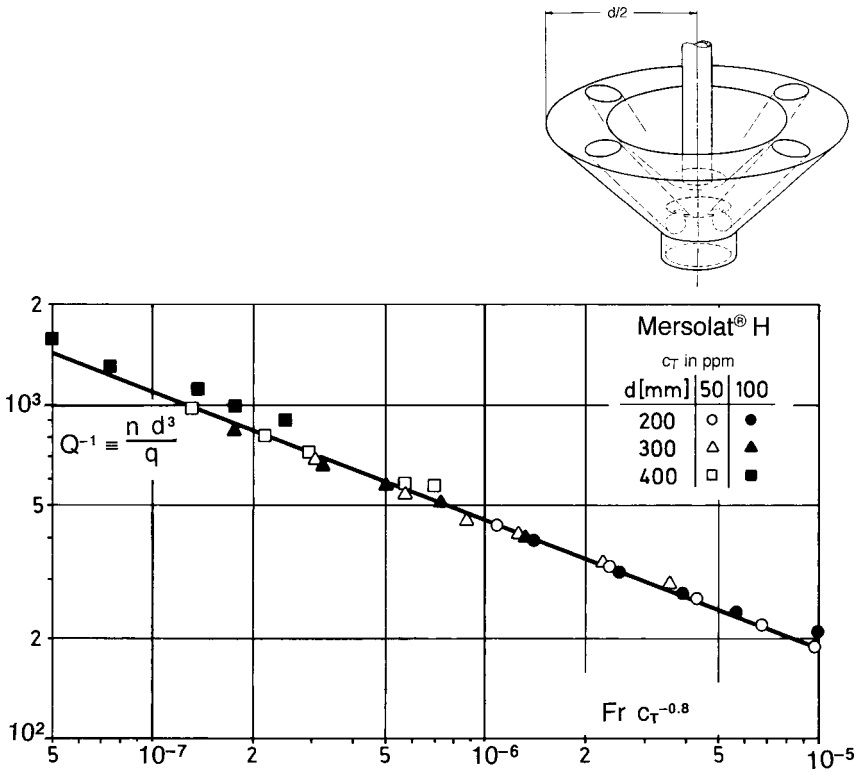


Fig. 4 Process characteristic of a foam centrifuge (sketch) for a given foaming agent (Mersolat®H) in two concentrations (c_T in ppm); form [177].

The model measurements [177] were performed on three geometrically similar appliances in the scale-ratio $\mu = 1 : 1.5 : 2$. Fig. 4 shows the dependence, eq. (6.2), for the foaming agent Mersolat®H (BAYER/Leverkusen, Germany). The process equation reads:

$$Q^{-1} = 1.37 \text{Fr}^{-0.40} c_T^{0.32} \quad (6.3)$$

For the five foaming agents examined the following exponents in

$$Q^{-1} = \text{const}_1 \text{Fr}^{-a} c_T^b \quad (6.4)$$

were found: $a = 0.40\text{--}0.45$ and $b = 0.1\text{--}0.36$.

The correlation $Q^{-1} \propto \text{Fr}^{-a}$ can be reduced after remodeling and, with respect to the exponent a , to the following dependences:

$$a = 0.5: n^2 d / g = \text{const}_2 c_T^{2b} \quad (6.5)$$

$$a = 0.4: n d = \text{const}_3 q^{0.2} c_T^b c_T^{2b} \quad (6.6)$$

In the first case, eq. (6.5), the necessary centrifugal acceleration $n^2 d$ only depends on the foam parameters S_i^* and c_T , these being independent of gas throughput $q \triangleq q_F$. These foams can be easily controlled. (With the exception of Mersolat[®]H ($a = 0.40$), the foaming agents investigated in [177] did not comply with this behaviour, here $a = 0.45$)

In the second case, eq. (6.6), which applies to Mersolat[®]H (Fig. 4), the tip speed, $n d$, necessary to control the foam is, to a small degree, also dependent upon q .

Since the exponent, a , of Fr in the process equation (6.4) exhibits a numerical value of $a = 0.45$ for four foaming agents and only $a = 0.40$ for Mersolat[®]H, see [177], the numerical value of the constant at $\text{Fr} = \text{const}$ can be plotted for all five foaming agents as a function of the foaming agent concentration. In this way the mechanical stability of foams produced by individual surfactants can be quantified. (It is shown in [177] that the effect of the type of foaming agent on the foam stability is surprisingly low.)

In order to design and scale-up a foam centrifuge of the type presented, measurements with the material system in question would have to be performed on an appropriate model appliance. In this manner, the design of the full-scale foam centrifuge presents no problem. Apart from that, these tests and their representation in the sense of Fig. 4 give the manufacturers of foaming agents a quantifiable assessment of the mechanical stability of the foams produced.

We learn from this example that, if material properties of the systems under treatment are not known, one is compelled (a) to perform the measurements with the given system in differently sized models – i.e., under a change of scale – and (b) to indicate the chemical or trade name of the product and its concentration instead of its unknown (or unmeasurable) physical properties.

6.2

Scale-up Under Conditions of Partial Similarity

When appropriate material systems are not available for model experiments, accurate simulation of the working conditions of an industrial plant on a laboratory- or bench-scale may not be possible. Under such conditions, experiments on differ-

ently sized equipment are customarily performed before extrapolation of the results to the full-scale operation. Sometimes this expensive and basically unreliable procedure can be replaced by a well-planned experimental strategy. Namely, the process in question can be either divided up into parts which are then investigated separately (Example 9: Drag resistance of a ship's hull after *Froude*) or certain similarity criteria can be deliberately abandoned and then their effect on the entire process checked (Example 41/2: Simultaneous mass and heat transfer in a catalytic fixed bed reactor after *Damköhler*).

Several "rules of thumb" for dimensioning different types of process equipment are, in fact, scale-up rules based unknowingly on partial similarity. These rules include the so-called *mixing power per unit volume*, P/V , widely used for scaling up mixing vessels, and the *superficial gas velocity*, v , which is normally used for scaling up bubble columns and fluidized beds (Example 10).

Example 9: Drag resistance of a ship's hull

This problem represents the dawn of scale-up methodology and is closely linked to the name of *William Froude* (1810 – 1879). We are very much indebted to this brilliant researcher because he treated this significant scale-up problem with a clear physical concept and carefully executed experiments to go with it.

We shall first treat this problem by using dimensional analysis. The drag resistance, F , of a ship's hull of a given geometry (the characteristic length being its length, l , and a given displacement volume, V) depends on the speed of the ship, v , the density, ρ , and kinematic viscosity, ν , of the water and – because of the bow wave formation – also on the acceleration due to gravity, g . The list of relevant quantities is thus:

$$\{F; l, V; \rho, \nu, g\} \quad (6.7)$$

The following – conveniently drawn up – dimensional matrix leads to, after only two linear transformations, the unity matrix (rank $r = 3$) and the residual matrix:

	ρ	l	ν	F	ν	g	V
mass M	1	0	0	1	0	0	0
length L	–3	1	1	1	2	1	3
time T	0	0	–1	–2	–1	–2	0
	core matrix			residual matrix			

	ρ	l	v	F	ν	g	V
M	1	0	0	1	0	0	0
$3M + L + T$	0	1	0	2	1	-1	3
$-T$	0	0	1	2	1	2	0
	unity matrix			residual matrix			

From this, after the well known procedure, the following four pi-numbers result:

$$\Pi_1 \equiv F/(\rho l^2 v^2) \equiv \text{Ne} \quad (\text{Newton number})$$

$$\Pi_2 \equiv \nu/lv \equiv \text{Re}^{-1} \quad (\text{Reynolds number})$$

$$\Pi_3 \equiv gl/v^2 \equiv \text{Fr}^{-1} \quad (\text{Froude number})$$

$$\Pi_4 \equiv V/l^3 \quad - \text{dimensionless displacement volume}$$

The problem is consequently completely defined by the pi-set

$$\{\text{Ne}, \text{Re}, \text{Fr}, V/l^3\} \quad (6.8)$$

The model theory requires that in the scale-up from model- to the full-scale not only the geometric similarity ($V/l^3 = \text{idem}$) is maintained, but also that all the other pi-numbers describing the problem keep the same numerical values ($\text{pi} = \text{idem}$). This implies that, for example, in the measurements with the boat and ship models both process numbers

$$\text{Fr} \equiv v^2/(l g) \quad \text{and} \quad \text{Re} \equiv \nu l/v \quad (6.9)$$

should be idem. However, this demand cannot be fulfilled.

Because the acceleration due to gravity cannot be varied on Earth, $\text{Fr} = \text{idem}$ is adjusted in model experiments by the model speed v_M . Thereafter, $\text{Re} = \text{idem}$ has to be met by the adjustment of the model fluid viscosity. Supposing that the size of the model is only 10 % of the full-scale size (scale-up factor $\mu = I_T/I_M = 10$), then the requirement $\text{Fr} = \text{idem}$ results in the speed of the model $v_M = 0.32 v_T$. From this, the kinematic viscosity, ν_M , of the model fluid can be calculated:

$$\nu_M/\nu_T = (v_M/v_T) (I_M/I_T) = 0.32 \times 0.1 = 0.032 \quad (6.10)$$

However, there is no fluid with a viscosity which is only 3 % that of water. (At $\mu = 100$ it follows that $\nu_M/\nu_T = 1 \times 10^{-3}$, i.e. the viscosity of the model fluid may be only 1×10^{-3} of the water value.)

If the scale-up factor did not have to be that small and models were not so expensive to build, experiments could be run with models of various sizes in water at the same value of Fr and then the results could be extrapolated to Ne_T at Re_T . In view of the powerful model reduction and the resulting extreme differences in the Reynolds number

$$\mu = 1; 10; 100 \rightarrow Re_M/Re_T = 1; 3.2 \times 10^{-2}; 1 \times 10^{-3} \quad (6.11)$$

extrapolation appears to be a risky undertaking, particularly when the cost of the motor used in the full-scale application is considered.

Naturally, the results of dimensional analysis discussed above and their consequences were not known to the ship builders of the 19th century. Since the time of *Rankine*, the total drag resistance of a ship has been divided into three parts: the surface friction, the stern vortex and the bow wave. However, the concept of *Newtonian* mechanical similarity, known at that time, only stated that for mechanically similar processes the forces vary as $F \propto \rho l^2 v^2$. Scale-up was not considered in assessing the effect of gravity.

Froude observed that the resistance due to the stern vortex was relatively small compared with the other two sources of resistance. Therefore, he decided to combine it with the bow wave resistance to obtain the *form drag*, F_f . As a result of careful investigations and theoretical considerations, he realized that the wave formation of the ship could be simulated by using scale models. He arrived at the law of appropriate velocities: "The wave formations at the ship and the model are (geometrically) similar, if the velocities are in the ratio of the square root of the linear dimensions". He also found that for similar wave formations, the hull drag (*friction drag* F_r) behaves not as $F_r \propto \rho v^2 l^2$, but as $F_r \propto v^{1.825} A \rho$ (A is the surface area). Consequently, he developed computational methods for scaling down models and ships by length and the type of the wetted surface. In this way, he was in the position of being able to calculate the form drag, F_f , from the total drag after subtracting the predictable friction drag. He found: "If we adhere to the law of the appropriate velocities in scaling-up the ship, the form resistances will correspond to the cubes of their dimensions (that is to say, their displacement volumes)" [84]^{*)}

In summary:

1. $F_{\text{total}} - F_r = F_f$
 2. If $v^2 \propto l$, then $F_f \propto l^3$
- (6.12)

Dividing this functional dependence (6.12) by $\rho l^2 v^2$, in order to transform F_f into the Newton number of form drag, leads to the following proportionality

*) The reference [84] includes the minutes of the session of The Institution of Naval Architects in London of April 7, 1870. During this session, W. Froude presented and

defended the results of his modeling with great steadfastness and conviction; this represents the sidereal hour of the theory of models.

$$\frac{F_f}{\rho l^2 v^2} \propto \frac{l^3}{\rho l^2 v^2} = \frac{l}{\rho v^2} \quad (6.13)$$

This means:

$$Ne_f = \text{idem} \quad \text{at} \quad Fr = \text{idem} \quad \text{with} \quad Fr \equiv v^2/lg$$

In order to verify these experimental results, the corvette “Greyhound” was towed by the corvette “Active” under the command of *Froude*, and the drag force in the tow rope was measured. *Froude* reported [32] that the observed deviations from the predictions of the model were in the range of only 7 – 10%.

M. Weber [152] pointed out that *Froude*’s procedure was not entirely correct and could never provide real proof, because complete similarity between the model and its full-scale counterpart could not be achieved. The described procedure can therefore represent nothing more than an excellent approximation of reality. He continued by saying: “The fact that *Froude* was able to achieve his goal with such a large measure of success, despite all the difficulties, lies in his ingenuity which enabled him to itemize and to assess all the practical and theoretical details of drag resistance and finally to trace a clear picture of this intricate phenomenon.” Nothing can be added to this assessment: *Froude*’s work represents a prime example of partial similarity. His performance can certainly be admired, especially in view of the measuring techniques available to him at that time.

J. Pawłowski [102] discussed an interesting alternative experimental approach to this scale-up problem. He also started by splitting the drag resistance into friction, depending only on Re , and a bow wave resistance, depending only on Fr :

$$Ne_T = f_1(Re_T) + f_2(Fr_T) \quad (6.14)$$

However, he proposed a different strategy from that of *Froude*. In the *first* experiment, measurements were made with the model ship in water at $Fr_1 = Fr_T$, consequently $Re_1 = Re_T \mu^{-3/2}$, i.e., the measurement was carried out at a correct Fr value and a false Re value. As a result, a false value of Ne_1 was also obtained from the relationship:

$$Ne_1 = f_1(Re_1) + f_2(Fr_T)$$

Two *additional* experiments were carried out, not with the model ship, but with a totally immersed form (**Fig. 5**) whose shape was given by reflecting the immersed portion of a ship’s hull at the water line (at $V/l^3 = \text{idem}$). In these experiments, the Froude number is irrelevant; the friction corresponding to the surface area of the model must be divided by 2.

The measurements in water were carried out at Re_1 and Re_T , thereby obtaining

$$Ne_2 = f_1(Re_T) \quad \text{and} \quad Ne_3 = f_1(Re_1)$$

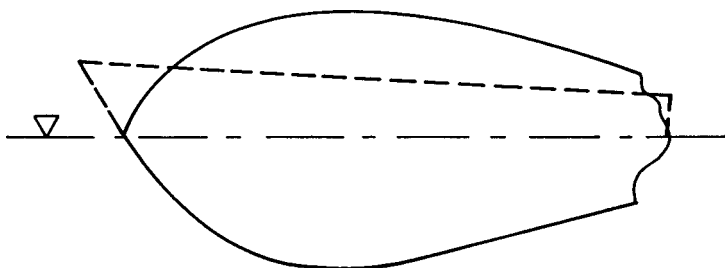


Fig. 5 Sketch of the completely submersed streamlined body.

The desired Ne_T could then be calculated:

$$Ne_T = f_1(Re_T) + f_2(Fr_T) = Ne_1 - Ne_3 + Ne_2$$

The above alternatives for the determination of a ship's hull resistance illustrate an application of the method of partial similarity in which the process is divided into parts, thus allowing them to be investigated independently.

Example 10: Rules of thumb for scaling up chemical reactors: Volume-related mixing power and the superficial velocity as design criteria for mixing vessels and bubble columns

In the introduction to this chapter it has already been pointed out that, for the design of chemical process equipment, “rules of thumb” exist. Upon closer examination, these rules provide conditions which subconsciously accept partial similarity. Actually, one cannot expect that complicated processes of fluid dynamics occurring during mass and heat transfer can be adequately described by criteria such as power per unit volume, P/V , for mixing vessels and superficial gas velocity, $v \equiv q/S$, for bubble columns.

Each unit operation in process engineering obeys specific laws which demand a separate pi-space. It cannot be expected that different processes would be depicted by the same pi-space.

Stirring vessels. Upon the examination of different stirring operations it was indeed found that the *intensively* formulated process parameter P/V represented the pertinent scale-up criterion only if the stirring power has to be dissipated in the volume as evenly as possible (micro-mixing, isotropic turbulence). Examples of this are the dispersion of a gas in a liquid or the dispersion of immiscible liquids; [184].

In the most important stirring operation – the homogenization of liquid mixtures – the convective transport of liquid balls (macro-mixing) is of predominant importance. Thus, this process depends to a large degree on space geometry and

type of stirrer. It is influenced by the *extensive* parameters such as stirrer speed, n , and stirrer diameter, d . Here, the similarity with respect to fluid dynamics is given by $Re \equiv n d^2 \rho / \mu = \text{idem}$.

Convective bulk transport is also an extremely important factor in the suspension of solids in a stirred tank (this is also responsible for the flow pattern at the tank bottom). P/V cannot be used as a scale-up criterion in this process either. Measurements have shown that the minimum rotational speed, n_{crit} , of the stirrer which is necessary for the suspension (whirling-up) of particles in the turbulent regime is given by the appropriate Froude number:

$$Fr_{\text{crit}} \equiv n_{\text{crit}}^2 d \rho / (g \Delta \rho) \quad (6.18)$$

What is the consequence of the “dimensioning criterion” P/V if the correct scale-up criterion is represented by $Fr = \text{idem}$?

Because the Froude number is the scale-up criterion here, we formulate it for a given material system by P/V as follows:

$$Fr \propto n^2 d = \text{idem} \quad (6.19)$$

$$P/V \propto n^3 d^2 \quad (P \propto n^3 d^5 \text{ in the turbulent flow range; } V \propto d^3) \quad (6.20)$$

$Fr \propto n^2 d = \text{idem}$ means that any power of Fr is equally idem :

$$Fr^{3/2} = (n^2 d)^{3/2} = (P/V) d^{-1/2} = \text{idem} \quad (6.21)$$

The answer to the question of the relationship between P/V and Fr , at $Fr = \text{idem}$, reads:

$$[(P/V) d^{-1/2}]_T = [(P/V) d^{-1/2}]_M$$

$$Fr = \text{idem} \rightarrow (P/V)_T = (P/V)_M \mu^{1/2} \quad \mu \equiv d_T/d_M \quad (6.22)$$

It should be pointed out that the scale-up rule $Fr = \text{idem}$ results in costly consequences: The power input per unit volume increases by the square root of the scale-up factor μ .

Bubble columns are often designed on the basis of the superficial gas velocity $v \equiv q/S$ (q – gas throughput, S – cross-sectional area of the column). Many authors have found that the gas/liquid mass transfer in bubble columns is indeed governed by this quantity (k_L – liquid side mass transfer coefficient; a – interfacial area per unit volume):

$$k_L a \propto v \rightarrow k_L a/v = \text{const.} \quad (6.23)$$

This interdependence is only understandable when one considers that the volume-related mass transfer coefficient is defined by $k_L a = G/(V\Delta c)$ and the superficial velocity, v , by $v = q/S$, as well the fact that volume $V = H \times S$ (H – height of the column):

$$\frac{k_L a}{v} = \frac{G}{H S \Delta c} \frac{S}{q} = \frac{G}{H q \Delta c} = \text{const.}$$

In words: The gas absorption rate $G[\text{MT}^{-1}]$ is directly proportional to the liquid height, H , the gas throughput, q , and the concentration difference, Δc [184]. (This is only valid under the condition that the gas is not completely absorbed: $\Delta c > 0$.)

In contrast with this – and in analogy to the corresponding findings in the mixing vessel – the mixing (back-mixing) in bubble columns cannot be described by the intensively formulated quantity v . For this scale-up task, the Froude number is effective here.

Experiments [165] performed in bubble columns of different sizes gave the following expression for the mixing time, θ :

$$\theta (g/D)^{1/2} \propto Fr^{-1/4} \quad (6.25)$$

$Fr \equiv v^2/D g$; D – diameter of the bubble column

It follows that

$$\theta \propto v^{-1/2} D^{3/4} \quad \text{resp.} \quad \theta = \text{idem} \rightarrow v^{-1/2} D^{3/4} = \text{idem}$$

This leads to the conclusion that

$$v_T = v_M \mu^{1.5} \quad (6.26)$$

Thus, in a bubble column which has been geometrically scaled up by a factor of $\mu = 10$, the same mixing time as in the model will be only obtained when the superficial velocity is increased by a factor of $10^{1.5} = 32$. Hence, v is not a scale-up criterion here.

- The correlations presented in Example 10 show emphatically that a particular scale-up criterion, that is valid in a given type of apparatus for a particular process, is not necessarily applicable to other processes occurring in the same device.

Finally, it must be noted that a series of processes exist in which the scale-up has to be performed under the conditions of partial similarity. All processes occurring on the surface, e.g., chlorine-alkali-electrolysis, catalytical off-gas treatment, surface fermentation as well as surface aeration in biological waste-water treatment, etc., belong to this group. If the process proceeds solely on the surface, then only the surface has to be scaled-up. This means that the scale-up has to be performed only by partial similarity as far as geometry is concerned.

7

Preliminary Summary of the Scale-up Essentials

7.1

The Advantages of Using Dimensional Analysis

The advantages made available by correct and timely use of dimensional analysis are as follows:

1. Reduction in the number of parameters necessary to define the problem. The pi-theorem states that a physical relationship between n physical quantities can be reduced to a relationship between $m = n - r$ mutually independent pi-numbers. Herein, r represents the rank of the dimensional matrix which is formed by the physical quantities in question and corresponds, in most cases, to the number of base dimensions contained in their dimensions.
2. Reliable scaling-up of the desired operating conditions from the model to the full-scale plant. This is based on the scale invariance of the pi-space. According to the model theory, two processes may be considered to be similar if they take place under geometrically similar conditions and all dimensionless numbers which describe the process have the same numerical value.
3. A deeper insight into the physical nature of the process. By representing experimental data in a dimensionless form, physical states (e.g., turbulent or laminar flow range, suspension state, heat transfer by natural or by forced convection, and so on) can be delimited from each other and the limits quantified. In this manner, the domain of individual physical quantities also becomes apparent.
4. A greater flexibility in the choice of parameters and their reliable extrapolation within the range covered by the dimensionless numbers. These advantages become clear if one considers the well-known Reynolds number, $Re = v l \rho / \mu$, which can be varied by altering the characteristic velocity, v , characteristic length, l , or the kinematic viscosity, $\nu = \mu / \rho$. By choosing appropriate model fluids, the viscosity can be very

easily altered by several orders of magnitude. Once the effect of the Reynolds number is known, extrapolation of both v and l is permitted within the examined range of Re .

7.2

Scope of Applicability of Dimensional Analysis

In order to describe a physico-technological problem by a complete set of pi-numbers, initially all essential (“relevant”) physical quantities which describe the problem must be known. A prerequisite of this requirement is a thorough and critical knowledge of the process in question.

In fact, the applicability of dimensional analysis depends heavily on the knowledge available. The following five steps can be outlined, see also **Fig. 6**.

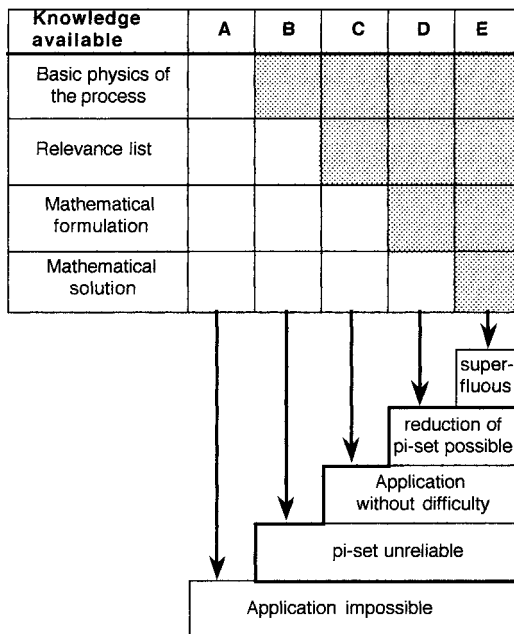


Fig. 6 Applicability of dimensional analysis, as dependent on the knowledge available; after J. Pawłowski [106].

In words:

1. The physics of the basic phenomenon is unknown.

Result: Dimensional analysis cannot be applied.

2. Enough is known about the physics of the basic phenomenon to compile a first, tentative relevance list.
Result The resultant π set is unreliable.¹⁾
3. All the relevant physical variables describing the problem are known.
Result: The application of dimensional analysis is unproblematic.
4. The problem can be expressed in terms of a mathematical equation.
Result: A closer insight into the π relationship is feasible and may facilitate a reduction of the set of dimensionless numbers.²⁾
5. A mathematical solution of the problem exists.
Result: The application of dimensional analysis is superfluous.

7.3

Experimental Techniques for Scale-up

In the Introduction, a number of questions were posed which are often asked in connection with model experiments. We are now in a position to answer them.

- *How small can a model be?* The size of a model depends on the scale factor $\mu \equiv l_T/l_M$ and on the experimental precision of the measurement. Where $\mu = 10$, a $\pm 10\%$ margin of error may already be excessive. A larger scale for the model will therefore have to be chosen to reduce the scale-up factor μ .
 - *Is one model scale sufficient or should tests be carried out with models of different sizes?* One model scale is sufficient if the relevant numerical values of the dimensionless numbers necessary to describe the problem (the so-called “process point” in the π -space describing the operational condition of the technical plant) can be adjusted by choosing the appropriate process parameters or physical properties of the model material system. If this is not possible, the process characteristics must be determined in models of different sizes, or the process point must be extrapolated from experiments in technical plants of different sizes.
-
- | | |
|---|--|
| <p>1) It must, of course, be said that approaching a problem from the point of view of dimensional analysis is still useful even if all the variables relevant to the problem are not yet known. The timely application of dimensional analysis may often lead to the discovery of forgotten variables or the exclusion of artefacts.</p> | <p>2) In principle, in constructing a relevance list, one should take into consideration all the available information to possibly reduce it. In this context, reference is made to Example 34 (Description of particle separation by inertial forces after Bürkholz) and to Example 44.2 (Catalytic gas reactions in fixed beds after Damköhler).</p> |
|---|--|

- *When must model experiments be carried out exclusively with the original material system?* When the material model system is unavailable (e.g., in the case of non-Newtonian fluids) or when the relevant physical properties are unknown (e.g., foams, sludges, slimes) the model experiments must be carried out with the original material system. In this case measurements must be performed in models of various sizes (cf. Example 8).

The unavailability of the model material systems can sometimes limit the application of dimensional analysis. In such cases it is of course absolutely wrong to speak of “*limits of the dimensional analysis*”.

7.4

Carrying out Experiments Under Changes of Scale

As a rule, model experiments are carried out in the laboratory in one and the same apparatus. There are certain reasons why this usage is often improper; here they are:

- a) If you assume that the process under consideration essentially depends on the acceleration due to gravity, g – which cannot be changed on the Earth – the experiments have to be performed under a change of scale. Examples concerning this are, e.g., the bow resistance of a ship (Example 9), scale-up rules for self-aspirating hollow stirrers (Example 21), for bubble columns (Example 33) and also for surface aerators for waste-water treatment (Example 38).
- b) If physical properties cannot be varied, one is compelled to perform the experiments under a change of scale in order to vary the operational (process) number. An example concerning this is given by the elaboration of the scale-up rules for a mechanical foam breaker (Example 8).
- c) If laboratory measurements are performed in a single vessel ($d = \text{const}$) filled with water or a liquid of similar viscosity ($\nu \approx \text{const}$), you cannot differentiate whether the Reynolds or the Froude number represents the process number. In mixing experiments in which d , ν and g remained practically constant, it follows that:

$$\text{Re} \equiv n d^2 / \nu \rightarrow \text{const} \times n$$

$$\text{Fr} \equiv n^2 d / g \rightarrow \text{const} \times n^2$$

To find out whether the process depends on Re or Fr numbers or on both of them, you must perform the experiments

in differently scaled models. (The work of *Hixon & Baum* [52], which has had a big impact on the development of the technology of suspension of solids in liquids by mixing, has been interpreted in the wrong way: In suspending solid particles in watery liquids it is the Froude number and not the Reynolds number which counts.)

- d) In mass transfer in the G/L system in volume aeration, the target number $k_L a$ is an intensively (volume-related) formulated quantity. This implies that process parameters (mixing power P and the gas throughput q) also have to be formulated as intensive quantities. The question arises of whether the gas throughput q has to be related to the liquid volume V as q/V , or to the cross-sectional area S of the vessel as the superficial velocity $v \propto q/S$. To find this out, laboratory measurements had to be performed under change of scale! (It was found that the second possibility is correct, see Example 39.)

8

Treatment of Physical Properties by Dimensional Analysis

8.1

Why is this Consideration Important?

A complete similarity between the model and the technical realization implies geometric, material and process-related similarity. The geometric similarity can be realized without problems in most cases. The same is true for the process-related similarity, as can be seen in the examples in the second half of this book. Material similarity can cause difficulties if different materials must be used in model experiments from the ones used in the technical plant.

The scale invariance of the pi-space (Chapter 4) allows us to work with different material systems in the model compared to the industrial scale. This presents one of the essential advantages, offered by the Theory of Models. Many of the conditions of flow in the technical plant can only be scaled down to the model scale when an appropriate model material system has been chosen.

Example a: The Reynolds number for a mixing vessel is given by $Re \equiv n d^2 / \nu$. Due to the fact that the scale of length (stirrer diameter d) enters Re with its square, in an essential scale-down it will also be necessary to diminish the kinematic viscosity of the liquid essentially in order to work with a reasonable stirrer speed.

The process point of the technical plant is given by, e.g.:

$$d = 1 \text{ m}; n = 1 \text{ s}^{-1} (= 60 \text{ min}^{-1}); \nu = 1 \times 10^{-4} \rightarrow Re = 10^4.$$

In a laboratory device which is scaled down by the scale factor of $\mu = 10 : 1$ and which applies the same fluid ($\nu = 1 \times 10^{-4}$), the stirrer speed must amount to $n = 100 \text{ s}^{-1}$ ($= 6000 \text{ min}^{-1}$) in order to sustain the same flow condition ($Re = 10^4$). If on the other hand the measurements are performed with water ($\nu = 1 \times 10^{-6}$) the experiment can be carried out with a reasonable stirrer speed of $n = 1 \text{ s}^{-1}$ ($= 60 \text{ min}^{-1}$).

In this example it has been assumed that the liquid is a Newtonian one and the temperature is constant. So the physical properties of the fluid (viscosity μ and density ρ) remained constant.

There are, however, processes in which the physical properties cannot be expected to be constant. For example, a temperature field in a material system under consideration will often produce a viscosity and possibly also a density field. In the case of non-Newtonian (pseudoplastic or/and viscoelastic) fluids, a viscosity field will also be caused by the shear stress.

Example b: In producing tin soldiers, tin melt is pressed into appropriate moulds far above its melting point of $T \approx 232^\circ\text{C}$ and cooled down. To check the flow behavior (dead space) in such moulds, a model is first produced out of transparent plastics and as a model fluid, water at room temperature is used. Now the question obviously occurs of whether or not the model fluid (here: water) behaves with respect to the temperature dependence of its material parameters (viscosity and density) in a similar way to the tin melt.

Example c: In order to dimension petrochemical plants (Example 44) so-called “cold models” of the size of about 1/10 of the technical plant are used. Hydrocracking, thermocracking, etc., will be performed at temperatures of $270 - 500^\circ\text{C}$ and pressures of $50 - 200$ bar. The question is, which model fluid may be used here to investigate the hydrodynamics in a model plant at room temperature, in order to assure that this fluid behaves in a similar way to the petrol fraction with respect to the temperature dependence of their material parameters?

Example d: May breakers and grinding mills be dimensioned for grinding quartz on the basis of measurements made with limestone? (Example 13).

Example e: May absorption measurements be performed with a pure water/air system to obtain dimensioning rules for biological waste-water treatment plants? Is pure water similar to the civil or industrial waste water with respect to the absorption process? (This question will be treated separately in Section 13.3: Coalescence phenomena in the G/L system.)

- Similarity in physical properties between different materials can only be recognized if the variability of each of them is presented as a standard representation of the material function in a dimensionless form.
- Analogous processes, in which different materials have different variabilities in their physical properties, are only similar to each other (are presented by the same π relationship) if the respective material functions can be presented by the same dimensionless standard representation. These facts are particularly important in the choice of materials for model experiments.

8.2

Dimensionless Representation of a Material Function

The dimensional-analytical treatment of variable material properties will be presented in the following for different examples, e.g., for the standard representation of the temperature dependence of the viscosity and density of Newtonian fluids; for the dependence of the particle strength of solids on particle diameter and also the dependence of the diffusion coefficient in plastics on the moisture content and the temperature. The same procedure is obvious for every changeable material parameter (e.g., surface tension) and every influencing parameter (e.g., pressure, concentration, etc.).

Example 11: Standard representation of the temperature dependence of the viscosity

Fig. 7 presents the dependence of the dynamic viscosity μ on the temperature for eight different fluids, which differ in viscosity at room temperature for five decades. Five of them show a relatively small temperature dependence on the viscosity (water, and four Baysilon, which are silicon oils of the Bayer AG), while the remaining three (glycerine, steam engine cylinder oil and molasses) show a pronounced one.

From the representation $\mu(T)$ a similarity between the eight fluids cannot be concluded. To decide this, the dimensional space $\mu(T)$ must be transformed into a dimensionless one. This is possible by a standardization (normalization) of this dependence.

For this purpose we choose a reference temperature T_0 (preferably the middle temperature of the measuring range), to which a reference viscosity μ_0 is related. Further on we determine at the point T_0 the gradient $\Delta\mu/\Delta T$, with which the temperature coefficient of the viscosity

$$\gamma_0 \equiv \left(\frac{1}{\mu} \frac{\partial \mu}{\partial T} \right)_{T_0} < 0 \quad (8.1)$$

is calculated.

By doing this, the dimensional space $\mu(T)$ is widened by three parameters. Now the relevance list

$$\{\mu, \mu_0, T, T_0, \gamma_0\} \quad (8.2)$$

delivers three dimensionless numbers without any problem

$$\{\mu/\mu_0, T/T_0, \gamma_0 \Delta T\} \quad (8.3)$$

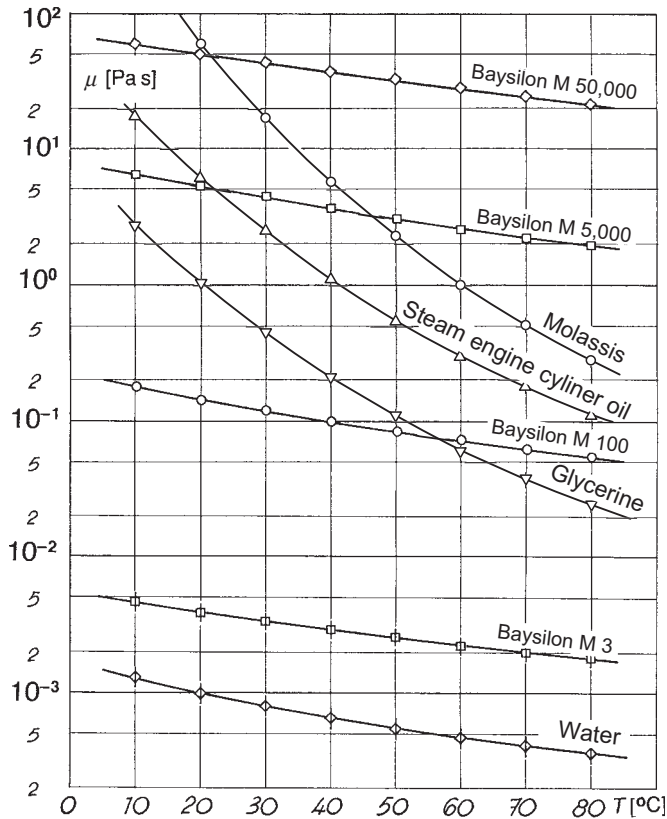


Fig. 7 The dependence $\mu(T)$ for eight different fluids, which differ markedly in their temperature coefficient of the viscosity; taken from [166].

Fig. 8 presents the measured values from Fig. 7 in the dimensionless space presented above.

The following facts can be realized:

- (1) With respect to the temperature dependence $\mu(T)$ all investigated fluids behave in a similar way to each other; they correspond to the same standard representation $\mu/\mu_0 = f(\gamma_0 \Delta T)$.
- (2) The standard representation proves to be invariant to the reference temperature T_0 : The parameter T/T_0 has no influence.
- (3) The fitting curve corresponds to the reference-invariant χ function after Pawlowski; see section 8.3. Water values are described with $\mu = -1.2$ and a logarithmic variance of 5.25×10^{-3} . For all remaining materials $\mu = -0.167$ and the log variance amounts to 1.51×10^{-2} .

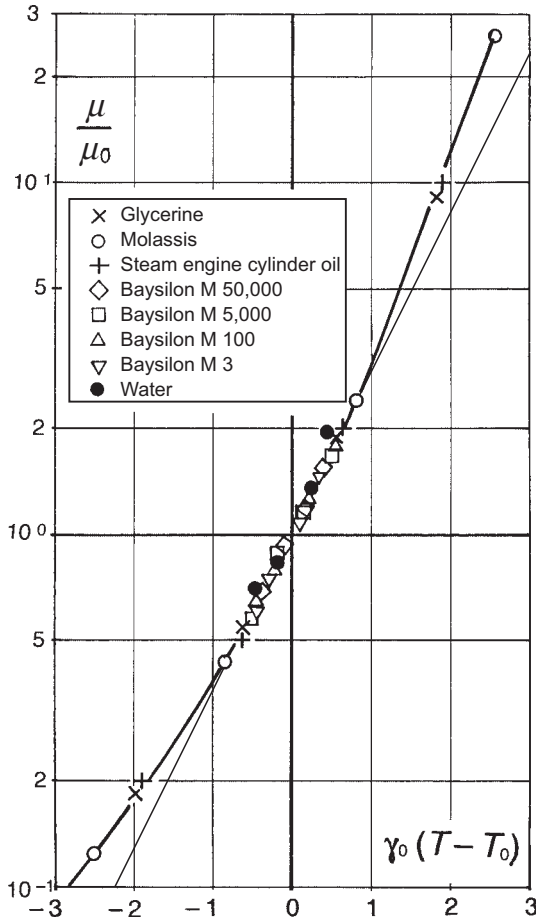


Fig. 8 Standard representation $\mu/\mu_0 = f(\gamma_0 \Delta T)$ for fluids from Fig. 7. Reference temperature $T_0 = 50^\circ\text{C}$. The fitting curve corresponds to the reference-invariant χ -function (see details in Section 8.3).

- (4) Water is obviously special. Its μ/μ_0 values coincide with the other values only in the range near to the reference point (i.e., at $\gamma_0 \Delta T \approx 0 \pm 1$). Outside this range they deviate more strongly.
- (5) The thinly plotted straight line is a very good approximation of this dependence in the range near the reference point (at $\gamma_0 \Delta T \approx 0 \pm 1$). It represents the engineering representation of the temperature dependence of the viscosity of a liquid

$$\ln(\mu/\mu_0) = \exp(\gamma_0 \Delta T). \quad (8.4)$$

Due to the equivalence between μ/μ_0 and $\gamma_0\Delta T$ the quotient μ/μ_0 is usually used in the engineering literature as the viscosity number $\text{Vis} \equiv \mu_w/\mu$ (μ_w – wall viscosity, μ – bulk viscosity); see Example 15.

To answer the question about the material similarity regarding $\mu(T)$ between water and tin melt, which was posed in section 8.1 under Example b; we should refer to the measurements of *Pawłowski* [109] (included also in [187]). They prove that both materials behave in a similar way regarding $\mu(T)$.

The dependence $\mu(T)$ can also be expressed in the form of the so-called Arrhenius relationship [109; 187]. (This kind of presentation was suggested by *Svante Arrhenius* (1889) to describe the temperature dependence of the reaction rate constant $k(T) = k_\infty \exp(E/RT)$.)

From

$$\frac{E_0}{R} \equiv \left(\frac{1}{\mu_0} \frac{\partial \mu}{\partial (1/T)} \right)_{T_0} = \left(\frac{\partial \ln \mu}{\partial (1/T)} \right)_{T_0} \quad (8.5)$$

it follows that

$$\mu/\mu_0 = \Phi_1 \left[\frac{E_0}{RT_0} \right] (T_0/T - 1) = \Phi_1 [\text{Arr}_0 (T_0/T - 1)], \quad (8.6)$$

whereas E_0 is the activation energy of the process and R is the universal gas constant.

Between the Arrhenius number Arr and $\gamma_0 T$ the following connection exists

$$\text{Arr} \equiv E/RT = -\gamma_0 T \quad (8.7)$$

In the first edition of this book [187] much space was devoted to performing model experiments with Newtonian fluids with temperature-dependent viscosity (see Chapter 8.5 of that edition). Therefore only a reference to the corresponding original works of *J. Pawłowski* [105, 109] is given here.

Finally, it should be pointed out that two different paths can be taken to describe the dependence $\mu(T)$ in a dimensionless form. The first method is to choose two reference quantities with dimension μ and T , respectively, contained in the material function itself. This has been accomplished here by the addition of μ_0 and γ_0 and E/R , respectively. This material-related representation has a greater importance in the dimensional-analytical treatment of processes than the second one, because its definition is based solely on material function.

In the second method, quantities with the dimension μ and T , respectively, are formed from appropriate influencing parameters of the relevance list of the process in question. In this case we speak of a process-related representation of a material function.

Example 12: Standard representation of the temperature dependence of density

The temperature dependence of the density $\rho(T)$ is expressed in an analogous way by introducing the temperature coefficient β_0 of the density:

$$\beta_0 \equiv \left(\frac{1}{\rho_0} \frac{\partial \rho}{\partial T} \right)_{T_0} \quad (8.8)$$

The standard representation of the material function $\rho(T)$ is produced in an analogous way to $\mu(T)$:

$$\rho/\rho_0 = f(\beta_0 \Delta T) \quad (8.9)$$

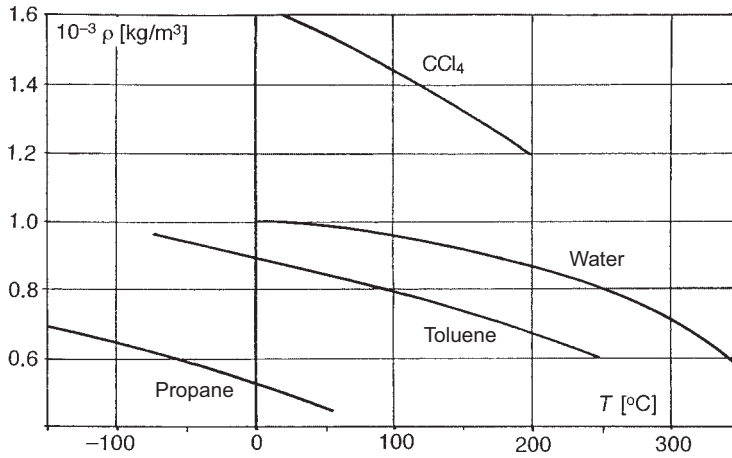


Fig. 9 $\rho(T)$ dependence for four liquids.

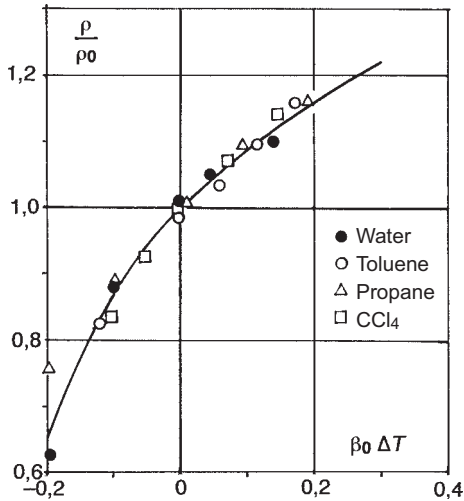


Fig. 10 Standard representation of $\rho/\rho_0 = f(\beta_0 \Delta T)$ for the liquids from Fig. 9.

Fig. 9 shows the dependence $\rho(T)$ for four fluids. Contrary to the viscosity, the density changes only slightly with the temperature.

Fig. 10 shows the standard representation of the facts discussed above. Here, too, the fitting curve represents the reference-invariant χ function; it was produced using $\mu = 3.94$ and it describes the measuring points with a variance of 4.69×10^{-2} . The $\rho(T)$ dependencies for propane, toluene and CCl_4 fall together. Their similarity in $\rho(T)$ is clearly seen (it was the aim of this presentation to prove this!) These three materials can be mutually used as model fluids in the whole recorded T range.

This does not apply to water. Both outer points for water indicate that at a larger distance from the reference point it will deviate from the $\rho(T)$ course of organic liquids.

For petrochemical investigations in the so-called “cold model” (Section 8.1, Example c) water is not suitable; see Example 46.

Example 13: Standard representation of the particle strength for different materials in dependence on the particle diameter

The particle strength $\sigma \equiv F/A$ of solids plays an important role in comminution. It is defined as the force F which is exerted at the breaking point, whereas the nom-

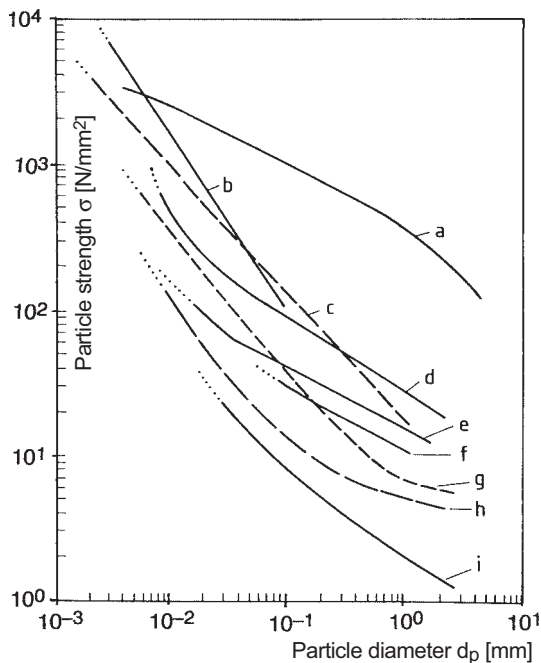


Fig. 11 Particle strength σ of different materials in dependence on the particle diameter d_p , from [49], a – Glass beads; b – Boron carbide B_4C ; c – crystallized Boron; d – cement clinker; e – marble; f – cane sugar; g – quartz; h – limestone; i – coal.

inal cross-sectional area $A = (\pi/4) d_v^2$ is related to the diameter, d_v , of a sphere of equal volume. It depends strongly on the particle diameter because, due to the progression of comminution, the defects in the solid decrease and thus it becomes more homogeneous and therefore stronger.

Fig. 11 shows the connection between the particle strength σ and the particle diameter d_p for a series of solids. The diversities in the dependencies are pronounced in such a way that one cannot conclude that there is any similarity with regard to particle strength between different solids. On the basis of this fact, e.g., you cannot decide whether or not the measurements of the comminution of limestone are suitable for dimensioning a grinder for quartz.

Fig. 12 shows the standard representation of this fact in the pi-space:

$$\sigma/\sigma_0 = \Phi \{-\phi (d_p - d_{p,0})\}, \quad (8.10)$$

where the coefficient ϕ of the particle strength has been obtained in a similar way to that in which the coefficients γ_0 and β_0 , were gained before:

$$\phi \equiv \left(\frac{1}{\sigma_0} \frac{\partial \sigma}{\partial d_p} \right)_{d_{p,0}} \quad (8.11)$$

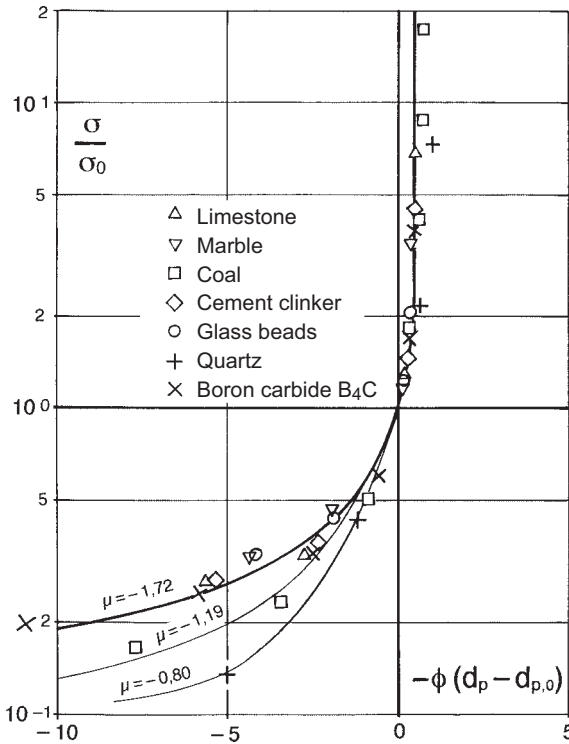


Fig. 12 Standard representation of particle strength σ of different materials in dependence on the particle diameter d_p . As reference parameter, without exception, $d_{p,0} = 0.1$ mm was applied from [189].

The curves in Fig. 12 represent the reference-invariant approximations (see Section 8.3) of individual point collectives. The thickly drawn curve, obtained by $\mu = -1.72$, describes most of the examined materials with a relative variance of 3.13×10^{-2} ; these are similar to each other. The thinly drawn curves apply to quartz ($\mu = -1.19$) and boron carbide ($\mu = -0.80$). They deviate more strongly than the other solids in respect of the particle strength in the range $\sigma/\sigma_0 < 1$.

It must be left to the experts to decide the importance that these results have on their experiments.

The third number, resulting from the relevance list of five parameters

$$\{\sigma, \sigma_0, d_p, d_{p,0}, \phi\} \quad (8.12)$$

– namely $\phi \times d_{p,0}$ – is obviously irrelevant, as the measuring data can be described by the reference-invariant approximations.

Example 14: Drying a wet polymeric mass. Reference-invariant representation of the material function $D(T, F)$

For further processing, the polymeric mass it must be thoroughly free of monomers. To achieve this, it is first shredded and submitted to water steam distillation. Then the particles are drained, dried, melted and further processed. In the case of Bayer-polyamide Dralon® (earlier: Perlon®) they are spun to acryl fibers.

For the pre-calculation of the drying course of Dralon shreds, the knowledge of the dependence of the diffusion coefficient D on the temperature T and the moisture content F is indispensable. Fig. 13 shows, on the one hand, the measured course of drying F/F_0 (●) and on the other hand, the pre-calculated ones. The pre-calculation was carried out for a sheet of Dralon of 0.9 mm thickness and of infinite extension, whereby constant diffusion coefficients were used which differed from one another by a factor of 20. The measured course of drying can only be

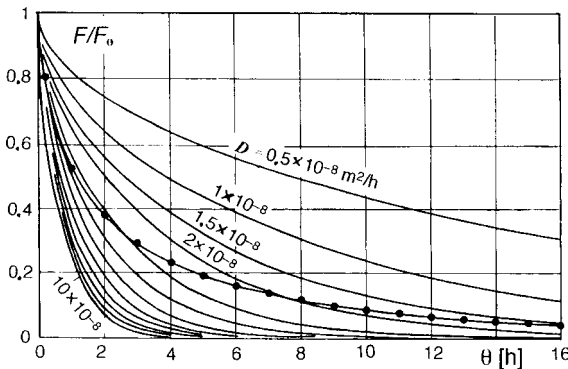


Fig. 13 Comparison of the measured course of drying (●) with the calculated one under the assumption of different, but constant diffusion coefficients; from [81].

approximated (fitting curve $\bullet\text{---}$), if the dependence $D(F, T)$ has been taken into account.

Fig. 14 shows the dependence $D(F, T)$ of the system Dralon/water. The standard representation of this is shown in **Fig. 15**:

$$\frac{D}{D_{F=0}} = \Phi(\xi F) \quad (8.13)$$

The coefficient ξ represents the dependence of the diffusion coefficient on the degree of moisture:

$$\xi \equiv \left(\frac{1}{D_0} \frac{\partial D}{\partial F} \right)_{F=0} \quad (8.14)$$

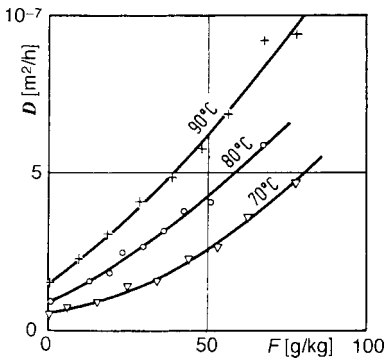


Fig. 14 Correlation $D(F)$ in Dralon® at three different temperatures [81].

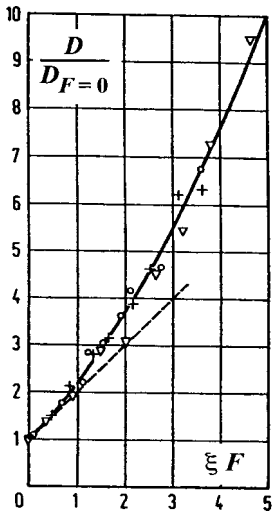


Fig. 15 Standard representation of the dependence $D(F, T)$ from Fig.14. The fitting curve is the reference invariant approximation with $\mu = 0.61$; from [103].

8.3

Reference-invariant Representation of a Material Function

In the standard representations of the facts in Examples 11–14 it has always been pointed out that the fitting curve corresponds to a reference-invariant function $\chi(u, \mu)$ and the corresponding numerical value of μ was indicated. In this section the necessary details will be explained.

In general, the standard representation depends upon the choice of the reference point. The question is posed: Do mathematical functions exist whose standard representations do not depend on the choice of the reference point and which could therefore be named “reference-invariant functions”? In the case of an affirmative answer on the one hand the reference point, e.g., T_0 , could be omitted (constriction of the pi-space by one pi-number) and, on the other hand, the dimensionless representation of the material function would stretch over the entire recorded range.

In [109] it is mathematically proven that *only one class* of reference-invariant standard representations exists and that this can be represented by one-parametric $\chi(u, \mu)$ functions:

$$\begin{aligned} w &= \chi(u, \mu) \equiv (1 + \mu u)^{1/\mu} & \text{for } \mu \neq 0 \\ w &= \chi(u, \mu) \equiv \exp(u) & \text{for } \mu = 0 \end{aligned} \quad (8.15)$$

μ is a numerical value which can be determined numerically or graphically.

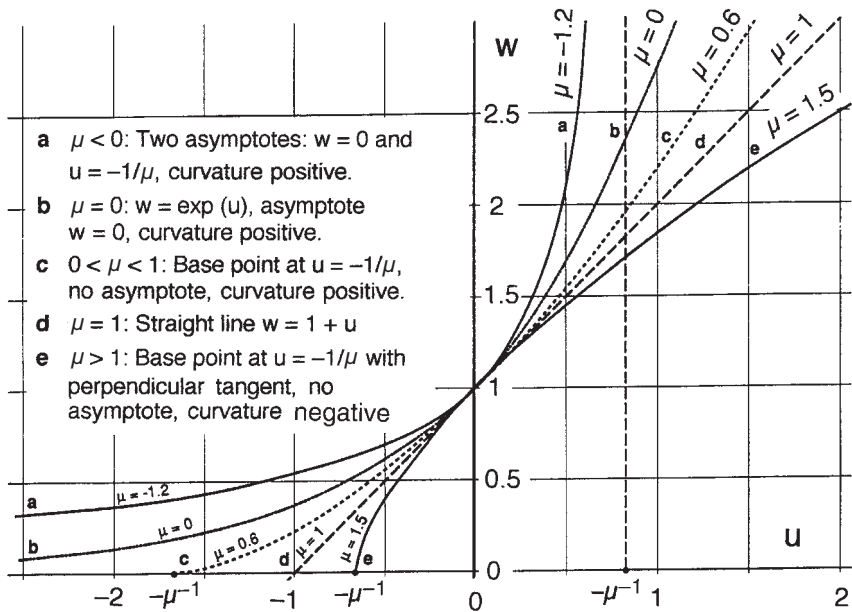


Fig. 16 Ranges of existence and appearance of reference-invariant functions $\chi(u, \mu)$ [109].

The regions of existence and appearance of reference-invariant functions $\chi(u, \mu)$ are represented in Fig. 16. Curves with maxima and minima cannot be described in a reference-invariant manner. In this case, both the dimensional-analytical representation and the model material system are confined to the region close to the “standardization point”.

The reference-invariant representation of the temperature dependence of the viscosity in Fig. 8 was obtained by $\mu = -0.167$ and only for water with -0.12 , respectively.

For the determination of μ , two procedures are discussed in [109]: the 3-point approximation ($N = 3$) and the approximation with the smallest relative standard deviation ($N > 3$).

8.4

Pi-space for Variable Physical Properties

If the scale-up is performed in the pi-space with *constant* physical properties, the requirement “idem” concerns all pi-numbers involved, whereby the dimensional quantities contained in them can be deliberately varied. The dimensional-analytical validity range includes all physically convenient numerical values of these dimensional quantities.

If the scale-up is performed in the pi-space with *variable* physical properties, the requirement “idem” also concerns the form of the dimensionless formulated material function. This aspect can make the choice of the model material system considerably more difficult. This requirement is fulfilled, *a priori*, only if the interesting range in the standard representation lies close to the standardization point, see the explanation concerning Fig. 8 in the text.

With variable physical properties the relevance list widens by both transformation parameters as well as by the reference point. In the dependence $\mu(T)$ this includes the quantities

$$\gamma_0, \mu_0, T_0$$

The original relevance list now contains two additional quantities, γ_0 and T_0 . Furthermore, μ_0 has to replace μ . By this it follows that the 3-parametric pi-space

$$\{\text{Nu}, \text{Re}, \text{Pr}\} \quad (8.16)$$

is transferred to the following 5-parametric one:

$$\{\text{Nu}, \text{Re}_0, \text{Pr}_0, \gamma_0 T_0, \Delta T/T_0\} \quad (8.17)$$

If one takes into consideration that the standard representation of $\mu(T)$ is reference-invariant, see Fig. 8

$$\mu/\mu_0 = \Phi(\gamma_0 \Delta T) \quad (8.18)$$

then the reference point T_0 is cancelled and the correlation (8.17) is reduced to

$$\text{Nu} = f\{\text{Re}_0, \text{Pr}_0, \gamma_0 \Delta T\} \quad (8.19)$$

whereby Re_0 and Pr_0 are formed by μ_0 .

Example 15: Consideration of the dependence $\mu(T)$ using the μ_w/μ term

In the treatment of heat transfer problems in the engineering literature, the process equation is, as a rule, extended by the parameter μ/μ_0 instead of $\gamma_0 \Delta T$. This is justified because between both terms, in accordance with the standard representation in Fig. 8, a simple correlation exists:

$$\mu/\mu_0 = \Phi(\gamma_0 \Delta T) \quad \text{or} \quad \mu/\mu_0 = \exp(\gamma_0 \Delta T) \quad (8.20)$$

Both terms are equivalent to each other.

In heat transfer problems, the liquid bulk temperature is normally taken as the reference temperature T_0 , whereas the temperature of the heat transfer surface (wall) T_w is taken to describe the effective temperature difference, ΔT , in the process. Therefore, the viscosity of the bulk liquid is taken as the reference viscosity: $\mu_0 = \mu$.

The inclusion of the *viscosity number*, $\text{Vis} \equiv \mu_w/\mu$, in the process equation for heat transfer in pipes goes back to *Sieder and Tate* [133]. These researchers succeeded in correlating experimental data obtained in pipe flow with the term $(\mu_w/\mu)^{-0.14}$. In this manner, the differences between the cooling and heating process were considered, manifesting themselves by the differences in the thickness of the boundary layers. During heating, practically no boundary layer is present as compared with cooling.

The heat transfer characteristics read:

Laminar range^{*)} ($\text{Re} \leq 2\,320$):

$$\text{Nu} (\mu_w/\mu)^{0.14} = 1.86 (\text{Re Pr } d/l)^{1/3} \quad \text{Re Pr } d/l = 10^1\text{--}10^4$$

Transition range ($\text{Re} = 2\,320\text{--}1 \times 10^4$):

$$\text{Nu} (\mu_w/\mu)^{0.14} = 0.12 (\text{Re}^{2/3} - 125) \text{Pr}^{1/3}$$

Turbulent range ($\text{Re} \geq 1 \times 10^4$):

$$\text{Nu} (\mu_w/\mu)^{0.14} = 0.1 \text{Re}^{2/3} \text{Pr}^{1/3}$$

^{*)} In this flow range the inlet effects make themselves felt until $d/l \approx 200$ (pipe diameter d , pipe length l).

However, the power -0.14 of the Vis term in heat transfer process characteristics has not been confirmed by later researchers.

Hruby [54] found in cooling measurements with a Newtonian oil that the power m of the Vis term depends on its numerical value:

$$m = -0.215 \text{Vis}^{-0.08} \quad 0.32 < \text{Vis} < 320$$

For $\text{Vis} = 3$ it then follows that $m = -0.20$ for $\text{Vis} = 200$, in contrast to $m = -0.14$.

Hackl [41] found insignificantly lower values:

$$m = -0.265 \text{Vis}^{-0.14} \quad 80 < \text{Vis} < 900$$

For $\text{Vis} = 80$ it is found that $m = -0.14$ for $\text{Vis} = 900$, in contrast to $m = -0.10$.

These facts were examined in a later work [42], where two viscous mineral oils ($\text{Vis} \approx 1\text{--}10^4$) were used in cooling. It was found that the power m attained for $\text{Vis} = 1\text{--}100$ was larger and for $\text{Vis} = 10^2\text{--}10^4$ was lower than -0.14 . A comparison of these findings is given graphically in Fig. 17.

For Pawlowski [109] a factorial inclusion of the Vis term into the heat transfer characteristic would make sense only if the temperature field is essentially restricted to the wall boundary layer, whereas the core flow would remain practically isothermal. This is certainly the case in the pipe flow at $\text{Re} > 2.300$.

According to this expectation, in stirring vessels equipped with close clearance anchor stirrers, the effect of the Vis term should not be observed. Zlokarnik [166] examined the heat transfer at cooling and heating in a stirring vessel equipped with a close clearance anchor stirrer. The eight liquids used displayed viscosities between 1 and $10^5 \text{ mPa}\cdot\text{s}$ at 20°C and had extremely different temperature coefficients: $\gamma_0 = (1.5\text{--}11.2) \times 10^{-2} \text{ K}^{-1}$. It was found that the data for cooling could be correlated only insignificantly by $\text{Vis}^{-0.02}$ and for heating by $\text{Vis}^{0.073}$.

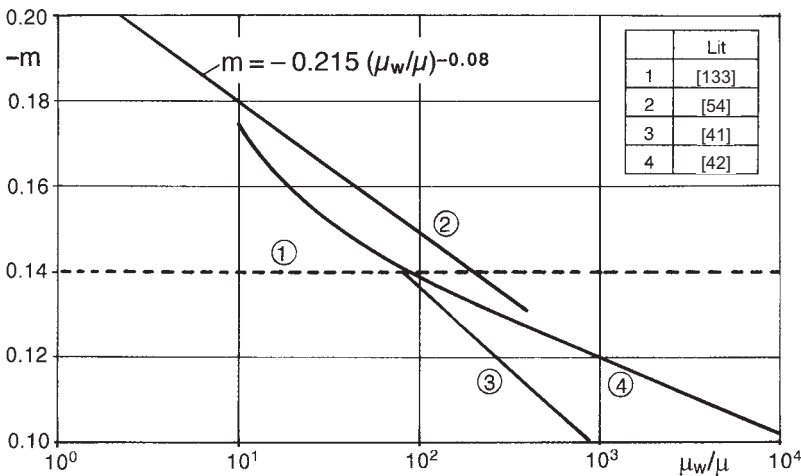


Fig. 17 Comparison of the findings $m(\text{Vis})$ of different authors.

In contrast, *Dunlap* and *Rushton* [23] succeeded in correlating their measurements at cooling and heating in a vessel with a turbine stirrer and pipe bundle heat exchangers with $Vis^{-0.4}$ because a thick boundary layer developed around the tubes.

Example 16: Consideration of the dependence $\rho(T)$ by the Grashof number Gr

In contrast to $\mu(T)$, the relevance of $\rho(T)$ is considered in the engineering literature exclusively by the number $\beta_0 \Delta T$. The ratio ρ/ρ_0 is used in heterogeneous material systems (solids/liquid or solids/gas) in which the density differences $\Delta\rho$ are occurring independently of the temperature differences.

Due to the fact that density differences can only have an effect when associated with acceleration due to gravity, both $\beta \Delta T$ and ρ/ρ_0 are consistently combined with the Galileo number $Ga \equiv Re^2/Fr \equiv g l^3/\nu^2$. Therefore, in the heat transfer under natural convection the

$$\text{Grashof number } Gr \equiv \beta \Delta T Ga \equiv g \beta_0 \Delta T l^3/\nu^2 \quad (8.21)$$

plays a role, whereas in processes where buoyancy or sedimentation occurs, the

$$\text{Archimedes number } Ar \equiv (\Delta\rho/\rho) Ga \equiv g \Delta\rho l^3/(\rho \nu^2) \quad (8.22)$$

is added to the pi-space.

Examples of the application of the Grashof number in heat transfer problems can be found, for example, in [40]. For examples where the Archimedes number is applied in solid/liquid systems (suspensions) see [184] and for sedimentation, as well as fluidization examples, please refer to textbooks dealing with unit operations in chemical process engineering.

8.5

Rheological Standardization Functions and Process Equations in Non-Newtonian Fluids

The dimensionless presentation of the flow behavior of a liquid is given by its rheological standardization function. There are two reasons that we need to know the rheological standardization functions in connection with scale-up.

One reason is based on the fact that the recognition of the rheological similarity of the liquids is only possible if their rheological standardization functions are known. Only when their flow behavior is described with the same material function (“master curve”), are they similar to one another and can then mutually be used as model substances.

The second reason is the fact that the rheological behavior of non-Newtonian fluids is compulsorily described by further additional parameters as compared

with Newtonian fluids. It must be proved which of them have to be included in the relevance list of a given process to assure that the process equation is also valid for the treatment of a non-Newtonian liquid.

8.5.1

Rheological Standardization Functions

In Newtonian liquids it is the dynamic viscosity μ [Pa s], which combines the shear stress τ [Pa] with the shear rate $\dot{\gamma}$ [s⁻¹] in the rheological equation of state:

$$\tau = \mu \dot{\gamma} \quad (8.23)$$

μ is a constant, depending only on the material and the temperature.

In non-Newtonian fluids, μ depends on the actually effective shear rate and occasionally also on history. Such fluids are subdivided into classes [22] according to their flow behavior, only two of which – the pseudoplastic and the viscoelastic ones – are treated in the following. (A thorough treatment of the rheology with regard to technological aspects is given in [13] and [14].)

8.5.1.1 Flow Behavior of Non-Newtonian Pseudoplastic Fluids

The pseudoplastic fluids are those fluids in which the stress viscosity is *decreased* due to shear. Under shear stress solid aggregates of a dispersion L/S or L/L (e.g., dye pigments) decompose into single particles which then orientate themselves towards the flow direction. Intermingled chains of macromolecules of a polymer solution or melt are stretched to the flow direction; spheres of erythrocytes of blood are elongated. In all these cases the viscosity is decreased due to shear.

The rheological material function $\mu(\dot{\gamma})$ is formulated in a dimensionless way with material standardization parameters ("shift parameters") H and $\dot{\Gamma}$ and is then transformed into a rheological standardization function. H represents μ_0 or μ_∞ (or similar) and $\dot{\Gamma}$ represents $\dot{\gamma}_0$ or $\dot{\gamma}_\infty$ (or similar):

$$\mu/H = f(\dot{\gamma}/\dot{\Gamma}) \rightarrow \mu/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0) \quad (8.24)$$

Liquids, whose rheological equations of state coincide in this dimensionless frame – whose standardization functions are identical – are rheologically similar to one another. (For choice of standardization parameters see Section 8.5.1.3.)

An important subclass of pseudoplastic fluids consists of those fluids whose material function is represented in the whole measured range in a log-log scale by a straight line with negative slope. In this case one speaks of an *Ostwald-de Waele* fluid, whose viscosity curve obeys the so-called "*power law*" (K – consistency index, m – flow index):

$$H^{1-m} = K \rightarrow \tau = K \dot{\gamma}^m \rightarrow \mu = K \dot{\gamma}^{m-1} \quad (8.25)$$

Therefore one equation exists for μ and $\dot{\gamma}$ with only two standardization parameters in which μ_0 and $\dot{\gamma}_0$ are linked to each other:

$$\mu_0 \dot{\gamma}_0^{1-m} = K \quad (8.26)$$

To demonstrate this flow behavior, eq. (8.25) in the form of a standardization function, $\dot{\gamma}_0$ is arbitrarily chosen and μ_0 , according to eq. (8.26), is assigned to it. This degree of freedom facilitates the choice of reference points for the standardization of the material function (8.25).

Pawłowski [103; page 124] has already pointed out that eq. (8.25) disobeys the principle of consistency of physical quantities because the dimension of $[K] = M L^{-1} T^{m-2}$ depends on the numerical value of the exponents. This means that, in the case of a temperature dependent viscosity in a temperature field, the quantity K changes its dimension from point to point and therefore neither $grad K$ nor K/K_0 can be built.

The physical consistency of eq. (8.25) is still preserved if it is transformed by eq. (8.26) into the relationship

$$\mu/\mu_0 = (\dot{\gamma}/\dot{\gamma}_0)^{m-1} \quad (8.27)$$

whereas K is eliminated.

In most pseudoplastic fluids at sufficiently low and sufficiently high shear rates $\dot{\gamma}$ Newtonian flow behavior can be observed, see Fig. 18. Viscosity which is approaching a constant value with low shear rates is called zero-shear viscosity, μ_0 , and its constant value at very high shear rates is called the infinite shear viscosity, μ_∞ . Classic model fluids of this type are aqueous solutions of macromolecular organic compounds as, e.g., Carboxy-methyl-cellulose (CMC), Poly-acryl-amide

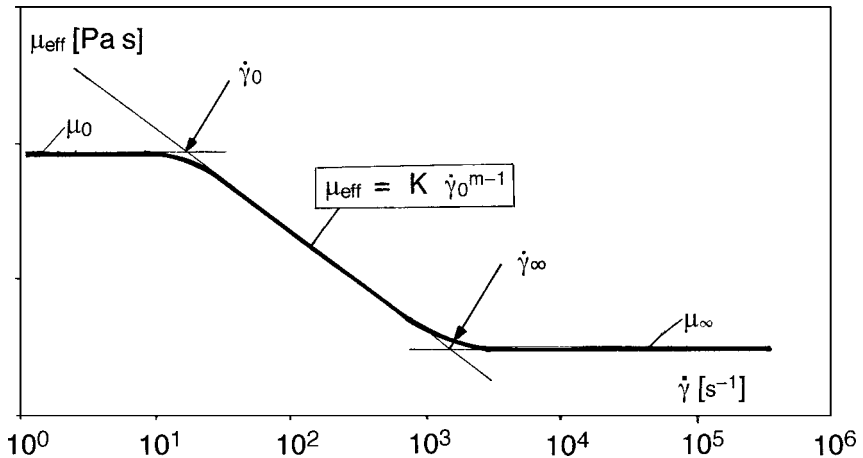


Fig. 18 Principal course of viscosity of a pseudoplastic fluid which, in a certain range, behaves as an *Ostwald-de Waele* fluid.

(PAA), Carbopol® (very sour acrylic acid polymerisates of the Goodrich Co), but also of biopolymers, such as Xanthane, etc.

In mixing vessels shear rates usually appear in the range $\dot{\gamma} = 50\text{--}500\text{ s}^{-1}$ in which many pseudoplastic fluids behave as *Ostwald-de Waele* fluids. This explains why the “power law” is often used to describe rheological behavior, see e.g. [47, 157].

In these pseudoplastic fluids the rheological standardization function can be presented in the pi-space

$$\{\mu/\mu_0, \dot{\gamma}/\dot{\gamma}_0, \Pi_{\text{rheol}}\} \quad \text{or} \quad \{\mu/\mu_\infty, \dot{\gamma}/\dot{\gamma}_\infty, \Pi_{\text{rheol}}\} \quad (8.28)$$

In the publication of *Henzler* [48], Π_{rheol} is represented by the exponent m of the straight line, which stands for an upsetting/stretching of the course of the curve belonging to the power law. In **Fig. 19** CMC and Xanthane solutions of various concentrations are correlated in this pi-space.

These materials are obviously not completely similar to one another. They would be, if the exponent m were constant and the dependencies could be correlated in the space $\mu/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0)$. Here, the exponent varies by factor two: $m = 0.35 - 0.6$.

In the introduction to this chapter it was pointed out that the importance of the rheological standardization function consists in recognizing materials which are similar to one another and thus building a class of materials belonging to a common *master curve* – which are therefore suitable as model substances. **Fig. 20** presents a class of aqueous poly-acryl-amide (PAA) solutions of different concentrations which correlate excellently with one another in the space $\mu/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0)$.

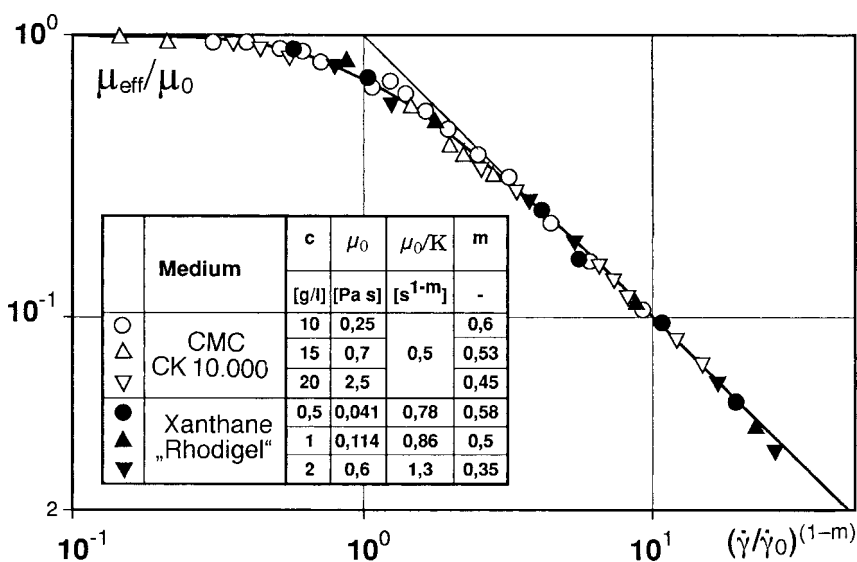


Fig. 19 Dimensionless standardization functions of some pseudoplastic fluids; from [48]. For the meaning of $\dot{\gamma}_0$ see Fig. 18.

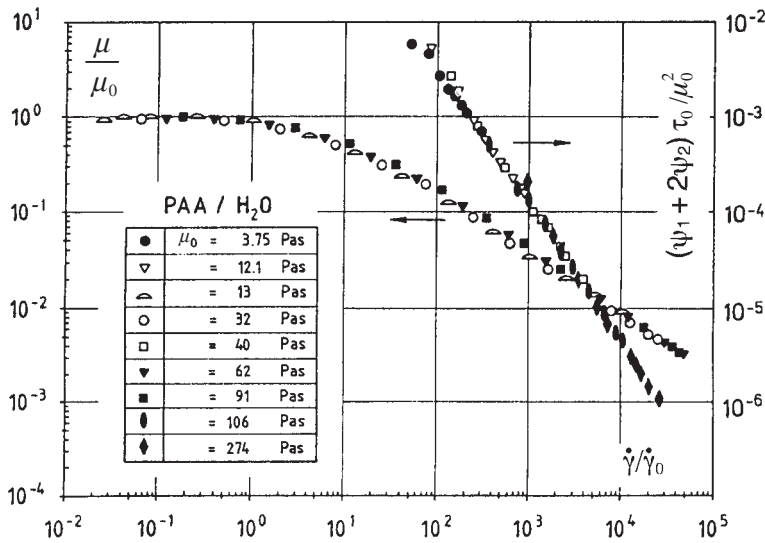


Fig. 20 Master curves μ/μ_0 and $(\psi_1 + 2\psi_2) \dot{\gamma}_0/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0)$, resp., for a class of aqueous PAA solutions of different concentrations and viscosities; from [12].

The finding and working out of model substances of a family of these, building a master curve, is a task of prime importance, when model measurements are necessary for dimensioning a process in which non-Newtonian fluids occur. Section 8.5.1.4 is dedicated to the presentation of their rheological behavior in the form of rheological standardization functions.

8.5.1.2 Flow Behavior of Non-Newtonian Viscoelastic Fluids

In viscoelastic fluids the distortion work performed by shear stress is not completely, immediately and irreversibly transformed into friction heat, but is partially elastically stored in order to relax after a delay. In this respect they are similar to solids. The liquid strains give way to mechanical shear stress as do elastic bonds by contracting. This is shown in shear experiments as a restoring force acting against the shear force which, at the sudden ending of the force effect, moves the plate back to a certain extent. In mixing a viscoelastic fluid it moves up the stirrer shaft, in order to evade the shear stress, which the rotating stirrer shaft exerts on the liquid on its surface (“Weissenberg’s phenomenon”); see Example 17.

For steady-state laminar flow, besides the shear stress $\tau = \sigma_{21} = \mu \dot{\gamma}$, normal stress is observed in viscoelastic fluids in all three directions:

$$\begin{array}{ll} \text{in the direction of flow:} & \sigma_{11} + p \\ \text{and perpendicular to the direction of flow} & \sigma_{22} + p \text{ and also } \sigma_{33} + p \end{array}$$

The isotropic pressure, p , can be eliminated by forming differences in normal stress:

1. difference in the normal stress $N_1 = \sigma_{11} - \sigma_{22}$
2. difference in the normal stress $N_2 = \sigma_{22} - \sigma_{33}$.

As N_2 values are always much smaller than N_1 values [48], it will be sufficient for most processes to take into consideration only N_1 . The differences in normal stress are independent of the direction of flow and in the case of laminar flow (low $\dot{\gamma}$) are proportional to $\dot{\gamma}^2$. In Newtonian viscosity, following $\mu = \tau/\dot{\gamma}$, occasionally normal stress coefficients $\psi_1 \equiv N_1/\dot{\gamma}^2$ and $\psi_2 \equiv N_2/\dot{\gamma}^2$ are used, and their dependence on the shear rate describes the nonlinear viscoelastic behavior of the fluid.

For the dimensionless presentation of the viscoelastic behavior of a fluid, normally the ratio of normal stress to shear stress is used. The so-called Weissenberg number Wi is defined as

$$Wi_1 \equiv Wi \equiv N_1/\tau \quad (8.29)$$

The material function of a viscoelastic fluid is presented in space $Wi(\dot{\gamma})$, the standardization function here is $Wi/Wi_0 = f(\dot{\gamma}/\dot{\gamma}_0)$.

On the right-hand side of Fig. 20 the correspondence $(\psi_1 + 2\psi_2)\dot{\gamma}_0/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0)$ is shown. This figure does not represent a standardization function. The proof of similarity for two material functions (congruence separated for each kind of function as well as for the relative position to each other) can only be obtained if these functions refer to quantities of equal dimension. Material functions $(\psi_1 + 2\psi_2) = f(\dot{\gamma})$ do not fulfill the condition of congruence of the relative position to the corresponding material functions. Only after the transformation to $(\varphi_1 + 2\varphi_2) = f(\dot{\gamma})$ with $\varphi \equiv \mu\dot{\gamma}$ does it follow that the fluids in Fig. 20 can really be used as model substances to one another; details are given in Section 8.5.1.3.

Occasionally a characteristic relaxation time λ is also used for the description of viscoelastic behavior. It is a measure for the time which is needed to transform the reversibly-elastically stored energy into the friction heat:

$$\lambda \equiv N_1/(2\tau\dot{\gamma}) = Wi/(2\dot{\gamma}) \quad (8.30)$$

If, according to the concept of *Metzner* and *Otto* (see Section 8.5.2.1), $\dot{\gamma}$ is replaced by the stirrer speed n , eq. (8.30) can also be written as

$$De \equiv \lambda n \triangleq \lambda \dot{\gamma} = Wi/2 \quad (8.31)$$

The expression λn is called the Deborah number De .

8.5.1.3 Dimensional-analytical Discussion of Viscoelastic fluids [111a]

The attempt to discuss viscoelastic fluids by two rheological material functions

$$\mu = f(\dot{\gamma}) \quad \text{and} \quad (\psi_1 + 2 \psi_2) = f(\dot{\gamma}) \quad (8.32)$$

similarly to pseudoplastic fluids, which possess only one rheological material function, first leads to contradictions. It is understandable that the condition of similarity – in the log-log space – is not extended only to the congruence of both material functions (8.32) but also over and above that to the congruence of their relative position to each other (“position congruence”).

One should now consider an alternative sequence of rheological material quantities $\omega_m = N \dot{\gamma}^m$ with the normal stress N (m – whole numbers). For the power characteristics of a stirrer, the relevance list reads

$$\{P; d; \rho, \dot{V}_1, H, \dot{V}_2, \Omega_m; n\} \quad (8.33)$$

where $(\dot{V}_1, H)$ and $(\dot{V}_2, \Omega_m)$ are shift parameters (see explanations to eq. (8.24)) for both material functions $\mu = f_1(\dot{\gamma})$ and $(\varphi_1 + 2 \varphi_2) = f_2(\dot{\gamma})$. From this an alternative pi-set follows

$$\{Ne, Re, B, A, C_m\} \quad (8.34)$$

with

$$\begin{aligned} Ne &\equiv P/(\rho n^3 d^5); Re \equiv \rho n d^2/H; B \equiv \rho \dot{V}_1 d^2/H; A \equiv \dot{V}_2/\dot{V}_1; \\ C_m &\equiv \Omega_m/(H \dot{V}_1^{m+1}) \end{aligned} \quad (8.35)$$

In the case of concurring power characteristics (Ne, Re), conditions $B, A, C_m = \text{idem}$ apply. While $B = \text{idem}$ can be fulfilled by an adjustment of d , the conditions $A, C_m = \text{idem}$ concern the form of both rheological curves and their relative position to each other.

Because the positional congruence of two congruent curves is given only at $A \equiv \dot{V}_2/\dot{V}_1 = \text{idem}$ and $\Omega_m/H = \text{idem}$ and the expression Ω_m/H must be dimensionless, it follows for the subscript m in (8.34) that it should be specified as $m = -1$. Consequently, in the above mentioned alternative sequence $\omega_m = N \dot{\gamma}^m$ only the quantity $\omega_{-1} = N \dot{\gamma}^{-1}$ with the dimension $[N/\dot{\gamma}] = [\mu] = [ML^{-1}T^{-1}]$ leads to an appropriate criterion for the selection of viscoelastic model fluids. Ignoring this fact would lead to a faulty selection and thus to confusion in model experiments.

With $\varphi \equiv \omega_{-1} \equiv \psi/\dot{\gamma}$ in the material function $(\varphi_1 + 2 \varphi_2) \equiv (\psi_1 + 2 \psi_2)\dot{\gamma} = f_2(\dot{\gamma})$ and the corresponding reference point $(\dot{V}_2, \Phi)$ with $[\Phi] = [\mu] = [ML^{-1}T^{-1}]$ the equations (8.33 – 8.35) obtain the valid form:

$$\{P; d; \rho, \dot{V}_1, H, \dot{V}_2, \Phi; n\} \quad (8.36)$$

$$\{Ne, Re, B, A, C\} \quad \text{with} \quad C \equiv \Phi/H \quad (8.37)$$

Dimensionless numbers A and C do not belong to the Π_{rheol} group, because they are freely selectable and do not contain material parameters of the material functions.

- Viscoelastic materials are therefore rheologically similar to one another, if not only their material functions $\mu = f(\dot{\gamma})$ and $(\psi_1 + 2\psi_2)\dot{\gamma} = f(\dot{\gamma})$ but also their relative positions to one another, are congruent.

A further evaluation of the rheological data from Fig. 20 shows that the demand for congruence is optimally fulfilled here. This result gives a dimensional-analytical explanation for the excellent correlation of the power characteristics $Ne = f(Re, B)$ shown in Fig. 23 (apart from the Re_{MO} -Abscissa after Metzner–Otto used in [12]). At the same time, this result represents an experimental confirmation of the theoretical dimensional-analytical considerations given above.

Pawłowski [111a] gained the original values of μ and φ from the flow and stress curves of Fig. 20. The result is presented in Fig. 21. The φ points are $(\varphi_1 + 2\varphi_2) = (\psi_1 + 2\psi_2)\dot{\gamma}$ values of individual fluids, which were transformed in combination with μ values. By doing this, the idem demand of the positional congruence $C \equiv \Phi/H$ is accomplished.

It is shown that φ points correlate very well. It is verified by this that the aqueous PAA solutions, used in model experiments, are indeed similar to each other in their viscoelastic behavior. Contrary to this, the $(\psi_1 + 2\psi_2)$ values, gained by the same transformation, show a considerable positional drift. A material selection according to the ψ criterion, in which the values $(\psi_1 + 2\psi_2)$ coincide, would lead to a false material selection.

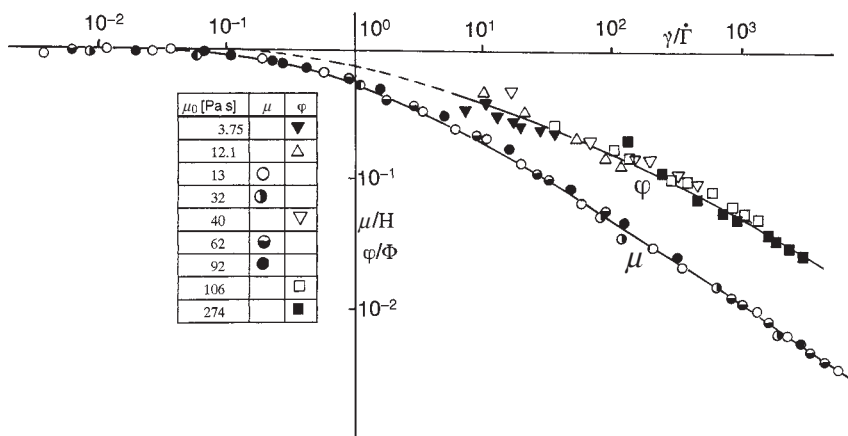


Fig. 21 μ and $(\psi_1 + 2\psi_2)\dot{\gamma}$ values gained from Fig. 20 show the positional congruence of both dependencies. This confirms that the aqueous PAA solutions behave in a similar way to each other with respect to their viscoelastic behavior. Evaluation from [111a].

8.5.1.4 Elaboration of Rheological Standardization Functions [111a]

In a dimensional-analytical discussion of hydrodynamic processes in non-Newtonian fluids, all rheological material functions such as, e.g., $\mu(\dot{\gamma})$ or $N_1(\dot{\gamma})$, are treated in the same way. It is assumed that they are presented in a double-logarithmic (x, y) field and can be transformed by shifting parallel to the axes. In doing this, the standardization parameters, e.g., μ_0 , $\dot{\gamma}_0$, $N_{1,0}$ or (μ/μ_0) , $(\dot{\gamma}/\dot{\gamma}_0)$, $N_1/N_{1,0}$, ... are related to the (x, y) coordinates.

The principal significance of the log-log presentation is based on the multiplicative structure of the pi numbers. This enables us to present dimensional material functions, e.g., $\mu(\dot{\gamma})$ or $N_1(\dot{\gamma})$ – as dimensionless standardized functions (“standardization functions”) $\mu/\mu_0 = f_1(\dot{\gamma}/\dot{\gamma}_0)$; $N_1/N_{1,0} = f_2(\dot{\gamma}/\dot{\gamma}_0)$, etc., congruently – i.e., without changing their shape.

By doing this, for each of these material functions, e.g., $\mu = f_1(\dot{\gamma})$ or $N_1 = f_2(\dot{\gamma})$, first a reference point $(\dot{\gamma}_{0,1}, \mu_0)$ or $(\dot{\gamma}_{0,2}, N_{1,0})$ is fixed and then their dimensionless representations – the standardization functions $(\mu/\mu_0) = f_1(\dot{\gamma}/\dot{\gamma}_{0,1})$ and $N_1/N_{1,0} = f_2(\dot{\gamma}/\dot{\gamma}_{0,2})$, respectively – are produced by axes-parallel shifting of material functions in the (x, y) field.

In this way, the reference points are transferred into the field point $(x, y = 1)$, in order to apply their reference coordinates $(\mu_0, \dot{\gamma}_{0,1}, N_{0,1}, \dot{\gamma}_{0,2})$ at the same time as the transformation parameters (shift parameters) of each corresponding function. This also applies for material functions with dimensionless variables, such as, e.g., $Wi = f_3(\dot{\gamma})$, with a dimensionless shift parameter Wi_0 .

In a dimensional-analytical discussion of flow processes, the rheological influence of the materials involved is only effective by their dimensionless standardized functions. All shift parameters of the process in discussion have to be included into the relevance list. Together with the other relevant parameters they determine the pi-space in which this process is presented.

A flow process and its process equation – e.g. $Ne = f(Re_0, \dot{\gamma}/\dot{\Gamma})$ – also depend on the form of the standardization functions. Such a dependence, in which a standardization function acts as an argument instead of a variable, is named a functional. In this case the f function is not only a function of variables, but at the same time it is a functional of the standardization function $(\mu/\mu_0) = f_1(\dot{\gamma}/\dot{\gamma}_0)$. The same is also valid for the entire flow process.

This gives a reason for the fact why the idem demand for standardized functions is a necessary prerequisite of a similar rheological behavior. By the way, this functional dependence is an analogue of the influence, which all geometric parameters of an appliance exert on a hydrodynamic process.

Substances, whose material functions can be portrayed using appropriate shift parameters by a common standardization function (“master curve”), are rheologically similar to each other and are described by the same process equation. They can be mutually used as model substances.

The goal is to achieve optimum congruence of the standardized functions of the substances investigated. In order to do this, analogous corresponding points have to be chosen on material functions. Prominent places are preferred, such as, e.g., the transition from the zero-shear viscosity, μ_0 , to the declining straight line $\mu(\dot{\gamma})$.

As a reference point the point of intersection of both tangents also can be defined, see Fig. 18.

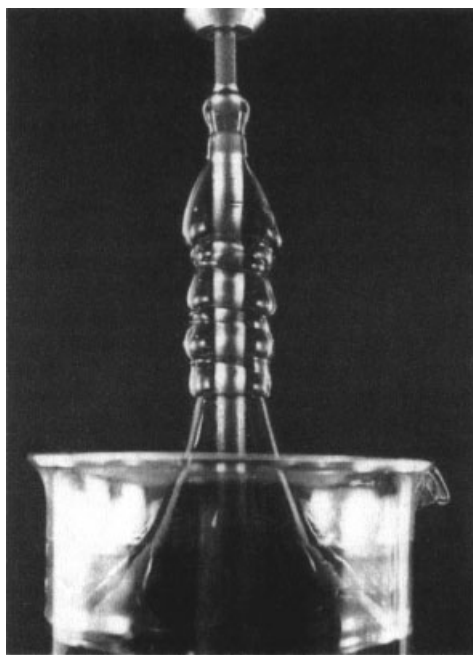
If under flow conditions a fluid behaves as an *Ostwald–de Waele* fluid, any point on the logarithmic $\mu(\dot{\gamma})$ straight line can be taken. If chosen conveniently, the number of shift parameters can possibly be diminished and thus the complete pi-set is reduced.

If flow processes are simultaneously influenced by several different rheological material functions, e.g., $\mu(\dot{\gamma})$ and $N_1(\dot{\gamma})$, they have to be correlated in the common field point $(x, y = 1)$ by using the appropriate shift parameters. The degree of rheological similarity of the model substances depends on the degree of congruence of the standardized functions.

Example 17: Dimensional-analytical treatment of *Weissenberg's* phenomenon – Instructions for a PhD thesis [111]

Background

Due to an increased presence of biotechnology, the viscoelastic fluids have also increased in importance. These mostly relatively low viscous solutions of water soluble polymers (CMC, HEC, PAA, PAN, etc.) and biopolymers (Xanthane, etc.), on one hand elastically store the part of the distortion work which was influenced by stress and on the other hand, evade the mechanic shear stress by contracting like rubber bands. In this way a viscoelastic fluid climbs up the stirrer shaft in



The *Weissenberg* phenomenon: a viscoelastic liquid overcomes gravity to creep up the rotating stirrer shaft; from [14].

order to evade the shear stress, which is exerted on it by the rotating stirrer shaft on the liquid surface ("Weissenberg's phenomenon").

This phenomenon has not yet been treated by dimensional analysis. Therefore it can be expected that, from this point of view, its treatment will deliver valuable insights for process engineering in relation to these fluids (e.g., for mixing processes).

Experimentally, Weissenberg's effect can easily be dealt with. It is a process in which model-theoretical handling with rheological material functions of viscoelastic fluids can be studied with a minimum of expenditure for experiment.

The gradient of the rotational speed around the rotating rod forms a stress field which causes the viscoelastic fluid, when the relaxation time is sufficiently long, to wind itself round the rod, then climb up the rod and form a liquid collar of a steadily renewing liquid.

With increasing collar height over the liquid surface, this stress field is gradually dismantled and the fluid begins to flow down. This process proceeds in either a pulse-like way more or less periodically, or a dynamic steady-state liquid column is formed.

An experimental determination of the collar height H will be particularly easy to measure, under steady-state conditions, as a function of the problem-relevant material and process-determining conditions.

Problem-relevant rheological material functions

It is assumed that the rheological influence of the fluid on Weissenberg's effect is restricted to the *viscoelastic* flow behavior according to the material function

$$N_1/N_{1,0} = f(\dot{\gamma}/\dot{\gamma}_0^*) \rightarrow Wi/Wi_0 = f(\dot{\gamma}/\dot{\gamma}_0^*) \quad (8.38)$$

as well as the *pseudoplastic* flow behavior according to the material function

$$\mu/\mu_0 = f(\dot{\gamma}/\dot{\gamma}_0), \quad (8.39)$$

Liquids whose flow behavior corresponds to the same material functions as (8.38) and (8.39), are rheologically similar to each other. The standardization parameters μ_0 , Wi_0 , $\dot{\gamma}$, $\dot{\gamma}_0^*$ are of material specific nature. Of the standardization parameters $\dot{\gamma}_0$ and $\dot{\gamma}_0^*$ only the first is taken into the relevance list, because the dimensionless number $\dot{\gamma}_0/\dot{\gamma}_0^*$ is a constant and cannot function as a variable argument.

Relevance list

The target quantity of the process is the collar height, H . The geometric parameters are the shaft (rotating rod) diameter, d , (and possibly also the immersion depth h of the rod.)

The material parameters are the density of the fluid, ρ , the material standardization parameters μ_0 , $\dot{\gamma}_0$, $N_{1,0}$, the surface tension, σ , and the contact angle, θ . Process parameters are the rotational speed of the rod, n , and the acceleration due to gravity, g .

Thus the relevance list consists of 10 parameters:

$$\{H, d, \rho, \mu_0, \dot{\gamma}_0, N_{1,0}, \sigma, \Theta, n, g\} \quad (8.40)$$

Dimensional analysis

With these 10 parameters $10 - 3 = 7$ known, partly named numbers are formed:

$$\frac{H}{d}, \text{ Re} \equiv \frac{n d^2 \rho}{\mu_0}, \text{ Fr} \equiv \frac{n^2 d}{g}, \text{ We} \equiv \frac{\rho n^2 d^3}{\sigma}, \text{ Wi}_0 \equiv \frac{N_{1,0}}{\mu_0 \dot{\gamma}_0}, \frac{n}{\dot{\gamma}_0}, \Theta \quad (8.41)$$

They will be transformed into equivalent numbers, to separate process parameters (n, d, g) from the material ones and at the same time to produce dimensionless numbers of which every one represents only one process variable:

Process numbers:

$$\Pi_n \equiv \frac{n}{\dot{\gamma}_0}, \quad \Pi_d \equiv \frac{\text{We}}{\text{Re}(n/\dot{\gamma}_0)} \equiv \frac{d \mu_0 \dot{\gamma}_0}{\sigma}, \quad \Pi_g \equiv \frac{\text{Re}^2(n/\dot{\gamma}_0)^2}{\text{Fr We}} \equiv \frac{g \rho \sigma}{(\mu_0 \dot{\gamma}_0)^2} \quad (8.42)$$

Material numbers:

$$\text{Wi}_0 \equiv \frac{N_{1,0}}{\mu_0 \dot{\gamma}_0}, \quad \Pi_\rho \equiv \frac{\text{Re}^3(n/\dot{\gamma}_0)}{\text{We}^2} \equiv \frac{\rho \sigma^2}{\mu_0^3 \dot{\gamma}_0}, \quad \Theta$$

The complete pi set with the target number $\Pi_H \equiv H/d$ reads:

$$\Pi_H, \Pi_n, \Pi_d, \Pi_g, \text{Wi}_0, \Pi_\rho, \Theta \quad (8.43)$$

Π_g contains ρg and thereby represents the influence of the specific gravity, whereas in Π_ρ only ρ appears. This number concerns mass inertia, which is only effective in combination with the centrifugal force. In creeping movement Π_ρ is irrelevant and can be cancelled.

Therefore, the target number H/d depends only on two process numbers Π_n and Π_d , whereas the four material numbers remain constant when working with the same material system.

Measurement execution and evaluation

(1) Apparatus

In a vessel ($D = 100$ mm, $H = 100$ mm) a shaft ($d = 10 - 20$ mm) rotates with fully adjustable velocity ($n = 10 - 100 \text{ min}^{-1}$). Shafts should be of different material, e.g., steel, Al, PVC, Polymethacrylate, in order to investigate the influence of the contact angle, Θ , as well.

The collar height H will be measured under respective conditions (shaft speed n ; shaft diameter d ; rheological material properties). To facilitate the determination of H , the shafts should be graded in millimeters.

(2) Leading variables and idem parameters

When the measurements are made, each of the dimensionless numbers presented in (8.43) represents a “leading variable”, whose influence on the target number Π_H is examined, whereas all the remaining numbers are kept constant (“idem”). This can easily be accomplished for both process variables Π_n and Π_d by an appropriate adjustment of n and d . With the material numbers, on the other hand, this adjustment can be accomplished only with difficulty.

(3) Material systems and material data maps

It is therefore advisable to use polymer solutions as model substances. By varying their concentration c and test temperature T , the idem demand can be realized for two of the three material numbers (Π_g, Wi_0, Π_ρ). (For Θ see later.)

To facilitate the search for appropriate material properties of each fluid, it is sensible to create material data maps. For this, experimentally determined c - T curves are plotted as a net on log-log diagrams with coordinates (Π_g, Wi_0) , (Wi_0, Π_ρ) and (Π_ρ, Π_g) .

For the verification of the measured process equations in pi-space (8.43), measurements should be executed in at least two different material systems.

(4) Relevance of Θ

Due to the fact that Θ is a genuine pi variable, it could be cancelled from the test program and from the relevance list, as was done with the relative immersion depth h/d of the rod. However, attention should be paid to the fact that a connection exists between Θ and σ . If σ is problem relevant, in changing the model fluids, Θ will also change. To check this, two series of measurements $\Pi_H = f(\Pi_n)$ should be performed at material and $d = \text{idem}$ with rods of different material ($\Theta \neq \text{idem}$). If Θ is not significant, it has to be cancelled in (8.43).

If, in contrast, Θ proves significant, it should be additionally examined as to whether Θ and σ can be combined to $\sigma \cos \alpha$ [$= \sigma_{SG} - \sigma_{SL}$ (1. Laplace sentence)]. This can be proved by two tests with the same fluid but with rods of different material, whereby all arguments in (8.43) are kept idem. The question regarding for which of the pi variables (Π_d, Π_g, Π_ρ) the coupling ($\sigma \cos \alpha$) exists and proves necessary, should be examined for each of these pi variables. In the case when the ($\sigma \cos \alpha$) assumption proves correct only for one variable in (8.43) alone, then Θ is cancelled as a genuine argument in (8.43). The quantity ($\sigma \cos \alpha$) can be determined experimentally by measurement with a capillary of the same material as the rod, because there is a correlation between it and the capillary ascension/depression h :

$$(\sigma \cos \Theta) = \rho g h R/2 \quad (R - \text{radius of the capillary}) \quad (8.44)$$

The ($\sigma \cos \Theta$) assumption suggests itself by the possible analogy between h and H .

(5) Realization of the idem conditions

Instructions for the determination of the correlation $\Pi_H = f(\Pi_{lead})$ for material numbers (Π_g, Wi_0, Π_ρ) , if a is irrelevant and the coupling $(\sigma \cos \Theta)$ exists, resp.

Denotations:

Π_{lead} – one of the variables (Π_g, Wi_0, Π_ρ) ,

Π_i, Π_k – both remaining variables of (Π_g, Wi_0, Π_ρ) in free assignation.

1. All arguments are fixed with the starting experiment (freely chosen).
2. In each of the following tests, each time using another material system, the procedure is as follows:
 - Rod material: different.
 - Determination of c and T , ensuring $(\Pi_i \text{ and } \Pi_k) = \text{idem}$. Thus the model fluid is fixed.
 - Proceeding from Π_n and Π_d , n and d are calculated for the next experiment.
 - With data obtained in this way the assigned value of Π_{lead} is calculated.

With successive tests conducted in this way, the new value for $\Pi_H = f(\Pi_{lead})$ is determined.

(6) Conclusions concerning the test program and the measuring device

1. Preparation of rheologically similar viscoelastic materials and systematic working out of material data maps is an indispensable prerequisite for the experimental program.
2. The material and diameter of the rods result from the concrete idem demands in the course of the test.
3. A fully adjustable electric motor ($10 < n < 100 \text{ min}^{-1}$) with coupling which enables a quick exchange of rods is required.
4. A temperature controlled cylinder of ca. 100 mm diameter $\times$ 100 mm liquid height is necessary.
5. A measuring device to measure the capillary ascension/depression and to determine $(\sigma \cos \Theta)$ – a temperature controlled capillary of the same material as the respective rod is (optionally) required.

8.5.2

Process Equations for Non-Newtonian Fluids

In the introduction to this chapter it was pointed out that certain parameters of the rheological normalization functions have to be included into the relevance list of a given process in order to assure that the process equation is also valid for the treatment of non-Newtonian liquids. This will be shown here for some examples from mechanical and thermal process engineering in connection with pseudo-plastic and viscoelastic fluids.

Unfortunately it is still current engineering practice to build the process number Re according to the *Metzner–Otto* concept [85] with a so-called effective viscosity μ_{eff} as $Re_{\text{eff}} \equiv v l \rho / \mu_{\text{eff}}$, and to ignore clear dimensional-analytical rules for non-Newtonian fluids, which have been discussed in section 8.5.1 of this chapter and demonstrated by *J. Pawlowski* 35 years ago [101, 103]. This fact makes it necessary to discuss this quantity first. (A detailed critical argument using this concept is given in [110].)

8.5.2.1 Concept of the Effective Viscosity μ_{eff} According to *Metzner–Otto*

This concept has been developed in association with the determination of the stirrer power in a non-Newtonian liquid [85]. (For stirrer power see Example 21 in section 9.1). It is assumed that in the immediate surroundings of the stirrer a representative shear rate $\dot{\gamma}_{\text{MO}}$ exists which is proportional to the stirrer speed, n :

$$\dot{\gamma}_{\text{MO}} = k_{\text{MO}} n \quad (8.45)$$

k_{MO} is a constant, depending only on the type of stirrer and on geometric conditions in the stirring vessel. The determination of $\dot{\gamma}_{\text{MO}}$ proceeds as follows:

1. A stirrer arrangement is used for which the power characteristics $Ne(Re)$ for Newtonian liquids is known.
2. Using the non-Newtonian fluid, the stirrer power is determined for some stirrer speeds and the Ne numbers are calculated.
3. With these Ne values the corresponding Re values are taken from the $Ne(Re)$ correlation for Newtonian liquids.
4. With these Re values the effective viscosity μ_{eff} is determined.
5. From the flow curve $\mu = f(\dot{\gamma})$ of the liquid used, the representative shear rate $\dot{\gamma}_{\text{MO}}$ is found.

The concept of the “effective viscosity” is well known in rheology. For the “viscosimetric flow” in a capillary viscosimeter it is theoretically justified, but it is valid only for creeping flow and fluids without viscoelastic properties. In the case under discussion (stirring vessel!) we are dealing with a *quantity*, which cannot even describe the flow behavior in the immediate surroundings of the stirrer where the power is dissipated; see **Fig. 22**.

It cannot be expected that a Reynolds number Re_{eff} , which is built from μ_{eff} , could describe the state of flow in the entire treated volume which is responsible for the liquid homogenization (“macro mixing”) [98]. This is demonstrated in a deterrent way in [66] by **Fig. 14.10**.

Here, two papers [12, 110] should be pointed out which convincingly disprove the concept of *Metzner–Otto*.

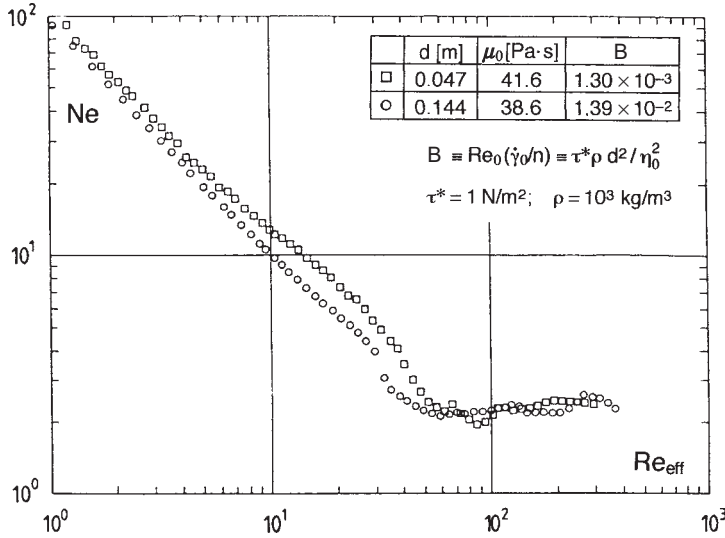


Fig. 22 Power characteristics of a turbine stirrer ($D/d = 2$, $h/d = 1$) under geometrically similar conditions in two mixing vessels of different size, $\mu = 1 : 3.43$. The aqueous PAA solutions used had practically the same

viscosity. From [12]; Re_{eff} is built with μ_{eff} according to Metzner–Otto. $\tau^* = 1 \text{ Pa}$; for the transformation to τ it goes as: $H \dot{\Gamma} = \text{const} \times 1 \text{ Pa}$.

8.5.2.2 Process Equations for Mechanical Processes with Non-Newtonian Fluids

In the relevance list for a process, there is no place for variables such as, e.g., μ_{eff} because these are results and not arguments known in advance. These can only be the rheological normalization parameters μ_0 , $\dot{\gamma}_0$, $N_{1,0}$, etc.

As has already been pointed out, with regard to the pi-space, the transition from a Newtonian to a non-Newtonian fluid has the following consequences:

- All pi numbers of the Newtonian case also appear in the non-Newtonian case, whereby instead of μ the normalization parameter of the same dimension, H , usually μ_0 , appears.
- An additional pi number appears which contains the normalization parameter $\dot{\Gamma}_0$, usually as $\dot{\gamma}_0$.
- Pure material numbers are increased by $\Pi_{rheol} = m$, Wi_0 , etc.

Example 18: Power characteristics of a stirrer

The relevance list for the power P of a given stirrer type (see Example 22) in a Newtonian liquid is given by

$$\{P; d, \rho, \mu, n\}$$

It leads to the dimensionless dependence

$$Ne = f(Re) \quad \text{with} \quad Ne \equiv P/(\rho n^3 d^5) \quad \text{and} \quad Re \equiv n d^2 \rho/\mu \quad (8.46)$$

In a pseudoplastic liquid both normalization parameters $\dot{\gamma}_0$ and μ_0 (if necessary also Π_{rheol}) have to be introduced. This means that the relevance list is enlarged by $\dot{\gamma}_0$ and μ is replaced by μ_0 :

$$f(P; d, \rho, \mu_0, \dot{\gamma}_0, n) = 0 \quad (8.47)$$

Therefore, the pi set is enlarged, with regard to (8.33), by one pi number

$$Ne = f(Re_0, Q) \quad \text{with} \quad Re_0 \equiv n d^2 \rho/\mu_0 \quad \text{and} \quad Q \equiv \dot{\gamma}_0/n \quad (8.48)$$

This fact is shown in **Fig. 23** for a given stirrer type and a series of pseudoplastic fluids of the Bingham type.

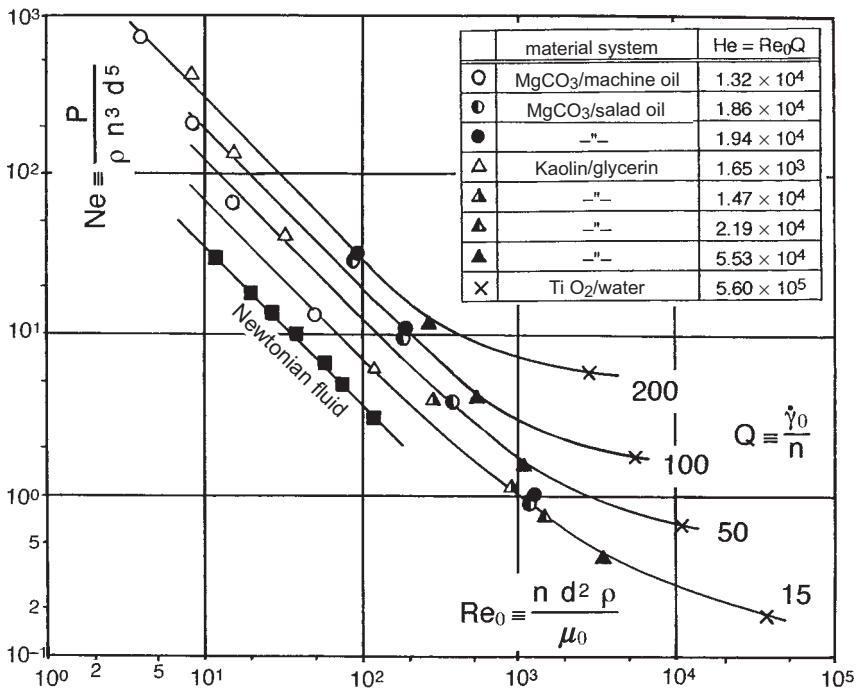


Fig. 23 Power characteristics of a double helical ribbon stirrer of a given geometry ($d/D = 0.95$; $h/d = 1$; $b/D = 0.1$; d – stirrer diameter; D – vessel diameter; b – ribbon width; $D = 20.3$ cm). All model fluids are of Bingham type; from [91], evaluated in [110].

Because both process numbers Re_0 and $Q \equiv n/\dot{\gamma}_0$ contain the stirrer speed, it is advisable to combine them in such a way that the stirrer speed is eliminated. The number consequently produced is:

$$B \equiv \frac{Re_0}{n/\dot{\gamma}_0} \equiv \frac{d^2 \rho \dot{\gamma}_0}{\mu_0} \equiv \frac{d^2 \rho \tau_0}{\mu_0^2} \quad (8.49)$$

Thus it follows:

$$Ne = f(Re_0, B) \quad (8.50)$$

Böhme and *Stenger* [12] have evaluated their results in this space. With regard to Re_{eff} they started from a wrong pi-space $\{Ne, Re_{eff}\}$, Fig. 22. They used the master curve in Fig. 20, the evaluation of which is founded on $\tau^* = 1 \text{ Pa} = \text{const.}$ Therefore in the model experiments, performed in differently shaped vessels ($\mu = 1 : 2$), $B = \text{idem}$ could be kept, if at doubling d μ_0 was also doubled. **Fig. 24** documents that at $B = \text{idem}$ the function $Ne = f(Re_{eff})$ has an identical course in differently shaped vessels.

Due to the positional congruence of the pseudoplastic and viscoelastic material function, Fig. 21, it is not surprising that in this representation the viscoelastic properties of the PAA solution do not need to be considered separately.

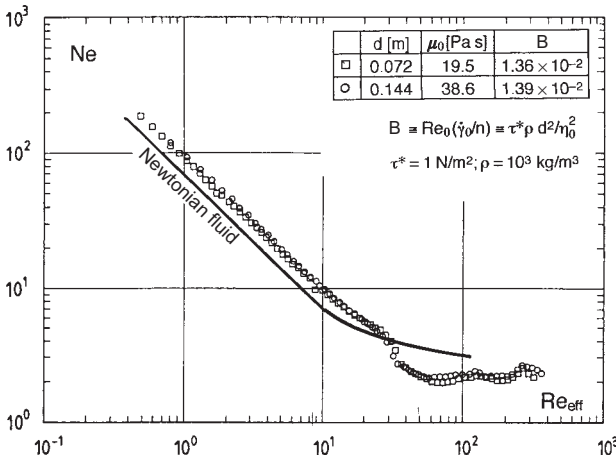


Fig. 24 Power characteristics $Ne(Re_{eff})$ of a turbine stirrer as in Fig. 21, determined in two geometrically similar mixing vessels of different shape (scale-up factor $\mu = 2.0$) with two aqueous PAA solutions of adopted viscosity ($B = \text{idem}$); from [12].

Example 19: Homogenization characteristics of a stirrer

For the duration θ which is the time needed for liquid homogenization (“mixing time”) of a given stirrer in a non-Newtonian liquid, the appropriate pi-space is analogous to that of the power demand, eq. (8.41):

$$n\theta = f(Re_0, n/\dot{\gamma}_0), \quad (8.51)$$

Its correctness is convincingly confirmed by measurements with aqueous HEC (hydroxy-ethyl-cellulose) solutions at a scale factor of $\mu = 1 : 2$ and with aqueous Carbopol solutions at a scale-factor of $\mu = 1 : 2 : 5.3$, see Fig. 25. Here, due to the positional congruence between pseudoplastic and viscoelastic properties it was not necessary to consider the latter by adding $f(Wi_0)$ separately.

It must be noted that, for the determination of mixing time by the chemical decolorization method, (“Method of the last color change”) the accuracy of measurements in relatively viscous fluids is reduced, due to the limited mobility of ions (see [184], section 3.3.2). On the other hand, measuring methods with selectively positioned sensors do not register the whole vessel contents.

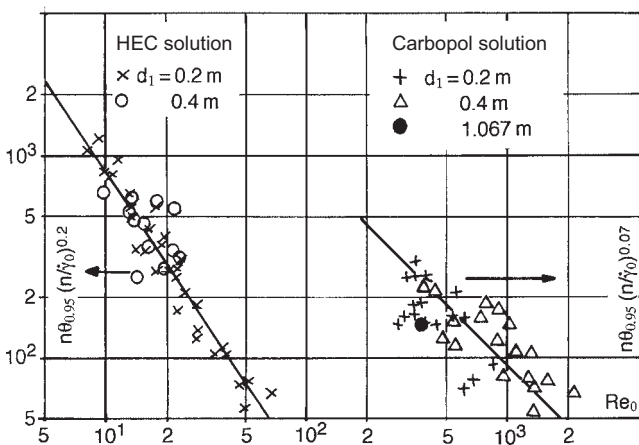


Fig. 25 Mixing time characteristics gained in three geometrically similar mixing vessels ($D = d_2 = 0.3; 0.6; 1.6$ m) with two different pseudoplastic fluids (above: HEC solutions, below: Carbopol solutions). A coaxial mixer arrangement of Stelzer/Warburg Co., rotating

countercurrently; $n_1 : n_2 = 4$, $d_2 : d_1 = 1.5$. Inside: pitched-blade stirrer (“Sigma stirrer”) with d_1 , outside: anchor stirrer with Teflon wiper blades. By courtesy of Henkel KGaA, Düsseldorf 2003.

8.5.2.3 Process Equations for Thermal Processes in Association with Non-Newtonian Fluids

As an example, the heat transfer characteristics of a smooth straight pipe of the diameter d are presented for non-Newtonian fluids. The superficial velocity v of the fluid is used. Position 1 is valid for the temperature independent and position 2 for the temperature dependent viscosity.

	a) Newtonian fluid	b) Pseudoplastic fluid	(8.52)
1	Nu, Re, Pr	$Nu, Re_0, Pr_0, v \dot{\gamma}_0/d, \Pi_{rheol} \triangleq m, Wi_0, \dots$	
2	$Nu, Re_0, Pr_0, \mu_w/\mu$	$Nu, Re_0, Pr_0, v \dot{\gamma}_0/d, \gamma_0 \Delta T, \Gamma, \Pi_{rheol}$	

Remarks:

- 1a) For the meaning of Nu and Pr see Example 5 in Section 4.
- 2a) Re_0 and Pr_0 are formed, as in 1 a), by the viscosity of the bulk liquid; for $\mu_w/\mu \triangleq \gamma_0 \Delta T$ – see Example 15.
- 2b) Re_0 and Pr_0 are formed by μ_0 at $\dot{\gamma}_0$ and T_0 , $\gamma_0 \Delta T$ at $\dot{\gamma}_0$;
 $\Gamma \equiv \partial \ln \dot{\gamma}_0 / \partial \ln \gamma_0$; $\Pi_{rheol} \triangleq m$

8.4.2.4 Scale-up in Processes with Non-Newtonian Fluids

Due to the fact that the rheological properties are usually not fully known, one is forced to perform model experiments with the same substance which is used in the full-scale process. Thanks to

$$\Pi_{\text{material}} \text{ (here } Pr_H, \Pi_{\text{rheol}}) = \text{idem}$$

the process therefore takes place in a pi-space which is enlarged by only one pi-number (namely $v\Theta/L$) with respect to the Newtonian case, see (8.52).

However, in the transition from model to full-scale, a complete similarity cannot be achieved. This is because in using the same material system $Re_H \equiv \rho v L/H = \text{idem}$, $v \Theta/L = \text{idem}$ cannot be ensured at the same time. It is recommended to use the same material system, but to change the model scale. An exception to this is represented by pure hydrodynamic processes in the creeping flow region (ρ irrelevant) under steady-state and isothermal conditions. Here mechanical similarity can be obtained in spite of constant physical properties; see Example 26: Single-screw machines.

9

Reduction of the Pi-space

Let us once more recall the statement of the pi-theorem (Section 2.7):

Every physical relationship between n physical quantities can be reduced to a relationship between $m = n - r$ mutually independent dimensionless groups, where r stands for the rank of the dimensional matrix, made up of the physical quantities in question and is generally equal to (or in some few cases smaller than) the number of the base quantities contained in their secondary dimensions.

In fact, this actually means that the pi-set could be reduced if one succeeds in enlarging the base dimensions of the dimensional system. However, it must be considered that in the enlargement (reduction) of a dimensional system the relevance list must also be enlarged (reduced) by the corresponding dimensional constant by which the number of the resulting pi-numbers is not changed. However, it can turn out that in the enlargement of the dimensional system the additional dimensional constant is, *a priori*, irrelevant to the problem. In this case, it need not be incorporated into the relevance list and the number of pi-numbers is, in fact, reduced by one.

9.1

The Rayleigh – Riabouchinsky Controversy

In his famous and extremely short essay entitled “The principle of similitude” Lord Rayleigh [118] discussed 15 different physical problems which can be condensed to laws by using dimensional analysis without performing any experiments. The last of these examples concerned the “*Boussinesq* problem” of the steady-state heat transfer from a fixed body to a ideal liquid flowing with velocity v .

He considered, in his own words, “somewhat in detail, *Boussinesq*’s problem of the steady passage of heat from a good conductor immersed in a stream of fluid moving (at a distance from the solid) with velocity v . The fluid is treated as incompressible and, for the present as inviscid, while the solid has always the same *shape* and presentation to the stream. In these circumstances the total heat, Q , passing in unit time is a function of the linear dimension of the solid, l , the temperature difference, ΔT , the stream velocity, v , the capacity for heat of the fluid per

unit volume, ρC_p , and the conductivity, k . The density of the fluid clearly does not enter into the question." Thus, we obtain the following relevance list:

$$\{Q, l, \Delta T, v, \rho C_p, k\} \quad (9.1)$$

These six quantities contain four base dimensions $[L, T, \Theta, H]$, wherein H means the amount of heat with the calorie as a measuring unit. According to the pi-theorem, a dependence between two pi-numbers will result. *Rayleigh* obtained the following two pi-numbers which are today named: The Nusselt number Nu and the Péclet number Pe , the latter being the product of Reynolds and Prandtl numbers, $Pe \equiv RePr$:

$$\frac{Q}{l k \Delta T} \equiv \frac{h l}{k} = f\left(\frac{l \rho C_p v}{k}\right) \rightarrow Nu = f(RePr) \quad (9.2)$$

Real liquids display a physical property which is named viscosity. Only after the kinematic viscosity, $\nu \equiv \mu/\rho$, is introduced can the product $RePr$ be taken apart:

$$\frac{Q}{l k \Delta T} \equiv \frac{h l}{k} = f\left(\frac{l \rho C_p v}{k}; \frac{\rho C_p \nu}{k}\right) \rightarrow Nu = f(Re/Pr, Pr) \rightarrow Nu = f(Re, Pr) \quad (9.3)$$

Out of 7 x-quantities and 4 base dimensions we now obtain $7 - 4 = 3$ pi-numbers.

Lord Rayleigh pointed out that one can deduce valuable information from the above interdependence (9.3) although it is an arbitrary function of the two variables. The latter of these appeared to be constant for any given kind of gas (compare with Example 5) and seemed to vary only moderately from one gas to another. Therefore, we are left with $Nu = f(Re)$; for each $v l = \text{const}$, so the functional dependence f remains unchanged.

Four months after this publication a "Letter to the Editor" of *D. Riabouchinsky* [122] appeared in *Nature*. He pointed out that *Lord Rayleigh* considered heat, temperature, length and time as four independent units. If we suppose that only three of these quantities are really independent, we obtain a different result. For example, if the temperature is defined as the mean kinetic energy of molecules, the principle of similitude allows us only to affirm that

$$\frac{Q}{l k \Delta T} = f\left(\frac{l \rho; C_p v}{k}, \rho C_p l^3\right) \quad (9.4)$$

If this is considered, then equation (9.2) is extended by another argument and, as a consequence, with two arguments is much more arbitrary than with one argument.

In his reply, *Lord Rayleigh* [119] referred this objection to the field of logic and explained that his conclusion followed on the basis of the usual Fourier equations of heat, in which heat and temperature are regarded as *sui generis*. He added: "It would indeed be a paradox if the further knowledge of the nature of heat afforded by molecular theory put us in a worse position than before in dealing with a particular problem. The solution would seem to be that the Fourier equations

embody something as to the nature of heat and temperature which is ignored in the alternative argument of Dr. Riabouchinsky.”

(*Lord Rayleigh* is completely right in his last sentence, as will be shown later.) After that, this problem was put aside as being unsolved and was called the “*Riabouchinsky paradox*”. Obviously, this was too much for engineers, and physicists, who were approached, were not interested. However, the solution to this problem is, in fact, rather easy.

It is correct to state that, according to the Theory of Gases, energy can be expressed as temperature. However, this is advantageous and reasonable only if the physical process is governed by molecular events. For macroscopic interrelations like *Boussinesq’s* problem, the molecular nature of the gas is irrelevant. Here, the microscopic parameters are replaced by mean values of the macroscopic ones, these appearing in measurable physical properties such as specific heat and heat conductivity. To equate energy with temperature as *Riabouchinsky* did, introduced irrelevant physics to the problem. See also the remarks of *L.I. Sedov* [132, p. 40+].

Furthermore, one has to bear in mind that relating energy with temperature implies the consideration of *Boltzmann’s* constant, **k**. However, this natural constant will play a role in a physical problem only if the molecular nature of matter is involved, otherwise it is irrelevant.

Example 20: Dimensional-analytical treatment of *Boussinesq’s* problem

In dealing with *Boussinesq’s* problem, *Lord Rayleigh* used the amount of heat *H* (measuring unit: calorie) as one of the, then-used, base dimensions. Only since the introduction of SI (*Système International d’Unités*) was it required to make no distinction between heat and mechanical energy, because both were considered to be equal. In order to comply with this requirement, the *Joule* equivalent of heat **J** [$\text{M L}^2 \text{T}^{-2} \text{H}^{-1}$] had to be introduced as a natural constant in the relevance list. If we proceed from the assumption of an “inviscid”, *ideal* liquid, no mechanical heat can be converted into heat. In this case, **J** is irrelevant.

a) Procedure of *Lord Rayleigh*

Relevance list:

target quantity:	amount of heat, <i>Q</i>
geometric parameters:	characteristic measurement of length, <i>l</i>
material parameters:	volume-related heat capacity, ρC_p heat conductivity, <i>k</i>
process-related parameters:	velocity of flow, <i>v</i> temperature difference, ΔT

$$\{Q, l, \rho C_p, k, \Delta T, v\} \quad (9.5)$$

	l	k	ΔT	ρC_p	Q	v
L	1	-1	0	-3	0	1
T	0	-1	0	0	-1	-1
Θ	0	-1	1	-1	0	0
H	0	1	0	1	1	0
L + 2T + 3H	1	0	0	0	1	-1
-T	0	1	0	0	1	1
$\Theta + H$	0	0	1	0	1	0
H + T	0	0	0	1	0	-1

Formation of pi-numbers:

$$\Pi_1 \equiv \frac{Q}{l \Delta T k} \equiv \text{Nu} \quad \Pi_2 \equiv \frac{v l \rho C_p}{k} \equiv \text{RePr} \equiv \text{Pe} \quad (9.6)$$

Conclusion: The dimensional analysis carried out fully proves *Lord Rayleigh's* accuracy.

b) Procedure of *D. Riabouchinsky*

Relevance list: {Q, l, ρC_p , k, v, ΔT , k}

Dimensional matrix:

	l	k	ΔT	ρC_p	Q	v	k
L	1	-1	0	-3	0	1	0
T	0	-1	0	0	-1	-1	0
Θ	0	-1	1	-1	0	0	-1
H	0	1	0	1	1	0	1
L + 2T + 3H	1	0	0	0	1	-1	3
-T	0	1	0	0	1	1	0
$\Theta + H$	0	0	1	0	1	0	0
H + T	0	0	0	1	0	-1	1

Formation of pi-numbers:

$$\Pi_1 \equiv \frac{Q}{l k \Delta T} \equiv \text{Nu} \quad \Pi_2 \equiv \frac{v l \rho C_p}{k} \equiv \text{RePr} \equiv \text{Pe} \quad \Pi_3 \equiv \frac{k}{l^3 \rho C_p} \equiv k^* \quad (9.7)$$

k^* is the pi-number, which *Riabouchinsky* would also have obtained, only it must be cancelled here because of the irrelevance of k . Consequently, we are left with the pi-set obtained by *Lord Rayleigh*.

Example 21: Heat transfer characteristic of a stirring vessel

In *Rayleigh's* treatment of *Boussinesq's* problem, we realized that Joule's heat equivalent had to be cancelled, because in this problem the liquid was supposed to be an ideal one. This is – of course – as with many problems arising in the areas of the mechanical and thermal process engineering, not the case. One only has to think of screw machines, where a large mechanical power input is transformed into heat. In such cases the J containing pi-number must not be disposed of. In this context, the heat transfer characteristics will be discussed, where from one part the dimensional system $\{M, L, T, \Theta\}$ and from the other $\{M, L, T, \Theta, H\}$ will be used.

Relevance list:

target quantity: heat transfer coefficient at the inner wall, h
 geometric parameter: vessel and stirrer diameters: D, d
 material parameters: density ρ ; viscosity μ ; temperature coefficient of viscosity, γ_0 ; heat capacity C_p ; heat conductivity, k
 process parameters: stirrer speed, n
 temperature difference, ΔT , between wall and liquid

The complete 10-parametric relevance list reads:

$$\{h; D, d; \rho, \mu, \gamma_0, C_p, k, n, \Delta T\} \quad (9.8)$$

a) Dimensional system $\{M, L, T, \Theta\}$

	ρ	d	n	ΔT	h	C_p	k	μ	D	γ_0
M	1	0	0	0	1	0	1	1	0	0
L	-3	1	0	0	0	2	1	-1	1	0
T	0	0	-1	0	-3	-2	-3	-1	0	0
Θ	0	0	0	1	-1	-1	-1	0	0	-1
M	1	0	0	0	1	0	1	0	0	0
$L + 3M$	0	1	0	0	3	2	4	2	1	0
$-T$	0	0	1	0	3	2	3	1	0	0
Θ	0	0	0	1	-1	-1	-1	0	0	-1

The following six pi-numbers are produced:

$$\Pi_1 = \frac{h \Delta T}{\rho d^3 n^3} \quad \Pi_2 = \frac{C_p \Delta T}{d^2 n^2} \quad \Pi_3 = \frac{k \Delta T}{\rho d^4 n^3} \quad \Pi_4 = \frac{\mu}{\rho d^2 n} \equiv \text{Re}^{-1}$$

$$\Pi_5 = D/d \quad \Pi_6 = \gamma_0 \Delta T$$

Pi-numbers Π_1 , Π_2 and Π_3 have an unusual appearance. We shall first combine Π_1 and Π_3 :

$$\Pi_1 \Pi_3^{-1} = h d/k \rightarrow h D/k \equiv \text{Nu} \text{ (Nusselt number)}.$$

Then, Π_2 will be transformed by Π_3 and Π_4 into a pure material number

$$\Pi_2 \Pi_3^{-1} \Pi_4^{-1} = C_p \mu/k \equiv \text{Pr} \text{ (Prandtl number)}.$$

Alas, we do not recognize the pi-number Π_3 and, therefore, have no idea about its significance. Otherwise, Π_4 is the inverse value of the Reynolds number and Π_5 and Π_6 speak for themselves.

The dimensional analysis performed with the dimensional system $\{M, L, T, \Theta\}$ delivers, for the heat transfer characteristic of a stirring vessel, the following 6-parametric pi-set:

$$\{\text{Nu}, \text{Pr}, \Pi_3, \text{Re}, \gamma_0 \Delta T, D/d\} \quad (9.9)$$

where the significance of the pi-number Π_3 is not obvious.

b) Dimensional system $\{M, L, T, \Theta, H\}$

	ρ	d	n	ΔT	k	h	C_p	J	μ	D	γ_0
M	1	0	0	0	0	0	-1	1	1	0	0
L	-3	1	0	0	-1	-2	0	2	-1	1	0
T	0	0	-1	0	-1	-1	0	-2	-1	0	0
Θ	0	0	0	1	-1	-1	-1	0	0	0	-1
H	0	0	0	0	1	1	1	-1	0	0	0
M	1	0	0	0	0	0	-1	1	1	0	0
$3M + L + H$	0	1	0	0	0	-1	-2	4	2	1	0
$-T - H$	0	0	1	0	0	0	-1	3	1	0	0
$\Theta + H$	0	0	0	1	0	0	0	-1	0	0	-1
H	0	0	0	0	1	1	1	-1	0	0	0

The following six pi-number result from this:

$$\Pi_1 \equiv \frac{h d}{k} \equiv \text{Nu} \quad \Pi_2 \equiv \frac{C_p \rho d^2 n}{k} \quad \Pi_3 \equiv \frac{J \Delta T k}{\rho d^4 n^3}$$

$$\Pi_4 \equiv \frac{\mu}{\rho d^2 n} \equiv \text{Re}^{-1} \quad \Pi_2 \Pi_4 \equiv C_p \mu / k \equiv \text{Pr} \quad \Pi_5 \equiv \frac{D}{d}$$

$$\Pi_6 \equiv \gamma_0 \Delta T \quad \Pi_3^{-1} \Pi_4 \equiv \frac{\mu d^2 n^2}{J \Delta T k} \equiv \text{Br}$$

In both dimensional systems the pi-number Π_3 has been produced which, after combination with the Reynolds number, can be recognized as the Brinkman number Br. As long as the heat production can be neglected as compared with the heat removal, Π_3 and Br, respectively, also remain negligible and can therefore be deleted. The complete pi-set then reads:

$$\{\text{Nu}, \text{Pr}, \text{Re}, D/d, \gamma_0 \Delta T\} \quad (9.10)$$

In spite of the fact that both dimensional systems yield the same pi-sets, it is the second one which allows, by the appearance of J in one of the pi-numbers, its interpretation and the decision about its relevance.

10

Typical Problems and Mistakes in the Use of Dimensional Analysis

Occasionally, one could have read about the *failures* of the Theory of Similarity or of its *limits*. However, this criticism has arisen when, due to some physical reasons, a complete similarity could not be achieved (see, e.g., the remarks of *Damköhler* [21] on page 218) or the scale-up criterion could not have been worked out with certainty because the measuring conditions did not allow it (false model scale, required sensitivity of the target quantity, non-availability of the model material system, ignorance about relevant physical properties, such as in foams and sludges, etc.).

When problems arise in model experiments for the above mentioned reasons, one cannot put the blame on an epistemologically and mathematically sound method.

The problems in connection with the non-availability of a model material system have already been discussed in Section 6.1. It has been shown how these problems can be handled by model measurements in differently scaled pieces of equipment. Only in cases where this option is too expensive (e.g., ship-building), can one depend on methods employing conditions of partial similarity (Section 6.2).

In contrast, this chapter deals with widespread problems related to measuring techniques. The majority of them have been taken from the area of stirring technology with which the author is particularly familiar, see [183] and [184].

10.1

Model Scale and Flow Conditions – Scale-up and Miniplants

Model experiments are customarily performed in laboratories and the size of the laboratory device will depend on the laboratory premises. Occasionally this can cause a problem with the flow behavior in the experimental device because it is extremely scale-dependent. It is interesting to observe that this situation has never been recognized as a problem by the universities.

In this chapter the principal aspects, which concern the model scale and the scale-up, will be summarized, see also [188].

10.1.1

The Size of the Laboratory Device and Fluid Dynamics

Within this viewpoint we are considering two of the most frequently used reactor types, the bubble column and the mixing vessel.

The state of flow in bubble columns is governed by the superficial velocity v_G of the gas, $v_G \equiv q_G/S$ (q_G – gas throughput; S – cross-sectional area of the column). With column diameters of $D \leq 8$ cm and enhanced values of v_G , so-called “piston bubbles” have been reported (essentially these are big bubbles formed by coalescence of tiny ones) that fill up the whole cross-sectional area S of the column. There is no doubt that such a state of flow cannot exist in an industrial bubble column of $D > 0.3$ m.

The state of flow is strongly scale-dependent. In *bubble columns* it is described by the extended Froude number (v_G – superficial velocity of the gas; g – acceleration due to gravity)

$$Fr' \equiv \frac{v^2 \rho_L}{D g \Delta \rho}$$

and in *mixing vessels* it is given by the Reynolds number

$$Re \equiv \frac{n d^2}{\nu}$$

The adjustment of $Fr'_M = Fr'_T$ (subscripts: M – model; T – technological, industrial scale) in small bubble columns may possibly cause problems due to the appearance of piston bubbles:

$$Fr'_M = Fr'_T = (v^2/D)_M = (v^2/D)_T \rightarrow v_M = v_T \sqrt{\mu^{-1}}$$

Assuming a customary industrial $v_T \approx 300$ m/h and a scale-up factor of $\mu \equiv D_T/D_M = 20$, we obtain $v_M = 67$ m/h. With $D_M = 0.05$ m this gives a gas throughput of $q_G = 130$ l/h.

The adjustment of $Re_M = Re_T$ in small mixing vessels causes difficulties due to the fact that the numerical value of Re is changed by d^2 . With a scale-factor $\mu \equiv d_T/d_M$ of, e.g., $\mu = 10$ and preserving the viscosity ν , laboratory measurements would have to be performed at extreme stirrer speeds:

$$(n d^2)_M = (n d^2)_T \rightarrow n_M = n_T \mu^2 \rightarrow n_M = 100 n_T$$

These circumstances can cause grave problems in the study of scale-dependent mixing operations, such as the homogenization of miscible liquids (determination of the desirable or required mixing time) and the suspension of solids in liquids, see [184].

10.1.2

The Size of the Laboratory Device and the Pi-space

It goes without saying that $Re_M = Re_T$ can be achieved in a laboratory device by intensive stirring. But then high shear stress will result in a high power dissipation per volume (P/V) which in turn is converted to heat. Then the pi-space will have to be extended by an additional pi-number, namely the Brinkman number Br :

$$Br \equiv \frac{n^2 d^2 \mu}{J \Delta T k}$$

(μ – dynamic viscosity; k – thermal conductivity; J – *Joule's* mechanical heat equivalent)

Result: pi-spaces in the laboratory and in an industrial device are not equal.

In discussing the power characteristics of a leaf stirrer (Example 22) it has been pointed out that pi-spaces differ for the creeping/laminar and for the turbulent flow regime. In the laminar flow range ($Re < 20$) the fluid density ρ is irrelevant, whereas in the turbulent flow range the viscosity is irrelevant. It has also been stressed that due to the dependence $Re \propto d^2$, a fully turbulent flow regime ($Re \geq 10^5$) cannot be adjusted in a laboratory scale ($D \ll 10$ cm) even if a liquid with the viscosity of water is treated.

The two subsequent examples concern heat transfer. They emphasize how the pi-space is influenced by the size of the laboratory device, which in turn is a determining factor for the state of flow.

In a creeping state of flow ($Re < 10^1$) fluids behave as solid bodies. The viscosity and the density have no influence. The customary pi-space $\{Nu, Re, Pr\}$ reduces to $\{Nu, RePr\}$:

$$Pe \equiv RePr \equiv \frac{n d^2 \rho}{\mu} \frac{\eta C_p}{k} \equiv \frac{n d^2 \rho C_p}{k} \equiv \frac{n d^2}{a}$$

(Nu – Nusselt number; Pr – Prandtl number; Pe – Péclet number; ρC_p – volume related heat capacity; k – thermal conductivity, a – thermal diffusivity).

In the combination $Pe \equiv RePr$ the viscosity is cancelled and the density ρ is linked to the mass-related heat capacity C_p to produce the volume-related heat capacity ρC_p .

The diametrically opposed example concerns the natural heat convection in storage vessels. In mixing vessels of usual size ($V \leq 50$ m³) the heat transfer is effected by stirrers (forced convection) and the process is depicted by the pi-space $\{Nu, Re, Pr\}$. However, for storage vessels ($V > 1000$ m³) outdoors, natural convection constitutes a contributing factor since the radiant heat of the sun leads to temperature gradients in the liquid. The natural convection is taken into account by the Grashof number Gr :

$$\text{Gr} \equiv \frac{g \beta \Delta T D^3}{\nu^2}$$

(β is the temperature coefficient of density). The corresponding pi-spaces read:

$$\{\text{Nu}, \text{Gr}, \text{Pr}\} \text{ and } \{\text{Nu}, \text{Re}, \text{Gr}, \text{Pr}\}, \text{ respectively.}$$

If a pipe is scaled-down to a capillary – as is customary in micro reactors – then the creeping flow ($\text{Re} \ll 1$) is necessarily affected. If capillaries are open to the environment (air), then capillary forces will arise due to the influence of the surface tension σ and the pi-space should be extended by the Weber number We :

$$\text{We} \equiv \frac{\rho v^2 D}{\sigma}.$$

10.1.3

Micro and Macro Mixing

According to the statistical Theory of Turbulence (*Kolmogorov*) the stirrer causes primary macro eddies. Their dimensions λ correspond to the diameter d of the stirrer head:

$$\lambda \propto d.$$

These eddies subsequently disintegrate, due to the liquid friction forces, into a spectrum of smaller and smaller eddies, until the dimension λ of the smallest eddy is reached. λ is called Kolmogorov's micro scale of turbulence. Its size depends solely on the kinematic viscosity of the liquid ν and on the power per unit mass $\varepsilon \equiv P/\rho V$:

$$\lambda = (\nu^3/\varepsilon)^{1/4}$$

Mixing operations, which depend on *macro* mixing (homogenization, suspension of solids from the vessel bottom), are therefore extremely scale-dependent. The state of flow in these operations is described by process numbers (Re , Fr), whose constituents contain *extensively formulated* quantities like the diameter of the device (vessel diameter D and stirrer diameter d , respectively) and process parameters like the superficial velocity v and the stirrer speed n .

Emulating such processes in the laboratory devices will, without doubt, cause great difficulty and in fact is completely impossible in mini plants, see Section 10.1.1.

In contrast, processes depending on *micro* mixing (dispersion processes in the systems L/L and G/L) are described by pi-numbers like $(P/V)^*$ and $(v \equiv q/S)^*$ which are comprised solely of *intensively formulated* quantities (P/V – power per

liquid volume; v – superficial velocity of the fluid). These processes are scale-independent. Therefore, they can definitely also be reliably emulated in mini plants.

In the dispersion process of the L/L system, the targets consist of the desirable particle or drop diameter d_p and of a narrow particle size distribution. To achieve the desired drop diameter d_p , the micro scale of turbulence λ should be of the same magnitude ([184], Section 6.2.5). From this it follows:

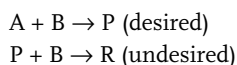
$$\lambda \approx d_p = (v^3/\varepsilon)^{1/4}.$$

The power per volume of liquid (P/V) necessary to achieve the desired drop diameter can be calculated from this relationship.

10.1.4

Micro Mixing and the Selectivity of Complex Chemical Reactions

It has often been overlooked that the steps in *transport-limited* chemical reactions in a production plant often proceed differently from those in the laboratory device. The selectivity of complex chemical reactions of the competitive-consecutive type



strongly depends on the degree of micro mixing. Due to the fact that the micro mixing is far more distinct on a small scale than it is in a technological plant, one should not be surprised if the selectivity of the reaction, achieved in the laboratory, cannot be met in the industrial plant.

These facts are well known from the textbooks of Chemical Reaction Engineering. The complex chemical reactions (e.g., azo-dye reactions) have been used for years in fundamental research in mixing [184, Section 1.4.6] to gain quantitative information about the extent of micro and macro mixing from the selectivity of the reaction.

10.1.5

Mini and Micro Plants from the Viewpoint of Scale-up

The above discussion gives a critical view of contemporary efforts to simulate entire processes on a mini and micro scale.

Mini plants are without doubt suitable for investigating the feasibility of a synthesis on a very small scale. All process steps, which proceed in a homogeneous phase and those depending on *micro* mixing, will be correctly executed and the experimental conditions can be used for a reliable scale-up.

Scale-dependent process steps will also be properly executed on the mini scale because their scale dependence cannot be visualized on the mini scale. But in this case the experimental conditions can by no means be used for a reliable scale-up.

In other words:

As a rule, mini plants are not suitable for obtaining scale-up rules for scale-dependent process steps.

In contrast to this, *micro plants* are not at all suitable for scaling-up by keeping the geometrical similarity because they would automatically lose their advantages (i.e., smallest mixing times, extreme heat transfer rates). An enlargement of these devices is only possible by coupling individual elements in parallel or in series [50, 156].

10.2

Unsatisfactory Sensitivity of the Target Quantity

Due to the highly developed field of chemical process engineering, there are possibly only a few targets left whose sensitivity is not satisfactory. These are from the area of stirring technology and two of them will be presented in the following as examples.

10.2.1

Mixing Time θ

In the leveling out of concentration differences in mutually soluble liquids to a molecularly homogeneous level by mixing (“homogenization”), the necessary homogenization or mixing time θ [T] is visually determined by a chemical decolorization method (e.g., neutralization reaction with phenolphthalein as indicator), see also Example 22. In the turbulent flow range ($Re > 10^4$) in the laboratory vessel, the mixing time only accounts for a few seconds. Because of this, the measuring accuracy and the reliable determination of the scale-up rule suffer.

In order to obtain a reliable scale-up rule one would have to perform measurements in larger vessels ($D > 1$ m) [154]. Unfortunately, in this case, this otherwise outstanding and comprehensive decolorization method fails completely: thick water layers glimmer pale blue and the white painted vessels make perception of the color change difficult. Consequently, in this field, computational fluid dynamics (CFD) could possibly provide a solution.

10.2.2

Complete Suspension of Solids According to the 1-s Criterion

A further exceedingly important mixing operation consists of whirling up solid particles (“suspension of solids”) to make their surfaces completely accessible to the surrounding liquid (dissolution of salts, solid catalyzed reactions in a S/L/G system, and so on). To work out the criteria important for this task, research was concentrated on measuring the “critical stirrer speed” necessary for the flow state in which no particle lingered longer than 1 second on the bottom of the vessel.

First, it has to be taken into account that criteria which characterize a state of flow have to be formulated in a dimensionless manner. In his experiments to determine the drag resistance of a ship's hull *W. Froude* had already found that the bow wave could only be reliably determined when the size of the ship model had the right proportions with respect to the travelling speed and channel width.

In the case of "1-s criterion", we are dealing with an easy to allocate, but an inaccurate quantity with a low sensitivity. Who is able to decide, by purely visual means, whether the time was 0.7 s or 1.3 s? In any case, who is able to use this criterion in a vessel of $D > 0.5$ m?

However, the fact that this criterion is not dimensionless is not of great importance because of its low sensitivity.

If one would chose a dimensionless time, e.g. $t^* \equiv t (g/D)^{1/2}$ instead of t [T], no advantage would be gained (see **Table 4**). In this representation, for four differently sized vessels of $D = 0.2$ – 2.0 m, the left-hand side indicates how the pi-number, t^* , is changed for $t = 1$ s. In the transition from $D = 0.2 \rightarrow 1.0$ m, t^* is changed only by a factor of 2.2. The right-hand side of the table shows how the 1-s-criterion should be changed in scale-up to satisfy $t^* = \text{const.}$

Table 4 Correspondence between 1-s criterion and the time number $t^* \equiv t (g/D)^{1/2}$ when D is changed by a scale factor of $\mu = 1:10$

D [m]	t [s]	t^*	t^*	t [s]
0.2	1	7.07	7.07	1.00
0.5	1	4.47	7.07	1.58
1.0	1	3.16	7.07	2.23
2.0	1	2.23	7.07	3.17

By the introduction of the 90 % "layer height ratio", $h^* \equiv h_s/H = 0.9$ (h_s – height of the suspension layer, H – liquid height in the vessel), a convenient attenuature to the 1-s criterion has been found; see [184].

10.3

Model Scale and the Accuracy of Measurement

In Section 7.3 (Experimental Techniques for Scale-up), it has already been mentioned that the model size depends on the scale factor $\mu \equiv l_T/l_M$ and on the measuring accuracy attainable in the tests. At $\mu = 10$ a measuring accuracy of ± 10 % will possibly not suffice and one will therefore have to chose a larger model scale to reduce the scale factor μ .

Measuring accuracy often depends on the model scale. This is demonstrated by two examples taken from the area of stirring technology.

10.3.1

Determination of the Stirrer Power

Before a torsion shaft with strain gauges was employed to determine the torque on a rotating stirrer shaft, a swiveling motor had to be used and the torque measured by weighing.

In the latter case, friction loss in the motor and shaft bearings was automatically included in the measurement. In small stirrers, this was of the same order of magnitude as the power consumption of the stirrer. Therefore, at that time, rapidly rotating stirrers had to have diameters of $d > 10$ cm.

10.3.2

Mass Transfer in Surface Aeration

The determination of the overall mass transfer coefficient, $k_L a$, using O_2 electrodes is accurate to $\pm 5\%$. This suffices to determine the process characteristic in the so-called volume (bulk) aeration in stirring vessels, as has been demonstrated by a comparative evaluation of measurements in vessel sizes covering liquid volumes from $V = 2.5$ liters to 906 m^3 [56], see Fig. 77.

Compared with this, so-called surface aeration brings about a comparatively modest oxygen uptake. Therefore, it is important to measure very accurately and/or to use a larger model scale. The latter is advisable because the diameters of the full-sized surface aerators amount to $d_T \geq 3$ m and therefore the scale factor surpasses $\mu \approx 10$, even if a relatively large laboratory stirrer diameter of $d_M = 0.3$ m is used.

In surface aeration, the absorption rate is also compulsorily measured with O_2 electrodes in the liquid volume. By this method, the liquid-side overall mass transfer coefficient, $k_L a$, is determined (a is the volume-related mass transfer area = the surface of all gas bubbles in the liquid volume). Due to the fact that the mass transfer in surface aeration occurs almost solely in the liquid surface, A , and not in the liquid volume, V , the measured $k_L a$ has to be multiplied by V to obtain the target quantity $k_L A = k_L a V$. (During the years 1960–1980 these facts were not known to civil engineers who were responsible for the municipal waste water treatment plants. As a result, innumerable plants equipped with surface aerators were completely wrongly dimensioned.)

The scale-up rules for surface aerators will be presented in Example 40. Here, only the impact of the measuring accuracy of the O_2 electrodes in the 70s, of ± 10 – 20% on the result, will be discussed.

The evaluation of own and foreign measurements [172, 174], which were performed on a laboratory scale of $d_M = 0.09$ – 0.27 m, showed (Fig. 76) that, due to the scattering of the measuring data, it could not be decided with certainty whether

the correspondence $(k_L A)^* = f(Fr)$ holds or whether an additional pi-number, containing d , plays a role.

If the correlation $(k_L A)^* = f(Fr)$ is taken as valid, then this leads, in association with the known power characteristic of the surface aerators, to the devastating result that the efficiency E [kg O₂/kWh] of the surface aerators *diminishes* proportionally to the square-root of the scale factor:

$$E_T = E_M \mu^{-1/2} \quad (10.2)$$

In the case of a realistic scale factor of $\mu = 10$, this means that the efficiency of the full-scale device would only be 32 % that of the model scale.

Not until the extremely accurate measurements of *Schmidtke and Horvat*, see [174], were published and, thereafter, evaluated by the author in the same pi-frame (Fig. 76), was it possible to guarantee that the sorption characteristic is not given by $(k_L A)^* = f(Fr)$. It thus turned out that an additional pi-number (the Galilei number $Ga \equiv Re^2/Fr \equiv d^3 g/\nu^2$), containing the stirrer diameter, d , had to be introduced in order to complete the correlation in a satisfactory manner. In plotting

$$(k_L A)^* Ga^{-0.115} = f(Fr) \quad (10.3)$$

the straight lines for different diameters coincide, whereas the correlation of the measuring points shown in Fig. 76 by different signs is not worth mentioning. This proves that their scattering is due to the inaccuracy of the measuring method.

From the accurate correlation, eq. (10.3), it follows that a completely different interrelation between the efficiency E [kg O₂/kWh] and the scale factor exists:

$$E_T = E_M \mu^{-0.155} \quad (10.4)$$

This states that, at a scale factor of $\mu = 10$, the surface aerator would still have an efficiency of 70 % of that measured on a laboratory device.

10.4

Complete Recording of the Pi-set by Experiment

In this section the following question has to be clearly answered:

“Under which circumstances does a pi-relationship hold true if certain x-quantities are not varied in the respective experiments?”

This important question has been answered by *Pawłowski* [107] as follows:

- A pi-relationship is *not* secured if a pi-number can be produced from parameters, which were not altered in the experiments, so that this pi-number remained unaltered.

This is illustrated by two examples:

- a) The pertinent relevance list for the heat transfer at the wall of a stirring vessel reads:

$$\{h; d; \rho, \nu, C_p, k; n\}$$

and the corresponding pi-set follows as:

$$\{Nu, Re, Pr\} \quad Nu \equiv h d/k, \quad Re \equiv n d^2/\nu, \quad Pr \equiv \rho C_p \nu/k$$

Under which test conditions is this pi-relationship secured?

Not varied x-parameters	Varied x-parameters	Pi-numbers remaining constant	Result
physical properties	d, n	Pr	negative
d, n	physical properties	no	positive

- b) Stirring power in an unbaffled vessel:

$$\{P; d; \rho, \nu; n, g\}$$

and the corresponding pi-set:

$$\{Ne, Re, Fr\} \quad Ne \equiv P/(\rho n^3 d^5), \quad Re \equiv n d^2/\nu, \quad Ga \equiv g d^3/\nu^2$$

Under which test conditions is this pi-relationship secured?

Not varied x-parameters	Varied x-parameters	Pi-numbers remaining constant	Result
d, g, ρ, ν	n	Ga	negative
d, n, g	physical properties	Fr	negative
d, g	physical properties, n	no	positive
g, physical properties	n, d	no	positive

10.5

Correct Procedure in the Application of Dimensional Analysis

In this section, a summary will be provided about how dimensional analysis can be properly utilized in order to obtain, with a minimum of time and expenditure, the results necessary for the determination of both the pi-relationship (process characteristic) of the process in question and a valid scale-up rule. In this context, nothing new will be imparted. All of this information has already been disclosed in previous chapters.

10.5.1

Preparation of Model Experiments

Compiling the relevance list

Dimensional analysis should be carried out *before* the experiments are executed. The relevance list should be compiled as completely as possible and, on its foundation, the pi-set can then be determined. Depending on the circumstances, some orientating preliminary experiments will be necessary in order to verify or falsify the relevance of one or other x-quantity.

Decision about the size of the model apparatus

The model scale depends on the size of the full-scale device and on the achievable measuring accuracy. Low measuring accuracy or a target quantity displaying a low sensitivity can possibly be counteracted by increasing the scale factor μ , in other words, the model scale should be as large as possible.

10.5.2

Execution of Model Experiments

It has already been pointed out that in model experiments the pi-number rather than the x-quantity should be varied. This results in various advantages. On the one hand, the pi-number is varied by changing the most available, the most manageable or the cheapest x-quantity constituting it (example: changing the Reynolds number by varying the kinematic viscosity of the fluid). In addition, the evaluation of the test results is made easier, because in varying a certain pair of pi-numbers, the numerical values of all the other pi-numbers remain constant ($\Pi_i = \text{idem}$).

10.5.3

Evaluation of Test Experiments

The evaluation of the test experiments will be shown using an example (Example 35: "Gas hold-up in bubble columns"). It must be noted that this purposeful approach is often not complied with. It is simply false to plot a pi-number Y as a function of a pi-number X and to note that, in doing so, the one or the other

dimensional quantity remained constant. In the representation $Y(X)$ all remaining pi-numbers have to be constant ($\Pi_i = \text{const}$)!

The author is aware of the fact that numerical data recording and processing enjoys great popularity these days. Numerical and graphical data processing can go well, but sometimes it is bound to go wrong.

A serious disadvantage of automated data processing is the fact that the researcher is deprived of those hours of leisure, in which he or she plots results in silence and seclusion on the double-log scale and this opportunity is used to think over whether or not the data evaluated in this way make sense or if they should rather be represented in another way. No graphical representation of test data is, in the first instance, more suitable than that on the double-log scale. This approach shows better and more quickly whether it is suitable for the description, or whether a simple-log scale would possibly fit better. Furthermore, curves on the double-log scale can easily be converted into straight lines ($Y = aX^a \pm b$) or simple analytical expressions of the form ($Y = aX^a \pm bX^b$). Of course, the statistical balancing of the coefficients and exponents can be left to the computer afterwards.

11

Optimization of Process Conditions by Combining Process Characteristics

In this chapter, three examples are introduced to exemplify the possibility of optimizing a process with respect to the desirable objective by the appropriate combination of process characteristics. A fourth example of this kind is presented at the end of Example 42, dealing with the efficiency of the oxygen uptake in a tower-shaped biological waste-water treatment plant.

Example 22: Determination of stirring conditions in order to carry out a homogenization process with minimum mixing work

In order to design and dimension stirrers for the homogenization of liquid mixtures – and this is by far the most common task when it comes to stirring! – it is vital to know the *power characteristic* and the *mixing time characteristic* of the type of stirrer in question. If this information is available for various types of stirrers, it is possible to determine both the best type of stirrer for the given mixing task and the *optimum operating conditions* for this particular type.

a) *Power characteristic* of a stirrer

The power consumption, P , of a given type of stirrer under the given installation conditions depends on the following variables:

- | | |
|-----------------------|--|
| geometric parameters: | stirrer diameter, d |
| physical properties: | density, ρ , and kinematic viscosity, ν , of the liquid |
| process parameters: | stirrer speed, n . |

The relevance list therefore reads as follows:

$$\{P; d; \rho; \nu; n\} \quad (11.1)$$

The (appropriately assembled) dimensional matrix undergoes only one linear transformation to produce the two pi-numbers (Ne – Newton number; Re – Reynolds number):

$$\Pi_1 \equiv \frac{P}{\rho d^5 n^3} \equiv Ne \quad \text{and} \quad \Pi_2 \equiv \frac{\nu}{d^2 n} \equiv Re^{-1} \quad (11.2)$$

	ρ	d	n	P	ν
M	1	0	0	1	0
L	-3	1	0	2	2
T	0	0	-1	-3	-1
M	1	0	0	1	0
L + 3 M	0	1	0	5	2
-T	0	0	1	3	1

The experimentally determined *power characteristic*, $Ne(Re)$, of a blade stirrer of a given geometry under the given installation conditions (see sketch in Fig. 28) is presented in Fig. 26.

From this representation a number of important points can be deduced:

1. $Re < 20$: $Ne \propto Re^{-1}$ and $NeRe \equiv P/(\mu n^2 d^3) = \text{const}$, respectively, is valid. The density is irrelevant here – we are dealing with the *creeping flow* region.
2. $Re > 50$ (vessel with baffles) or $Re > 5 \times 10^4$ (unbaffled vessel): $Ne \equiv P/(\rho n^3 d^5) = \text{const}$ is valid. In this case, the viscosity is irrelevant, we are dealing with a *turbulent flow* region.
3. The influence of the baffles is, understandably, nil in the laminar flow region. However, it is extremely strong at $Re > 5 \times 10^4$. The installation of baffles under otherwise unchanged operating conditions increases the stirrer power by a factor of 20 here!

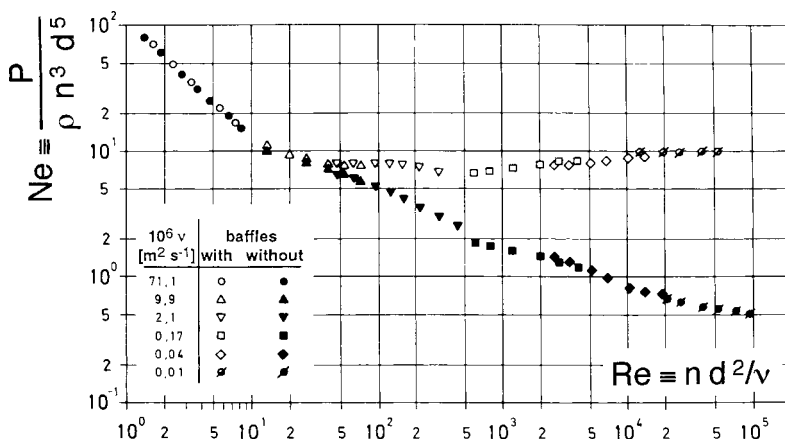


Fig. 26 Power characteristic of a blade stirrer of a given geometry under the given installation conditions; from [164].

4. When stirred in an unbaffled vessel, fluid begins to circulate and a vortex is formed. The question whether gravitational acceleration, g , and hence the Froude number, $Fr \equiv n^2 d/g$, plays a role under these circumstances can be safely answered as being negative on the basis of the test results shown in Fig. 26: For confirmation, one only needs to look at the points on the lower $Ne(Re)$ curve where the *same* Re value was set for fluids with *different* viscosities. This was only possible by a proportional alteration of the rotational speed of the stirrer. Where $Re = idem$, Fr was clearly not $idem$, but this has no influence on Ne : g is therefore irrelevant!

b) *Mixing time characteristic* of a stirrer

For our purposes, the mixing time, θ , is the time required to mix two fluids of similar density and viscosity until they are molecularly homogeneous. (See [184] for the practical determination of this quantity). In this case, the relevance list is as follows:

$$\{\theta; d; \rho, \nu, D; n\}$$

where $D[L^2 T^{-1}]$ is the diffusivity of one fluid into the other.

Dimensional analysis of this example is associated by a reduction of the rank of the matrix, because the base dimension of mass is only contained in the density, ρ . From this it does not follow that the density would not be relevant here, but that it is already fully considered in the kinematic viscosity ν , which is defined by $\nu = \mu/\rho$. Therefore

$$\{n\theta; Re \equiv nd^2/\nu; Sc \equiv \nu/D\} \quad \text{resp.} \quad n\theta = f(Re, Sc) \quad (11.4)$$

represents the appropriate pi-space here. The corresponding pi-relationship, the *mixing time characteristic* of the blade stirrer for the geometric conditions given in Fig. 28, is shown in Fig. 27.

The measured values demonstrate, in analogy with point 4 where the power characteristic was discussed, that D , and consequently the Schmidt number, are not as relevant as assumed. (In the same system of materials – a mixture of water and cane syrup – at almost the same D value, the Schmidt number is varied by the kinematic viscosity, ν , over many orders of magnitude.)

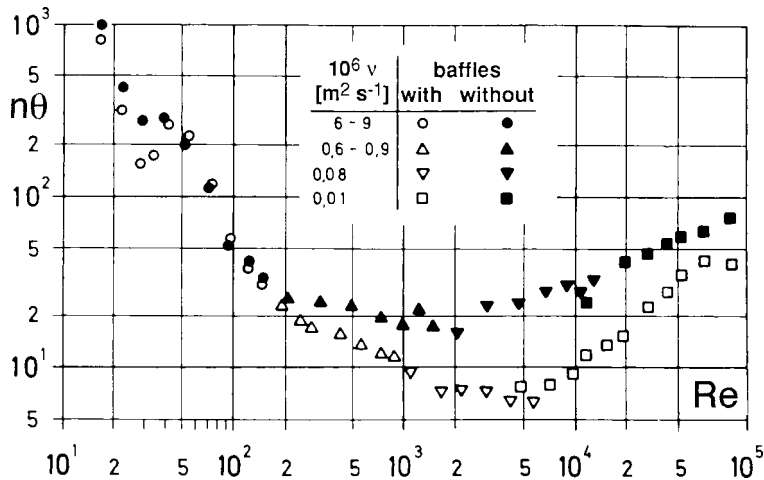


Fig. 27 Mixing time characteristic of a blade stirrer. For installation conditions see Fig. 28; from [164].

c) *Optimum conditions* for the homogenization of liquid mixtures

If the power and mixing time characteristics are known for a series of common stirrer types under favorable installation conditions, see [164], one can go on to consider the optimum operating conditions by asking the question: Which type of stirrer operates within the desired mixing time θ with the lowest power consumption and hence the minimum mixing work ($P\theta = \min$) in a given system of materials and a given vessel (vessel diameter D)?

In answering this question, we need not (at least for the moment) consider the diameter of the stirrer or its speed; the relevance list is as follows:

$$\{P, \theta; D; v, \rho\} \quad (11.5)$$

When the dimensional matrix has been assembled it becomes:

$$\Pi_1 \equiv \frac{P D}{\rho v^3} = \frac{P D \rho^2}{\mu^3} \quad \text{and} \quad \Pi_2 \equiv \frac{\theta v}{D^2} = \frac{\theta \mu}{D^2 \rho} \quad (11.6)$$

	ρ	D	v	P	θ
M	1	0	0	1	0
L	-3	1	2	2	0
T	0	0	-1	-3	1
M	1	0	0	1	0
$L + 3 M + 2 T$	0	1	0	-1	2
$-T$	0	0	1	3	-1

These two pi-numbers can be formed from the known numerical values of Ne , $n\theta$ and Re with the help of D/d . The following interrelations exist:

$$\Pi_1 \equiv Ne Re^3 D/d \quad \text{and} \quad \Pi_2 \equiv n\theta Re^{-1} (D/d)^{-2} \quad (11.7)$$

Fig. 28 shows this relationship for those stirrer types exhibiting the lowest Π_1 values within a specific range of the dimensionless number Π_2 , i.e., the stirrers requiring the least power in this range.

This graph is extremely easy to use. The physical properties of the material system, the diameter of the vessel (D) and the desired mixing time (θ) are all known and this is enough to generate the dimensionless number Π_2 . The curve $\Pi_1 = f(\Pi_2)$ in Fig. 28 then provides the following information:

1. The stirrer type and baffling conditions can be read off the abscissa. The diameter of the stirrer can be determined from data on the stirrer geometry in the sketch.
2. The numerical value of Π_1 can be read off at the intersection of the Π_2 value with the curve. The power consumption, P , can then be calculated from this.
3. The numerical value of Re can be read off the Re scale at the same intersection. This, in turn, makes it possible to determine the stirrer speed.

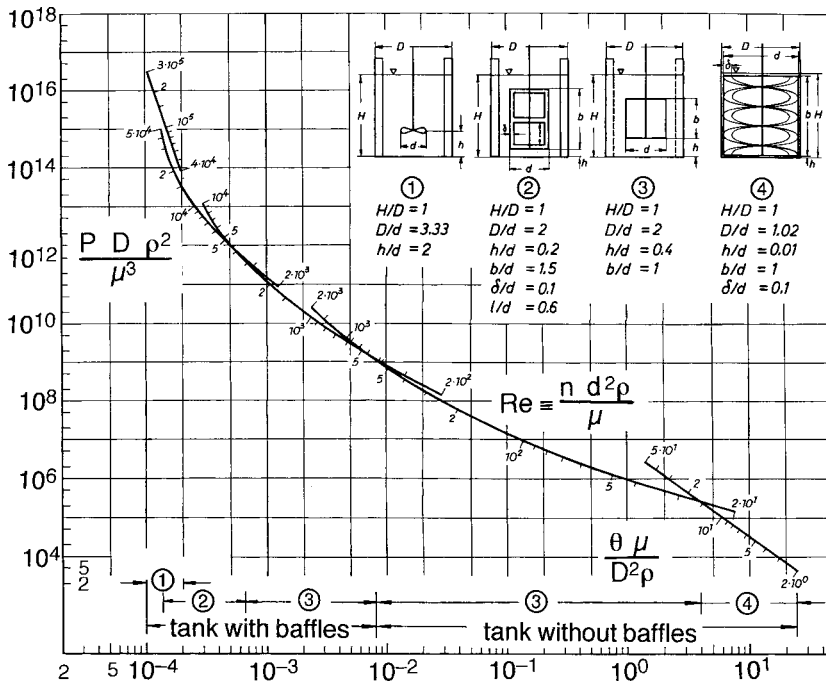


Fig. 28 Working sheet for the determination of optimum working conditions for the homogenization of liquid mixtures in mixing vessels; from [164].

Example 23: Process characteristics of a self-aspirating hollow stirrer and the determination of its optimum process conditions

As a result of their shape, hollow stirrers utilize the suction generated behind their edges (*Bernoulli* effect) to suck in gas from the head space above the liquid. As “rotating ejectors”, they are both stirrers and gas pumps in one and are therefore particularly suitable for laboratory use (especially in high pressure autoclaves) because they achieve intensive gas/liquid contacting via internal gas recycling without a separate gas pump [163/1]. A particularly effective type of this stirrer – the pipe stirrer – is depicted in **Fig. 29**.

In operating a hollow stirrer, both target quantities – gas throughput q and the stirrer power P – adapt themselves simultaneously. Both target quantities depend on the following parameters:

Geometry:	Stirrer diameter	d
Physical properties:	density	ρ
	viscosity	μ
Process parameters:	stirrer speed	n
	gravitational constant	g

Thus we obtain two separate relevance lists:

$$\{q; d, \rho, \mu; n, g\} \quad (11.8)$$

$$\{P; d, \rho, \mu; n, g\} \quad (11.9)$$

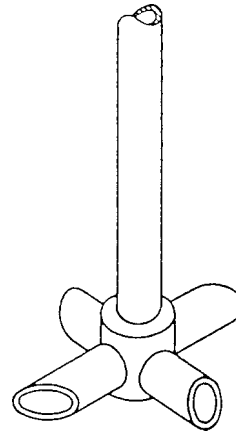


Fig. 29 Sketch of the pipe stirrer

	ρ	d	n	q	μ	g
M	1	0	0	0	1	0
L	-3	1	0	3	-1	1
T	0	0	-1	-1	-1	-2
M	1	0	0	0	1	0
3M + L	0	1	0	3	2	1
-T	0	0	1	1	1	2

The first relevance list (11.8) leads to the following three pi-numbers:

$$\Pi_1 \equiv \frac{q}{n d^3} \equiv Q \quad \Pi_2 \equiv \frac{\mu}{\rho d^2 n} \equiv \text{Re}^{-1} \quad \Pi_3 \equiv \frac{g}{d n^2} \equiv \text{Fr}^{-1} \quad (11.10)$$

Π_1 is named the gas throughput number Q , Π_2 is the inverse Reynolds number and Π_3 is the inverse Froude number. The gas throughput characteristics of a hollow stirrer then reads:

$$\frac{q}{n d^3} = f_1 \left(\frac{n d^2 \rho}{\mu}, \frac{n^2 d}{g} \right) \rightarrow Q = f_1(\text{Re}, \text{Fr}) \quad (11.11)$$

The power characteristics – obtained from the relevance list (11.9) by a similar dimensional matrix – containing P instead of q – leads to:

$$\frac{P}{\rho n^3 d^5} = f_2 \left(\frac{n d^2 \rho}{\mu}, \frac{n^2 d}{g} \right) \rightarrow \text{Ne} = f_2(\text{Re}, \text{Fr}) \quad (11.12)$$

The pi-number containing P is termed the Newton number.

Model experiments with another type of hollow stirrer (3-edged stirrer, see sketch in Fig. 31) were performed in water/air under defined experimental conditions and the scale factor $\mu = d_T/d_M$ was changed in the range of $\mu = 1 : 2 : 3 : 4 : 5$. The results demonstrate that the Reynolds number is irrelevant in the turbulent flow range ($\text{Re} > 10^4$) and the process is exclusively governed by the Froude number. Both process characteristics can therefore be represented as

$$Q = f_1(\text{Fr}) \quad \text{and} \quad \text{Ne} = f_2(\text{Fr})$$

see Fig. 30 and 31.

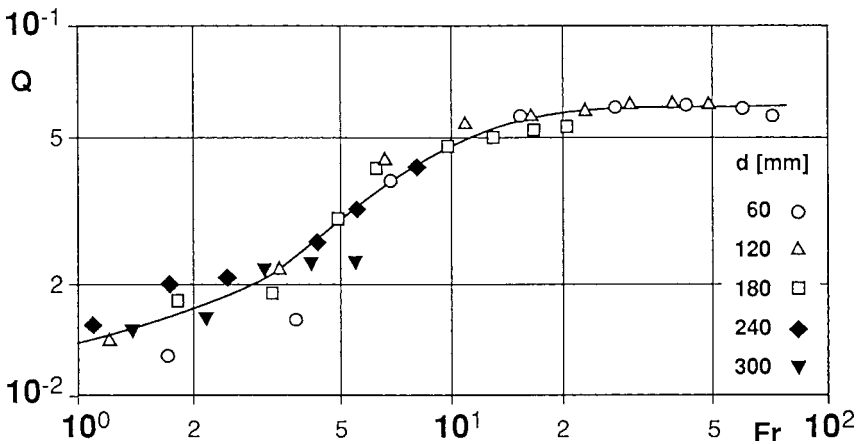


Fig. 30 Gas throughput characteristic of a 3-edged stirrer; from [163/1].

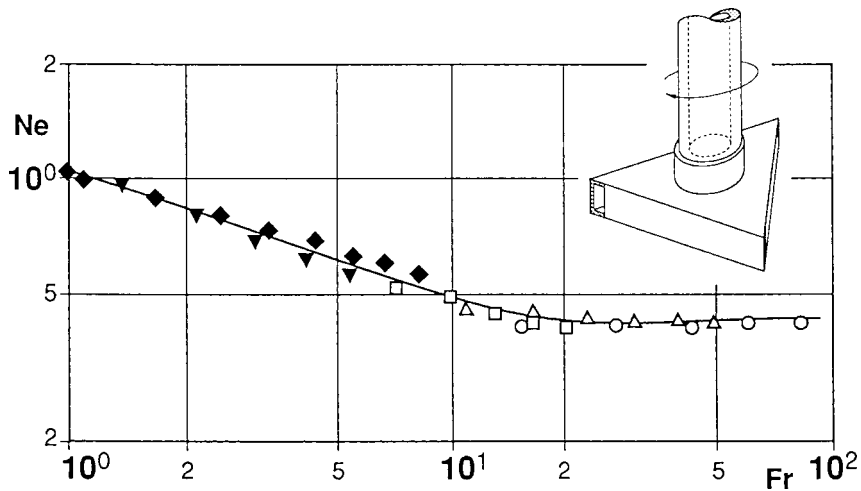


Fig. 31 Power characteristic of a 3-edged stirrer. For the legend see Fig. 30; from [163/1].

These results impressively demonstrate to which extent information can be compressed by dimensional analysis. These process characteristics present a reliable basis for the scale-up of this hollow stirrer under the given geometric conditions. But they also allow a further optimization of this process as will be demonstrated by the following additional treatment. A question must be answered with respect to the process conditions under which this stirrer will achieve a given gas throughput with the minimum power, $P/q = \min$! This answer can easily be found by combining the above characteristics in Fig. 30 and 31 in such a way that a dimensionless expression for P/q is produced. This is the pi-number combination

$$\frac{Ne \cdot Fr}{Q} \equiv \frac{P}{q \cdot d \cdot \rho g} = f_3(Fr) \quad (11.13)$$

This new pi-number is, as are its constituents, also a function of the Froude number. This dependency is represented by Fig. 32. It can be seen that two sections exist:

$$Fr \leq 10 : Ne \cdot Fr / Q = \text{const} \rightarrow P/q \propto d.$$

Here, P/q increases in direct proportion to scale (d).

$$Fr \leq 10 : Ne \cdot Fr / Q \propto Fr \rightarrow P/q \propto d^2.$$

In this range, the hollow stirrer is still less effective, because the power per unit gas throughput increases with the square of the scale (d^2).

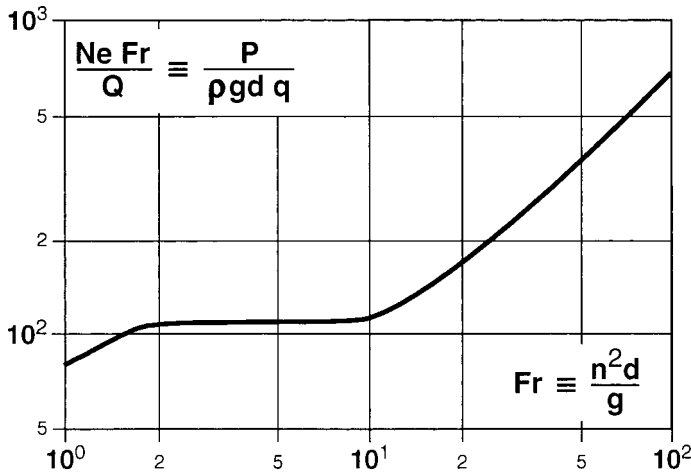


Fig. 32 Dimensionless interdependence of P/q and d under different process conditions (Fr).

Under these circumstances, which can be described by “small is beautiful”, it can be clearly shown that hollow stirrers are not suitable for sucking in large amounts of gas on a full scale. In this case, it is advisable to decouple the gas throughput and the power consumption by using a high speed stirrer (e.g., a turbine stirrer) and supply it with gas from underneath it via a blower.

In transport limited reactions in gas/liquid systems, mass transfer is usually dimensioned according to $P/V = \text{idem}$ and $v = q/S = \text{idem}$, see Section 13.2 and Example 41. In scaling up, these conditions also suggest the decoupling of the gas supply and stirrer speed, because processes with two mutually independent process parameters can be more easily optimized than those having only one process parameter.

However, there are many chemical reactions in the G/L system in which the gas throughput plays no role because micro-kinetics is rate determining. In such cases, the hollow stirrers, due to their dual role as stirrers and gas conveyers, play a prime role, particularly in high pressure chemical engineering.

Example 24: Optimization of stirrers for the maximum removal of reaction heat

In the optimization of stirrers for an optimal heat transfer, it should be remembered that the removable heat flow, Q [kW], increases according to the heat transfer characteristic with $Re^{2/3} \propto n^{2/3}$, whereas the associated stirrer power (stirring heat) increases substantially overproportionally according to the power characteristic of the stirrer, with P [kW] $\propto n^3$. From this it follows that there is an optimum stirrer speed at which a maximum of process heat, e.g., chemical reaction heat, R , can be removed:

$$R \equiv Q - P \quad (11.14)$$

Fig. 33 illustrates this situation with a concrete example. It shows that the optimum range with respect to the stirrer speed is very flat, 90% of the maximum value being removed in the range $n_{\text{opt}} = 20 \text{ min}^{-1} \pm 60\%$.

In the prediction of optimal conditions (n_{opt} , R_{max}), Pawlowski and Zlokarnik [104] applied the following procedure:

With $Q = h A \Delta T$ (in the laminar flow range the overall heat transfer coefficient $U \approx h$), the following expression follows from the relationship $R \equiv Q - P$

$$R/V = h A \Delta T/V - P/V \quad (11.15)$$

Formulation of this expression in terms of dimensional analysis yields:

$$\Pi_2 = \text{Nu} - (D/d) \Pi_1^{-1} \text{Re}^3 \text{Ne} \quad (11.16)$$

where Π_1 and Π_2 stand for

$$\Pi_1 \equiv \frac{D^2 \rho^2 k \Delta T}{\mu^3} \frac{A}{D H} \quad \text{and} \quad \Pi_2 \equiv \frac{(R/V) D^2}{k \Delta T} \frac{V}{A D} \quad (11.17)$$

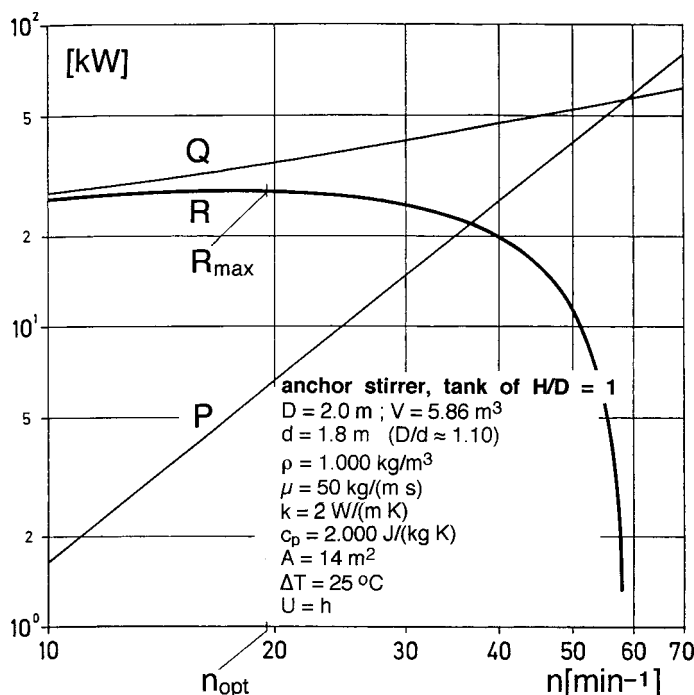


Fig. 33 Graphical representation of the courses of $R \equiv Q - P$ as a function of the stirrer speed for the example given in the plot; from [104].

If the known functions $Nu = Nu(Re, Pr)$ and $Ne = Ne(Re)$ are incorporated into the relationship (11.16), conditions are obtained for the sought optimum by differentiating this expression with respect to Re and setting its differential to zero.

The determination of these conditions for the optimum is made easier by the work-sheet in Fig. 34. It applies for two anchor stirrers with different wall clearances [$D/d = 1.00$ (no wiper blades!) and $D/d = 1.10$] in the laminar flow range ($Re < 100$).

The geometric parameters A/DH and V/DA for tanks with dished bottoms, which are necessary for utilizing the work-sheet, can be taken from the auxiliary diagram in inset (a) of Fig. 34 as functions of the aspect ratio H/D . Since the optimal stirrer speed determined can, in practice, only seldomly be realized, the Re and thereby the rotation speed range is given in the auxiliary diagram in inset (b) of Fig. 34. Within this range 90% of $(R/V)_{opt}$ (according to expression (11.15)) are attained.

Application example for Fig. 34:

The conditions used were those on which Fig. 33 is based. With $Pr = 5 \times 10^4$ and the abscissa value $\pi_1 Pr (H/D)^{-1} = 2.82 \times 10^8$, the optimum conditions $Re_{opt} Pr^{1/2} = 4.8 \times 10^3$ and the ordinate value $(\pi_2)_{opt} = 8 \times 10^1$ follow from the work-sheet, producing $n_{opt} \approx 20 \text{ min}^{-1}$ and $R_{max} = (R/V)_{opt} V = 28.5 \text{ kW}$ (see the optimum operating point in Fig. 33). At this stirrer speed the stirrer power amounts to ca. 6 kW, which is ca. 20% with respect to the maximum removal of reaction heat. From the auxiliary diagram in inset (b), it can be inferred that the rotation speed interval, in

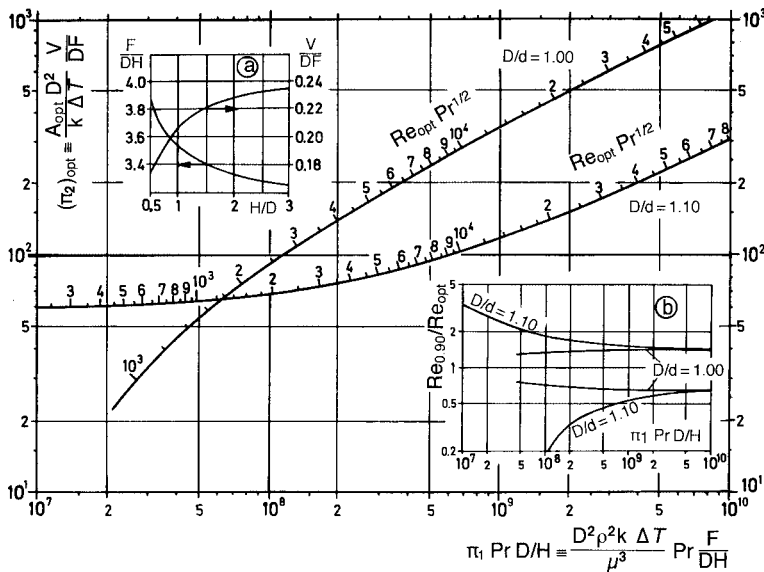


Fig. 34 Work-sheet for determining optimal operating conditions for heat removal in a vessel with an anchor stirrer with two different wall clearances ($D/d \approx 1.00$ – without wiper blades – and $D/d = 1.10$) in the laminar flow range ($Re < 100$); from [104].

which at least 90% of the maximum achievable value ($R_{90\%} = 25.6 \text{ kW}$) could be removed, lies between 8 and 32 min^{-1} .

If more than the determined amount $R_{\max}$ has to be removed, an anchor stirrer with $D/d = 1.1$ can be replaced with one with $D/d \approx 1.0$ (no wiper blades). Consequently, at a slightly lower optimum stirrer speed ($n = 17 \text{ min}^{-1}$), the removal of $R_{\max} \approx 60 \text{ kW}$ is possible.

Another option for raising $R_{\max}$, this being a simpler option from a technical point of view, involves choosing a tank with a higher aspect ratio. For $H/D = 2$ and the same volume ($V = 5.86 \text{ m}^3$) as above, a tank diameter $D = 1.57 \text{ m}$ is obtained. (For the given values of V and H/D the sought D is found with the assistance of the auxiliary diagram (a), if the H/D associated product $(A/DH)(V/DA)(H/D) \equiv V/D^3$ is generated. Therefore, in this case, $V/D^3 = 1.52$. With the new abscissa value $II_1 \text{ Pr } (H/D)^{-1} = 1.7 \times 10^8$, it follows that the quantity $R_{\max} \approx 39 \text{ kW}$ at $n_{\text{opt}} = 19.7 \text{ min}^{-1}$. For $H/D = 3$ ($D = 1.37 \text{ m}$, $d = 1.25 \text{ m}$), $R_{\max} = 45.5 \text{ kW}$ at $n_{\text{opt}} = 20.6 \text{ min}^{-1}$ can be removed. In our example, $R_{\max}$ increases with $(H/D)^{1/2}$. The calculated $R_{\max}$ value of 60 kW for an anchor stirrer with $D/d \approx 1.00$ is first achieved for $H/D = 6$.

In the range $Re > 200$ (turbulent range with respect to heat transfer), heat flux merely increases according to $R_{\max} \propto (H/D)^{1/3}$. An effective increase of $R_{\max}$ is only possible here by increasing ΔT or by using stirrers with wiper blades [184], see Fig. 72.

12

Selected Examples of the Dimensional-analytical Treatment of Processes in the Field of Mechanical Unit Operations

Introductory Remark

Fluid mechanics and mixing operations in various types of equipment, agglomeration as well as disintegration and mechanical separation processes, just to mention a few, are described by parameters, the dimensions of which only consist of *three* base dimensions: Mass, Length and Time. An isothermal process is assumed. The physical properties of the material system under consideration are related to a *constant* process temperature. The process relationships obtained in this way are therefore valid for *any* constant, random process temperature to which the numerical values of the physical properties are related. This holds true as long as there is no departure from the scope of the validity of the respective process characteristic verified by the tests.

The scale-up can only present problems when model substances are not available. (Prime example: Drag resistance of a ship's hull and *Froude's* approach of this problem, Example 9).

Example 25: Power consumption in a gassed liquid. Design data for stirrers and model experiments for scaling up

Gas/liquid contacting is frequently encountered in chemical reaction and bioprocess engineering. For reactions in gas/liquid systems (oxidation, hydrogenation, chlorination, and so on) and aerobic fermentation processes (including biological waste water treatment), the gaseous reaction partner must first be dissolved in the liquid. In order to increase its absorption rate, the gas must be dispersed into fine bubbles in the liquid. A fast rotating stirrer (e.g. a turbine stirrer), to which the gas is supplied from below, is normally used for this purpose (see the sketch in Fig. 35).

For a given geometry of the set-up, the relevance list for this problem contains the power consumption, P , as the target quantity, the stirrer diameter, d , as the characteristic length and a number of physical properties of the liquid and the gas (the latter are marked with an apostrophe); densities, ρ and ρ' , kinematic viscosities, ν and ν' , surface tension, σ , and an unknown number of still unknown phys-

ical properties, S_i , which describe the coalescence behaviour of finely dispersed gas bubbles and by this, indirectly, their hold-up in the liquid. The process parameters are the stirrer speed, n , and the gas throughput, q , which can be adjusted independently, as well as the gravitational acceleration, g , which is implicitly relevant because of the large density difference. (We should actually have written $g\Delta\rho$ here – see Section 5.1 – but, since $\Delta\rho = \rho - \rho' \approx \rho$, the dimensionless number would contain $g\Delta\rho/\rho \approx g\rho/\rho = g$.) The relevance list is therefore:

$$\{P; d; \rho, \nu, \sigma, S_i, \rho', \nu'; n, q, g\} \quad (12.1)$$

In this case, the relevance list contains at least eleven variables – more than twice the number required for the power consumption in the homogeneous liquid system (Example 22, eq. (11.1)).

Before employing dimensional analysis for generation of the dimensionless numbers, it is worthwhile anticipating obvious numbers such as ρ'/ρ and ν'/ν . S_i are physical properties of unknown dimension and number. Therefore they cannot be included in the dimensional matrix. However, this is no problem since, with the known relevant physical properties ρ , ν and σ , one will always be able to transform S_i to the dimensionless numbers S_i^* . The above relevance list can therefore be reduced to

$$\{P; d; \rho, \nu, \sigma; n, q, g\} \quad \text{for anticipated} \quad \rho'/\rho, \nu'/\nu, S_i^* \quad (12.2)$$

Here, the simplest dimensional matrix is also the best one because it leads directly to the common, named dimensionless numbers.

	ρ	d	n	P	ν	σ	q	g
M	1	0	0	1	0	1	0	0
L	-3	1	0	2	2	0	3	1
T	0	0	-1	-3	-1	-2	-1	-2
M	1	0	0	1	0	1	0	0
L + 3M	0	1	0	5	2	3	3	1
-T	0	0	1	3	1	2	1	2

$$\Pi_1 \equiv P/(\rho d^5 n^3) \quad \equiv \text{Ne} \quad (\text{Ne} - \text{Newton number})$$

$$\Pi_2 \equiv \nu/(d^2 n) \quad \equiv \text{Re}^{-1} \quad (\text{Re} - \text{Reynolds number})$$

$$\Pi_3 \equiv \sigma/(\rho d^3 n^2) \quad \equiv \text{We}^{-1} \quad (\text{We} - \text{Weber number})$$

$$\Pi_4 \equiv q/(d^3 n) \quad \equiv Q \quad (Q - \text{throughput number})$$

$$\Pi_5 \equiv g/(d n^2) \quad \equiv Fr^{-1} \quad (Fr - \text{Froude number}).$$

Taking the anticipated numbers into account, it follows that:

$$Ne = f(Re, We, Q, Fr, \rho'/\rho, v'/v, S_i^*) \quad (12.3)$$

However, we can now see that the important process parameter, the stirrer speed, n , is present in *all* numbers: the numerical value being changed with each change in speed. This is not advantageous for the planning and evaluation of experiments. Our aim is therefore to transform Re and We , which contain the physical properties v and σ , into dimensionless material numbers by combination with other numbers.

First, we form two combinations of the dimensionless numbers which do not contain n :

$$Re^2/Fr \equiv \frac{g d^3}{v^2} \equiv Ga \text{ (Galilei number)} \quad (12.4)$$

$$We/Fr \equiv \frac{\rho d^2 g}{\sigma} \quad (12.5)$$

Then we combine these two new numbers so as to eliminate d :

$$\{(Re^2/Fr)^{2/3}(Fr/We)\} \equiv (Re^4 Fr)^{1/3} We^{-1} \equiv \frac{\sigma}{(\rho^3 v^4 g)^{1/3}} \equiv \sigma^* \quad (12.6)$$

By this means it is possible to transform the We number into a pure material number, σ^* , the numerical value of which is dependent only on the material system. In contrast, the only advantage of the Ga number over Re is that it does not contain the stirrer speed. Let us stay with Re and base our considerations on the following pi-space:

$$Ne = f(Q, Fr, Re; \sigma^*, \rho'/\rho, v'/v, S_i^*) \quad (12.7)$$

In this case, the pi-space consists of a target number (Ne), three process numbers (Q, Fr, Re) and a series of pure material numbers ($\sigma^*, \rho'/\rho, v'/v, S_i^*$).

The first question, we must ask, is: Are laboratory tests performed in *one single piece* of laboratory equipment (i.e., on one single scale) capable of providing reliable information on the decisive process number (or combination of numbers)? Although we can change Fr by means of the stirrer speed, Q by means of the gas throughput and Re (or Ga) by means of the liquid viscosity, independently of each other, we must accept the fact that a change in viscosity will alter not only Re but also the numerical values of the material numbers, σ^* , v'/v , and, very probably, S_i^* .

In contrast to the gas density, ρ' (hydrogen is a factor of 16 lighter than air, its bubbles therefore have a stronger buoyancy and will escape the liquid more quickly!), an influence of the gas viscosity, ν' , on the stirrer power is not to be expected ($\nu'/\nu = \text{irrelevant}$). Preliminary tests with methanol/water mixtures showed [169] that σ does not influence the stirrer power either, therefore σ^* is irrelevant. Furthermore, measurements revealed that the coalescence behaviour of the material system is not affected if aqueous glycerol or cane syrup mixtures are used to increase viscosity. This means that the influence of S_i^* on Ne cannot be significant. These results alone give us the right to perform model experiments in *one single piece* of apparatus in order to elaborate the process relationship. The following pi-space is then used as the basis

$$Ne = f(Q, Fr, Re) \quad (12.8)$$

and only *one* gas (air) is used, $\rho'/\rho = 1.20 \times 10^{-3} = \text{const.}$

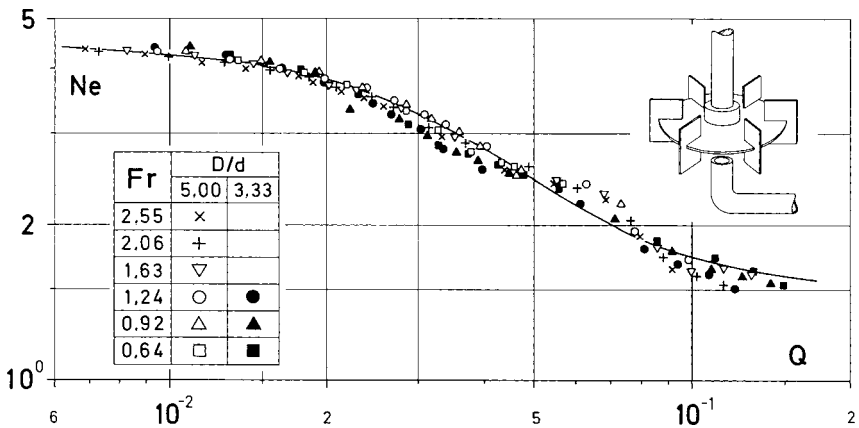


Fig. 35 Power characteristic of a turbine stirrer under industrially interesting flow conditions ($Re \geq 10^4$; $Fr \geq 0.65$; $\rho'/\rho = 1.20 \times 10^{-3}$); from [169].

The results of these model experiments are described in detail in [169]. For our consideration based on the theory of similarity, it is sufficient to present only the main result here. This states that, in the industrially interesting range ($Re \geq 10^4$ and $Fr \geq 0.65$), Ne is only dependent on Q; see **Fig. 35**.

Knowledge of this power characteristic, the analytical expression for which is

$$Ne = 1.5 + (0.5 Q^{0.075} + 1600 Q^{2.6})^{-1} \quad Re \geq 10^4; Fr \geq 0.65; \rho'/\rho = 1.20 \times 10^{-3} \quad (12.9)$$

can be used to reliably design a stirrer drive for conducting reactions in the gas/liquid system (e.g., oxidation with O_2 or air, fermentation, etc.) as long as the physical, geometric and process-related (Re and Fr range) boundary conditions comply with those of the model measurement.

At this point we should ask ourselves how we would perform the scale-up if we did not know anything about the above functional relationship! Let us therefore try to determine the stirrer power of a given stirrer for a specified, large fermenter (e.g., $V = 100 \text{ m}^3$; $H/D = 3$; $D = 3.5 \text{ m}$) on the basis of model measurements where the physical properties of the system and the gas throughput of interest are known. With a freely selectable stirrer speed, we can presuppose that, apart from the numerical value of Nu , the numerical values of all other dimensionless numbers are known. (Of course we do not know anything about the coalescence phenomena – this corresponds to the state of knowledge existing about 30 years ago.)

Our considerations are therefore based on the following relationship:

$$Ne = f(Q, Fr, Re; \sigma^*, \rho'/\rho, v'/v) \quad (12.10)$$

Naturally we will perform the model experiments (e.g., on a scale of $\mu = 1:10$, i.e., $V = 0.1 \text{ m}^3 \rightarrow D = 0.35 \text{ m}$) with the industrially interesting material system; in this way, at least, the numerical values of the three material numbers remain constant. However, this means that Re (resp. Ga) and Fr can no longer be adjusted *independently* of each other because only the rotational speed of the stirrer is available for their realization.

Therefore, we can only realize *partial similarity* in the model. We can either set Q and $Re = \text{idem}$ or Q and $Fr = \text{idem}$. We will opt for the second case (Q and $Fr = \text{idem}$) because we expect g and hence Fr to be more important than v , and hence Re , in an aeration process.

From the scale-up rules

$$Fr \propto n^2 d = \text{idem} \rightarrow n_M = n_T \mu^{1/2} \quad \text{and} \quad (12.11)$$

$$Re \propto n d^2 = \text{idem} \rightarrow n_M = n_T \mu^2 \quad (\mu = d_T/d_M) \quad (12.12)$$

it follows that, in order to maintain $Fr = \text{idem}$, the stirrer speed in the model is only faster by a factor of $\mu^{1/2}$ than in the full-scale application. However, the condition $Re = \text{idem}$ requires a stirring rate which is faster by a factor of μ^2 . This means that if we carry out the model measurement at $Fr = \text{idem}$, we are running the risk of the flow condition shifting substantially towards the laminar region with respect to Re . That is to say, if we perform the model experiments, which are characterized with $Fr = \text{idem}$, $n_M = n_T \mu^{1/2}$ and $d_M = d_T \mu^{-1}$, the following is valid for the Re number under these conditions:

$$Re_M \propto n_M d_M^2 = n_T \mu^{1/2} (d_T \mu^{-1}) \propto Re_T \mu^{-3/2} \quad (12.13)$$

In our example ($\mu = 1:10$), the Re number would only be approximately 1/30th of the value obtained in the full-scale application! In view of this fact, the following approach would seem to be sensible, see **Fig. 36**:

1. The first measurement point is determined at Re_{M1} and $Fr, Q = \text{idem}$ (*filled circle in the figure*).

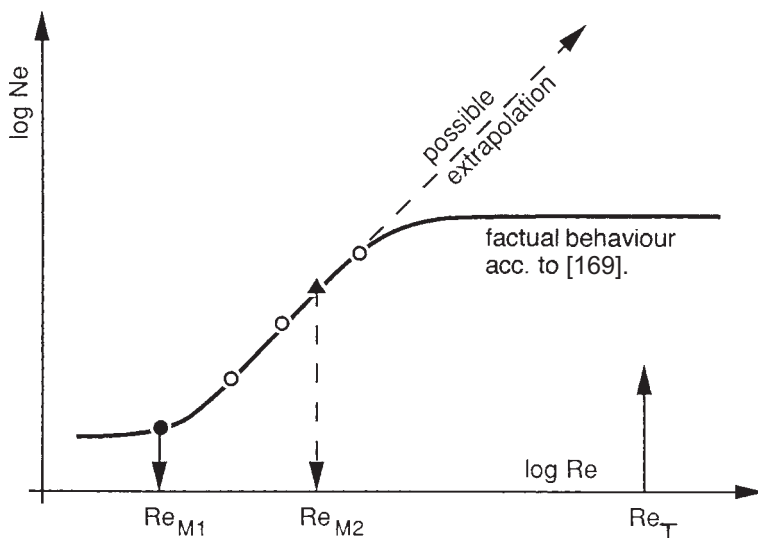


Fig. 36 Presentation of the $Ne(Re)$ curve for $Q, Fr = \text{idem}$ kept constant. Illustration of model experiments. For explanation see text.

2. In the course of further measurements at $Fr, Q = \text{idem}$, the viscosity of the experimental liquid is reduced stepwise to raise Re towards Re_T ; compare the three *hollow circles*. The smaller the selected model scale, the greater is the danger of this approach also finally leading by false extrapolation to $Ne(Re_T)$. We do not know that Re is no longer relevant at $Re \geq 10^4$!
3. Since the material numbers are changed by the approach described in point 2, we will perform another model experiment on a larger scale at Re_{M2} , just to be on the safe side (see the *filled triangle*). Although we may confirm preceding measurement results, this does not reduce the risk of *extrapolation* to Re_T as shown by comparison with the actual state of affairs (curve in Fig. 35).

This example is not intended as a general deterrent against measurements under the conditions of partial similarity since these – when performed with care – can frequently provide valuable information. The purpose of this consideration is merely to show that there can be no substitute for *complete* information about a technical matter in the pertinent pi-space.

Example 26: Scale-up of mixers for mixing of solids

In the final state, the mixing of solids (e.g., powders) can only lead to a stochastically homogeneous mixture. We can therefore use the theory of random processes to describe this mixing operation. In the present example from [89] we will concentrate on a mixing device in which the position of the particles is adequately given by the x coordinate. Furthermore, we will assume that the mixing operation can be described as a stochastic process without “after-effects”. This means that only the actual condition is important and not its history. The temporal course of this so-called *Markov* process can be described by the 2nd *Kolmogorov* equation. In the case of a mixing process without selective convective flows (requirement: $\Delta\rho \approx 0$ and $\Delta d_p \approx 0$; see [142]), the solution of *Fick’s* diffusion equation gives a cosine function for the local concentration distribution, the amplitude of which decreases exponentially with the dimensionless time $\theta D_{\text{eff}}/(\pi^2 L^2)$, see **Fig. 37**. (The variation coefficient or the relative variation, v , is defined as the standard deviation divided by the mean value: $v \equiv \sqrt{s^2/\bar{x}}$.)

Let us now consider this process using dimensional analysis. We have the following parameters:

Target quantity:	v	variation coefficient as a measure for the mixing quality
Geometric parameters	D, L	diameter and length of the drum
	d	diameter of the mixing device
	d_p	mean particle diameter
	ϕ	degree of fill of the drum
Material properties	D_{eff}	effective axial dispersion coefficient
	ρ	density of the particles
Process parameters	n	rotational speed of the mixer
	θ	mixing time
	$g\rho$	solid gravity

The relevance list contains 11 parameters

$$\{v, D, L, d, d_p, \phi; D_{\text{eff}}, \rho; n, \theta, g\rho\} \quad (12.14)$$

After the exclusion of the dimensionless quantities v and ϕ and the obvious geometric pi-numbers L/D , d/D and d_p/D , the following 3 pi-numbers are obtained:

	ρ	D	n	θ	D_{eff}	$g\rho$
M	1	0	0	0	0	1
L	-3	1	0	0	2	-2
T	0	0	-1	1	-1	-2
M	1	0	0	0	0	1
L + 3M	0	1	0	0	2	1
-T	0	0	1	-1	1	2

θn Mixing number
 $D_{\text{eff}}/D^2 n \equiv \text{Bo}^{-1}$ Bo – Bodenstein number
 $g\rho/\rho D n^2 \equiv \text{Fr}^{-1}$ Fr – Froude number

The complete pi-set reads:

$$\{v, L/D, d/D, d_p/D, \phi, \theta n, \text{Bo}, \text{Fr}\} \quad (12.15)$$

To obtain the rotational speed of the drum in only one number (the process number Fr), we combine the other two accordingly with Fr and get: $\theta D_{\text{eff}}/D^2$ and $g D^3/D_{\text{eff}}^2$.

The experimental results presented in **Fig. 37** were obtained in one single model ($D = 0.19$ m) with different lengths ($L/D = 1; 1.5; 2; 2.5$). The geometric and material numbers d/D , d_p/D , ϕ and $g D^3/D_{\text{eff}}^2$ remained unchanged as did Fr because of the constant rotational speed of the drum $n = 50 \text{ min}^{-1}$. As a result, the measurements can only be depicted in the pi-space

$$\{v, \theta D_{\text{eff}}/D^2, L/D\} \quad (12.16)$$

whereby d/D , d_p/D , ϕ , $g D^3/D_{\text{eff}}^2$, Fr = idem.

The result of these measurements is

$$v = f(\theta D_{\text{eff}}/L^2) \quad (12.17)$$

In other words, the mixing time (at Fr = const), required to attain a certain mixing quality, increases with the square of the drum length L. In order to reduce the mixing time, the component to be mixed would have to be added in the middle of the drum or simultaneously at several positions.

Fig. 37 shows experimental results in a single logarithmic graph. They are compared with the theoretical prediction of a stochastic *Markoff's* process. For details see [89].

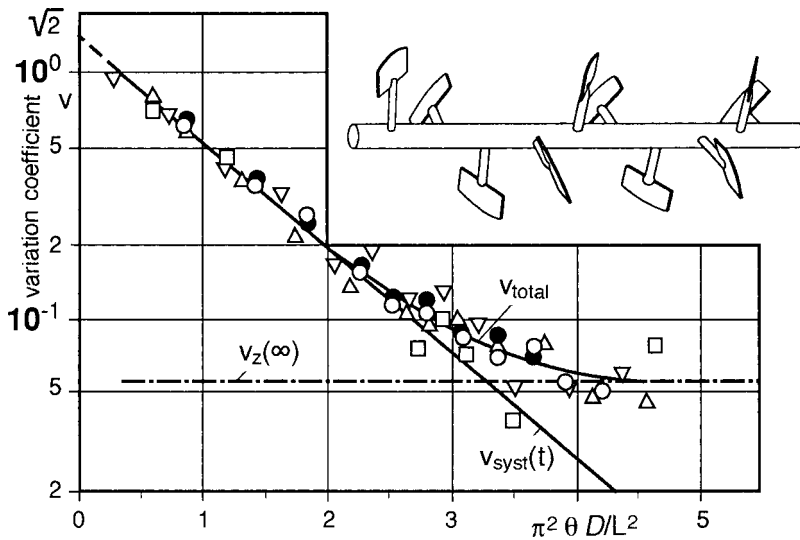


Fig. 37 Mixing quality as a function of the dimensionless mixing time for different l/D ratios. Copper and nickel particles of $\delta = 300 - 400 \mu\text{m}$, fill degree of the drum $\phi = 35\%$, Froude number of the drum $\text{Fr} = 0.019$. From [89].

Entrop [27] reported the process characteristics of the *Nauta*[®] mixer. The *Nauta*[®] mixer utilizes an orbiting action of a helical screw rotating on its own axis to carry material upward, while revolving about the center line of the cone-shaped shell near the wall for top-to-bottom circulation., see the sketch in Fig. 38. *Nauta*[®] mixers of different sizes are not build geometrically similar to each other. The diameter of the helical screw and its pitch are kept equal.

Mixing time characteristic of the *Nauta*[®] mixer

Relevance list. In the case of a pure convective mixing and $\Delta\rho \approx 0$ as well as $\Delta d_p \approx 0$, the particle size d_p has no influence.

Target quantity:	θ	mixing time
Geometric parameters	d, l	diameter and length of the helical screw
Material properties	ρ	solid density
Process parameters	n, n_b	rotational speed of the helical screw and of its beam
	g	gravitational acceleration

$$\{\theta; d, l; \rho; n, n_b, g\} \quad (12.18)$$

The base dimension M is contained only in the density ρ . Therefore, this quantity has to be deleted from the relevance list. $6 - 2 = 4$ numbers will be produced. The pi-set reads:

$$\{n\theta, l/d, n_a/n, Fr \equiv n^2 d/g\} \quad (12.19)$$

The measurements were executed under the following conditions: Mixer volume $V = 0.05\text{--}10\text{ m}^3$; diameter of the helical screw $d = 0.15\text{--}0.63\text{ m}$; rotational speed of the helical screw $n = 30\text{--}120\text{ min}^{-1}$; $n_a/n = 20\text{--}70$; $Fr = 0.24\text{--}4$. Material systems: sand and fine-grained limestone.

The mixing time characteristic of the *Nauta*® mixer is given in Fig. 38. It can be shown that the type of material has a negligible influence (proof that the density ρ is irrelevant indeed!). Likewise, the number n_a/n has, within the used range, no effect. In contrast, the influence of the parameter l/d is very pronounced. The process equation reads:

$$n\theta = 13 (l/d)^{1.93} \quad n_a/n = 20\text{--}70 \quad Fr = 0.24\text{--}4 \quad (12.20)$$

This means, in practice, that the mixing time is lengthened by the square of the length (compare to the eq. (12.17)).

For the power characteristic of the *Nauta*® mixer has been found [27]:

$$Ne Fr \equiv \frac{P}{n d^4 g \rho} \propto (l/d)^{1.62} \quad (12.21)$$

By multiplication of the process characteristics (12.20) and (12.21), the expression for the mixing work, W , can be obtained, this being necessary for a given mixing quality.

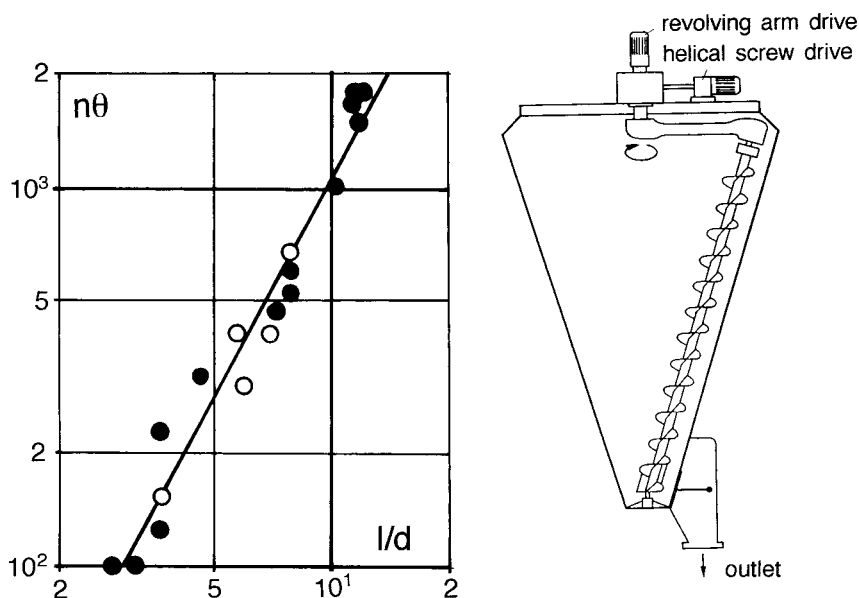


Fig. 38 Mixing time characteristic of the *Nauta*® mixer and its drawing; from [27]. (Sign legend is missing in the original publication.)

$$W = P \theta \propto d^{0.45} l^{3.55} \rho g \quad (12.22)$$

From the energetic point of view, it is therefore advantageous to construct mixers of low height and to provide them with helical screws of large diameter.

In the study of plow-share mixers (centrifugal mixers) a different data processing method has been used [90]. From the concentration distribution of the material travelling through the drum, an effective Bodenstein number, $Bo \equiv v L/M$ (v – travelling velocity of the material to be equalized; L – drum length; M – mixing coefficient, comparable to the dispersion coefficient $D_{\text{eff, ax}}$), is determined and M calculated from it. With these M values a different Bodenstein number, $Bo \equiv n D^2/M$, is now formed and plotted against the Froude number $Fr \equiv n^2 D/g$. **Fig. 39** shows that for $Fr \leq 0.038$ the Bodenstein numbers remain constant but afterwards they decrease proportionally to Fr^2 . At this border value of the Froude number, the flow behavior changed completely.

According to Müller [90], the manufacturers of mixing equipment comply with the scale-up rule: $u = \text{const}$ ($u = \pi n d$ – tip speed of the mixing device). From this, the following applies for the mixing time θ :

$$Fr \leq 0.038: \quad \theta = D/u \quad (L/D)^2 \quad (12.23)$$

$$Fr \geq 0.038: \quad \theta = D^3/u^5 \quad (L/D)^2 \quad (12.24)$$

At the same standardized L/D value and an equal tip speed, u , for $Fr \leq 0.038$ the mixing time increases proportionally to the drum diameter, D . For $Fr \geq 0.038$, the mixing time increases proportionally to D^3 . In this range, however, even a minute change in the rotational speed of the mixing device exerts a strong influence on the mixing time.

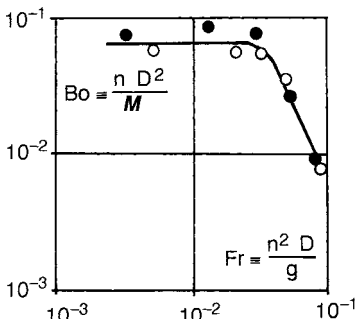


Fig. 39 Mixing time characteristic of a plow-share mixer. Material: Cooper shot and corn shoot. $D = 0.2$ and 0.4 m; degree of fill $\varphi = 0.35$ and 0.4 – 0.65 ; from [90]. (Sign legend is missing in the original publication.)

Example 27: Conveying characteristics of single-screw machines

Screw machines are important appliances used for the production (mass polymerization) and processing (mixing, extrusion) of plastics. They play also an important role in the food industry (chocolate, pasta). A distinction is made between sin-

gle-screw and multiple-screw machines (e.g., self-wiping twin-screws) and between conveying, mixing and kneading screw machines. The conveying characteristics are represented for all types of screw machine in the same π -space. In the following, they will be given and discussed for single-screw machines.

The conveying properties of a single-screw machine of given screw geometry are represented in the creeping range ($Re < 100$) of Newtonian liquids by the following characteristics [103, 108]:

$$\text{Pressure characteristic:} \quad Eu \, Re \, d/L \equiv \frac{\Delta p \, d}{\mu \, n \, L} = f_1(Q) \quad (12.25)$$

$$\text{Axial force characteristic:} \quad NeF \, Re \, d/L \equiv \frac{F}{\mu \, n \, d \, L} = f_2(Q) \quad (12.26)$$

$$\text{Power characteristic:} \quad NeF \, Re \, d/L \equiv \frac{P}{\mu \, n^2 \, d^2 \, L} = f_3(Q) \quad (12.27)$$

Q represents the flow rate number $Q \equiv q/(nd^3)$. These three characteristics are illustrated in **Fig. 40** for a screw of given geometry. They are linear dependences which are described by analytical expressions in the form:

$$\frac{1}{y_1}Y + \frac{1}{q_1}Q = 1 \quad (12.28)$$

where y_1 and q_1 are the respective axis intercepts.

In scrutinizing the conveying characteristics in **Fig. 40**, one discovers three typical ranges in which the screw machine can function. They are outlined in **Fig. 41**. In this representation, the throughput number Q is standardized by the intercept A_1 . In other words, this is the numerical value of Q where the screw machine is conveying without pressure formation. With this “flow-kinematic” parameter, $\mathcal{A} \equiv Q/A_1$, the state of flow of a screw machine can be outlined more distinctly.

The two intercepts A_1 and A_2 are called profile parameters of a screw machine because their numerical values depend on the screw geometry (“screw profile”), for details see [103, 108].

From the three ranges of the conveying characteristics, only the middle one $0 < \mathcal{A} < 1$ (the so-called active conveying range of the screw machine) can be implemented by suitable throttling and/or a change in the rotational speed alone, without an additional conveying device. At $\mathcal{A} = 0$ the screw machine is fully choked and the highest pressure builds up. At $\mathcal{A} = 1$ the highest throughput is achieved without pressure build-up.

In order to implement the other two ranges, it is necessary to couple the screw machine with an additional conveying device (e.g., a positive displacement gear-type rotary pump). If the pump transports the liquid in the same direction as the screw, the range $\mathcal{A} > 1$ results. Here, the conveying action of the screw machine is “run over” by the conveying action of the pump. At $\mathcal{A} < 0$, the pump pushes the liquid against the conveying sense of the screw. The screw machine is then only a mixing device.

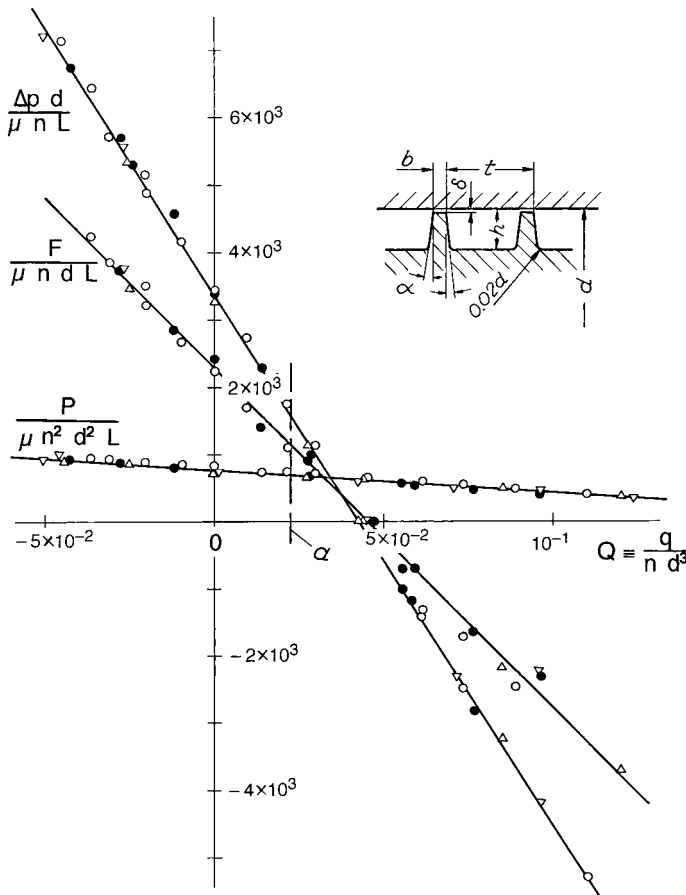


Fig. 40 Conveying characteristics of a single-screw machine of given screw geometry, taken from [103].

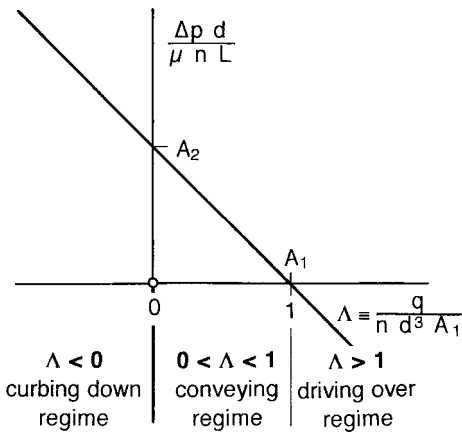


Fig. 41 Subdivision of the typical working ranges of a screw machine by the "flow-kinematic" parameter Λ .

It has already been pointed out that the conveying properties of a single-screw machine can be described by linear dependencies of the type shown in eq. (12.28), this only being valid in the creeping range ($Re < 100$) of Newtonian liquids. The proof for this is given in Fig. 42. It represents the pressure characteristics of a single-screw machine of a given screw geometry in dependence on the Reynolds numbers. It can be seen that at $Re \geq 240$ this linearity no longer exists.

In the case of a non-Newtonian liquid, the pressure characteristic is depicted by the pi-set:

$$\{\Delta p d / (H n L), Q, n\theta, \Pi_{\text{rheol}}\} \quad \text{with } H = \mu_{\infty} \quad \text{and} \quad \theta = 1/\dot{\gamma}_{\infty} \quad (12.29)$$

In this type of liquid, in which the viscosity curve corresponds to that shown in Fig. 18, the pi-space is given by

$$\{\Delta p d / (\mu_{\infty} n L), Q, n\dot{\gamma}_{\infty}\} \quad \text{with} \quad Q \equiv q / (n d^3) \quad (12.30)$$

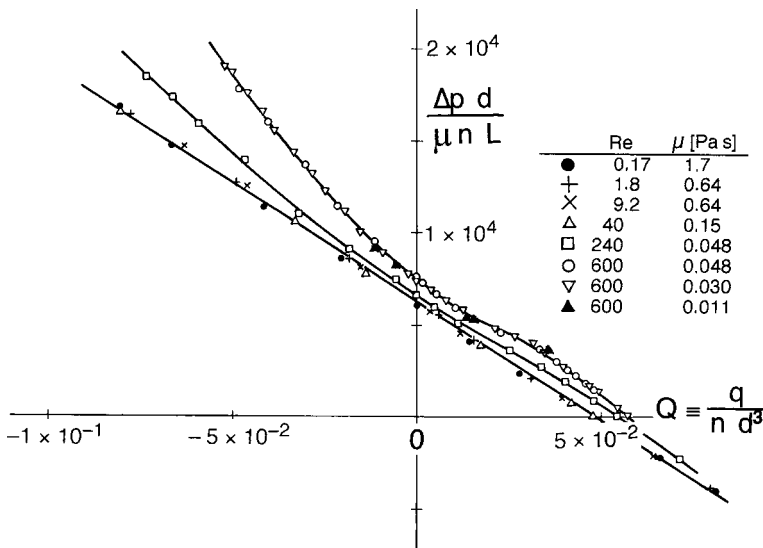


Fig. 42 Dependence of the pressure characteristics of a single-screw machine on the state of flow (Re); from [103].

Fig. 43 shows this relationship for such a liquid (lubricant for steam engines with ca. 7 % Al stearate, $\mu_{\infty} = 9.7$ Pa s, $\dot{\gamma}_{\infty} = 0.205$ s⁻¹). This was established with two differently sized ($d = 60$ and 90 mm) single-screw machines with the same profile geometry using two rotational speeds ($n = 1.65$ and 25 min⁻¹). Independent of the screw diameter, the curves coincide for $n/\dot{\gamma}_{\infty} = \text{const}$. The higher the rotational speed, the higher is the shear stress; the straight line (a), which is also valid for Newtonian liquids, adjusts as the limit case ($\mu = \mu_{\infty}$).

Without taking the pi number $n/\dot{\gamma}_\infty$ into account it would be impossible to correlate pressure drop characteristics obtained in two differently sized screw machines ($\mu = 1 : 1.5$).

When conducting extremely exothermic reactions in a screw machine as chemical reactor, the operating conditions at which a minimum of power is dissipated in the liquid ($\triangleq$ lowest thermal load of the liquid throughput) are of interest. The dissipated power, H , is obtained from the difference between the power, P , of the motor drive and that of the pump, $q\Delta p$:

$$H = P - q\Delta p \quad (12.31)$$

A dimensionless formulation of this relationship is possible using the conveying characteristics of the screw machine in question, see Fig. 40. In the active conveying range, $0 < A = 1$, the dissipation characteristic passes through a minimum in which the lowest power dissipation H occurs for the given values of q and Δp , this corresponding to $H/q = \min$.

In Section 8.7 it was pointed out that even using the same non-Newtonian liquid, complete similarity of the model and the full-scale counterpart can only be attained if there is a creeping, steady-state and isothermal flow condition. Scale-up is then carried out as follows:

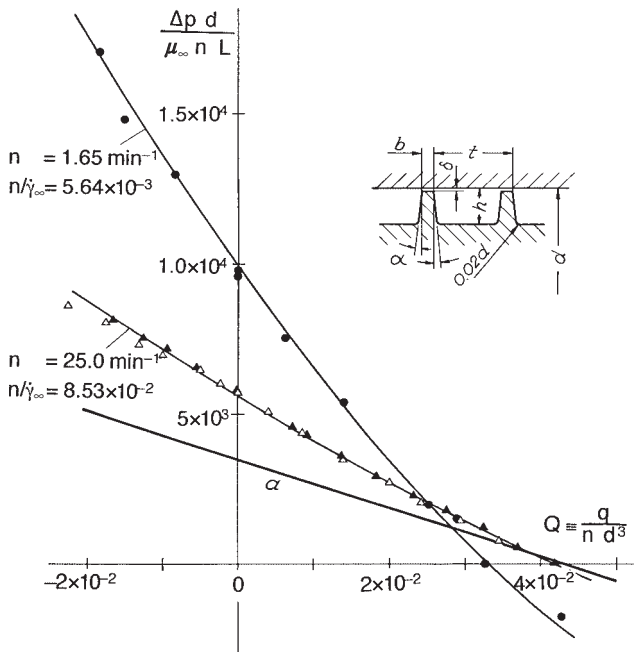


Fig. 43 Pressure characteristic of a screw machine of given screw geometry for a non-Newtonian liquid (see text). Full signs: $d = 60$ mm, hollow signs: $d = 90$ mm; for μ_∞ and $\dot{\gamma}_\infty$ see Fig. 18; from [103].

The non-Newtonian liquid and the parameters q_T and Δp_T of the industrial facility are given. We are searching for the variable d and the rotational speed n of the technical device, whereby $P = \min$ is required. Corresponding to the above pi-set for non-Newtonian liquids, it follows that:

$$L/d, n, \Delta p = \text{idem} \rightarrow q/d^3; P/q = \text{idem} \quad (12.32)$$

In the model screw machine, the dependences $q(n)$ and $P(q)$ are established and depicted as $P/q = f_1(n)$ and $q/d^3 = f_2(n)$. This situation also applies to the full-scale appliance. The possibly existing minimum of P/q gives the optimum rotational speed n_{opt} , which is also valid for the technical device. The corresponding values d_{opt} and P_{opt} for the technical device are obtained from the values $(q/d^3)_{\text{opt}}$ and $(P/q)_{\text{opt}}$ by setting $q = q_T$. We have therefore solved the task.

A consequent dimensional-analytical treatment of the homogenization process, residence time distribution and heat transfer behavior in single-screw machines was performed by Pawłowski and was published in a sequence of scientific papers. A summary of this work is printed as a monograph [108].

Example 28: Dimensional-analytical treatment of liquid atomization

Liquid atomization (liquid-in-gas-dispersion) is an important unit operation which is employed in a variety of processes. These include: fuel atomization, spray drying, the spraying of a lime suspension into combustion gases in power stations for SO_2 removal, powder metallurgy (metal powder production), coating of surfaces by spraying, and so on.

In all these tasks the achievable (as narrow as possible) droplet size distribution represents the most important target quantity. It is often described merely by the mean droplet size, the so-called “Sauter mean diameter” d_{32} [125], which is defined as the sum of all droplet volumes divided by their surfaces. Mechanisms of droplet formation are:

1. Liquid column formed by a pressure nozzle is inherently unstable. The break-up of the laminar column occurs by a symmetrical oscillation, a sinusoidal oscillation and, finally, atomization.
2. Liquid sheet formation by an appropriate nozzle is followed by rim disintegration, aerodynamic wave disintegration and turbulent break-up.
3. Liquid atomization by a gas stream.
4. Liquid atomization by centrifugal acceleration.

Equations exist for all of these operations process and some of them will be represented in the following.

As the discharge velocity at the nozzle outlet increases, the following states appear in succession: dripping, laminar jet break-up, wave disintegration and ato-

mization. These states of flow are described in a pi-space $\{Re, Fr, We_p\}$, where $We_p \equiv \rho v^2 d_p / \sigma$ represents the Weber number, formed by the droplet diameter, d_p . To eliminate the flow velocity, v , these numbers are combined to give

$$\text{Bond number} \quad Bd_p \equiv \frac{We}{Fr} \equiv \frac{\rho g d_p^2}{\sigma} \quad \text{and} \quad (12.33)$$

$$\text{Ohnesorge number} \quad Oh_p \equiv \frac{We^{1/2}}{Re} \equiv \frac{\mu}{(\rho v d_p)^{1/2}} \quad (12.34)$$

The subscript p indicates that these pi-numbers are formed with the droplet diameter.

For a liquid, dripping from a tiny capillary with diameter, d , it follows that:

$$\frac{d_p}{d} = 1.6 \left(\frac{\rho g d^2}{s} \right)^{-\frac{1}{3}} = 1.6 Bd^{-\frac{1}{3}} \quad (12.35)$$

Broader tubes ($Bd > 25$) exert no influence on d . Then we obtain:

$$Bd_p \equiv \rho g d_p^2 / \sigma = 2.9-3.3 \quad (12.36)$$

On the jet surface, waves are formed which, at wavelengths of $\lambda > \pi d_j$ (d_j – jet diameter), grow rapidly. The fastest wave disturbance takes place at the optimum wavelength of

$$\lambda_{opt} / \pi d_j = \sqrt{2 + 6 Oh} \quad (12.37)$$

For a low liquid viscosity $d/d_j \approx 1.9$ applies. If liquid output pulsates, uniformly spaced droplets are obtained; here $d/d_j \approx 1$. Another possibility to produce mono-sized droplets consists in using pneumatic extension nozzles.

With higher discharge velocities laminar jets are produced, which disintegrate to droplets at a certain distance from the capillary. The transition from dripping to liquid jet disintegration occurs at higher Weber numbers:

$$We \equiv \rho v^2 d / \sigma = 8-10 \quad (12.38)$$

At $We < 8$, gravitational acceleration also has to be considered, therefore the Bond number has to be included into the process equation.

The working principle of hollow cone nozzles is that the liquid throughput is subjected to rotation by a tangential inlet and is then further accelerated in the conical housing towards the orifice; see the sketch in Fig. 44. A liquid film with a thickness, δ , is thereby produced, which, at the discharge from the orifice, spreads to a hollow cone sheet and disintegrates to droplets.

At low discharge velocities and low film thicknesses, the sheet disintegration is due to the oscillations caused by air motion. In this case, the film thickness has a large impact on the droplet size. In contrast, it is insignificant whether a pure liquid or a lime-water suspension (mass portion $\phi = 16-64\%$) is treated, see Fig. 44.

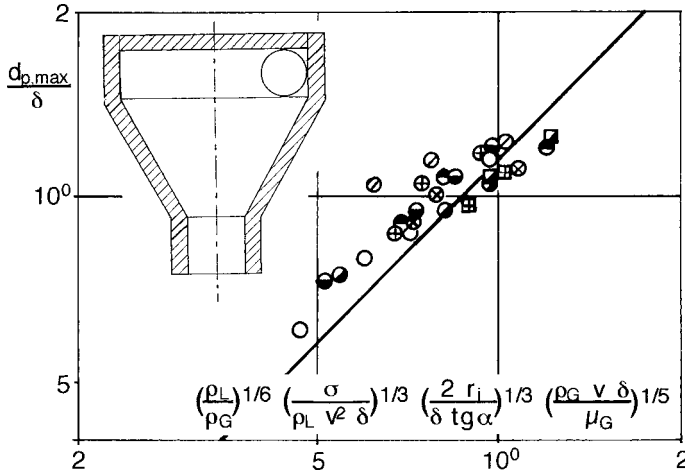


Fig. 44 Film disintegration by wave oscillation. Measurements with hollow cone nozzles of different geometry and with lime-water-suspensions. For an explanation of signs see the original publication [20]. The fitted line is valid for pure liquids.

The fitting line corresponds to the analytical expression for the wave disintegration of pure liquids by hollow cone nozzles:

$$\frac{d_{p,max}}{\delta} = 1.13 \left(\frac{\rho_L}{\rho_G} \right)^{1/6} \left(\frac{\sigma}{\rho_L v^2 \delta} \right)^{1/3} \left(\frac{2r_i}{\delta \tan \alpha} \right)^{1/3} \left(\frac{\rho_G v \delta}{\mu_G} \right)^{1/5} \quad (12.39)$$

(r_i – orifice radius; α – angle of the hollow conical liquid sheet produced)

This pi-equation, taken from [20], represents a physical correlation. It is *absolutely useless* for scaling up purposes: To predict the target quantity $d_{p,max}$, knowledge of another target quantity, namely the film thickness, δ , is also necessary!

By exceeding a certain discharge velocity, turbulence forces increase to such an extent that film disruption takes place immediately at the orifice. Now, the droplet size is independent of the film thickness. This state of atomization is described by the critical Weber number. Measuring data obtained with hollow cone nozzles of different geometry and pure liquids as well as lime-water-suspensions are represented in **Fig. 45**. $We_{p,crit}$ and the Ohnesorge number are formed by the largest stable droplet diameter, $d_{p,max}$. The pi-equation reads:

$$We_{p,crit} = 4.5 \times 10^4 Oh_p^{1/6} \quad (12.40)$$

This equation, also taken from [20], is equally useless for scaling up purposes, because the (unknown!) target quantity $d_{p,max}$ appears in both dimensionless numbers. In the combination

$$We_{p,crit} Oh_p^2 \equiv We/Re \equiv v \mu / \sigma$$

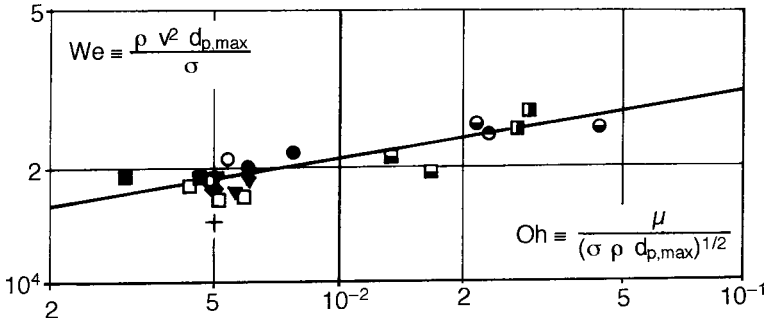


Fig. 45 Liquid film atomization by turbulent forces. For an explanation of signs see the original publication [20].

a new pi-number is obtained which does not contain $d_{p,max}$:

$$We_{p,crit} = 1.97 \times 10^4 (v \mu / \sigma)^{0.154} \quad (12.40a)$$

This process equation can now serve for scaling up $d_{p,max}$.

Example 29: The hanging film phenomenon

In a countercurrent flow of liquid and gas in a vertical tube, at a critical gas velocity, v_G , a situation arises at which the liquid supply to the tube is interrupted and the liquid film inside the wall is held at rest; see sketch, Fig. 46. Increasing the gas velocity above v_G causes the film attachment point to rise in the tube. Lowering the gas velocity below v_G causes the liquid film to move down the tube. Most of the researchers reported little or no difference between these two critical v_G values. However, Wallis and Makkenchery [150] stated that, for small tube diameters ($D = 6$ mm) and air-water mixtures, the velocities differed by a factor of about two. These researchers found that the lower critical v_G increased with pipe diameter, D , and was independent of it for large values of D . For acrylic glass the final result was found to be:

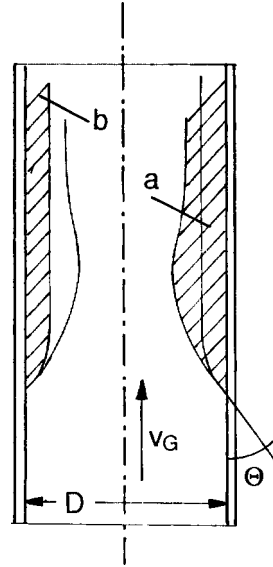


Fig. 46 Schematic representation of the hanging film phenomenon (as sketched in [150]).

$$Ku \equiv \frac{\rho_G^{1/2} v_G}{(\sigma g \Delta \rho)^{1/4}} \approx 3.2 \quad Bd \equiv D \left(\frac{g \Delta \rho}{\sigma} \right)^{1/2} > 40 \quad (12.41)$$

To obtain a linear expression for the characteristic lengths (D), the square-root of the usual Bond number was used. $Ku \equiv (Fr^*We)^{1/4}$ is the Kutateladze number. Physical properties of the liquid have no subscript, $g\Delta\rho \equiv g(\rho - \rho_G)$.

Russian researchers [115] have found that in glass tubes the same Ku value is obtained at $Bd > 6$. This discrepancy caused *Eichhorn* [26] to carry out a detailed dimensional-analytical examination. He first discovered that the lower critical v_G corresponded to a critical film thickness and to a critical shear rate in the phase boundary G/L. Therefore, there are three parameters independent of each other, which could be regarded as target quantities. However, v_G can be measured more accurately and more easily than the others, therefore, it is accepted as the target quantity.

The contact angle, Θ , between the liquid and the tube wall is viewed as an essential material parameter. With this quantity, the wettability of the tube wall is taken into account. Therefore, the relevance list reads:

$$\{v_G; D; \rho_G, \rho, \mu_G, \mu, \sigma, \Theta; g\Delta\rho\} \quad (12.42)$$

$9 - 3 = 6$ dimensionless numbers will be produced. Three of them are: $\rho_G/\rho, \mu_G/\mu, \Theta$.

With the remaining six x-quantities, a dimensional matrix is formed $\{\rho, D, \mu|v_G, \sigma, g\Delta\rho\}$ which is transformed, in three steps, to the matrix of unity. The following pi-numbers are produced:

$$\Pi_1 \equiv \frac{v_G \rho D}{\mu} \equiv Re; \quad \Pi_2 \equiv \frac{\sigma \rho D}{\mu^2}; \quad \Pi_3 \equiv \frac{g\Delta\rho \rho D^3}{\mu^2} \quad (12.43)$$

To obtain the two pi-numbers presented in (12.41), these numbers are combined as follows:

$$Bd \equiv (\Pi_3 \Pi_2^{-1})^{1/2}; \quad Ku \equiv \Pi_1 (\Pi_2 \Pi_3)^{-1/4} (\rho_G/\rho)^{1/2} \quad (12.44)$$

As the third pi-number, a pure material number can be produced out of the three:

$$\Pi_3 \Pi_2^{-3} \equiv \frac{g\Delta\rho \mu^4}{\sigma^3 \rho^2} \equiv CB \quad (12.45)$$

Eichhorn [26] names it the “capillarity-buoyancy number” CB.

From (12.42) we obtain the following pi-set:

$$\{Ku, Bd, CB, \rho_G/\rho, \mu_G/\mu, \Theta\} \quad (12.46)$$

Eichhorn [26] considers the air velocity, v_G^* , to be a more appropriate parameter, this corresponding to the gas-sided friction at a dry tube wall. (An explanation for this decision is not clear: in the turbulent flow range the gas friction number, ζ , is practically independent of the Reynolds number!)

$$\frac{\zeta}{2} \equiv \frac{\tau_w}{\rho_G v_G^2} = \left(\frac{v_G^*}{v_G} \right)^2 \quad (12.47)$$

Considering (12.47), the Ku number is transformed to

$$Ku^* \equiv Ku \sqrt{\zeta/2} \equiv \left(\frac{\tau_w^2}{\sigma g \Delta \rho} \right)^{1/4} \quad (12.48)$$

The experimental data [115, 150] are given in **Fig. 47** in the form $Ku^* = f(Bd)$. This means that the contact angle, θ , has not yet been considered. The discrepancy due to the different θ values can be clearly seen.

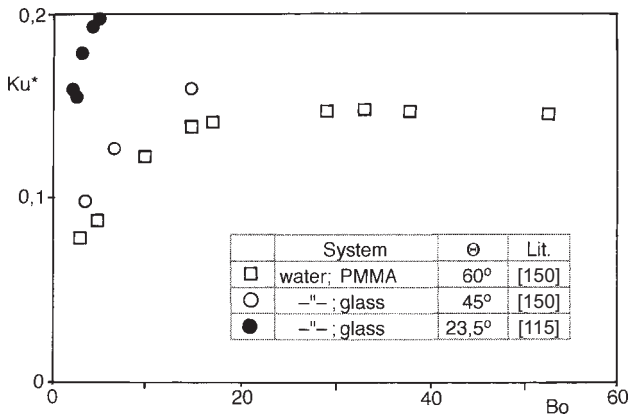


Fig. 47 The correlation $Ku^* = f(Bd)$ for the hanging film at the lower critical air velocity.

Fig. 48 clearly shows that the contact angle, θ , is satisfactorily taken into account by the function $Ku^* \times \sin \theta = f(Bd)$. Only now is it also obvious that the “capillarity-buoyancy number” CB exerts no influence on the hanging film. The liquid viscosity, μ , proves to be irrelevant. This is not surprising because of the fact that the respective measurement were executed in the turbulent flow range, $Re = 4.15 \times 10^3 - 1.42 \times 10^5$.

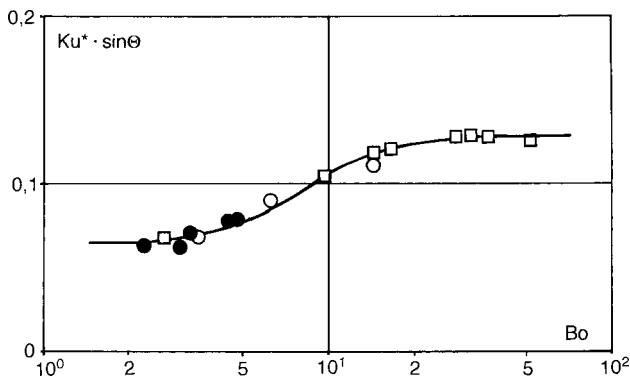


Fig. 48 The correlation $Ku^* \times \sin \theta = f(Bd)$. The fitted line corresponds to eq. (12.49), from [26].

The fitted line in Fig. 48 corresponds to the process equation

$$\text{Ku}^* \times \sin \Theta = 0,096 \left(1 + \frac{1}{3} \left[\frac{(\text{Bd}/8)^3 - 1}{(\text{Bd}/8)^3 + 1} \right] \right) \quad (12.49)$$

This example also proves that in physics minor causes (here the contact angle, Θ) can result in major effects.

Example 30: The production of liquid/liquid emulsions

Liquid/liquid emulsions consist of two (or more) non-miscible liquids. Classical examples of this are oil in water (O/W) emulsions, for example milk, mayonnaise, lotions, creams, water soluble paints, photo emulsions, and so on. As appliances, teeth-rimed rotor-stator emulsifiers and colloid mills, as well as high-pressure homogenizers, are used.

All of these utilise a high energy input to produce very fine droplets of the disperse (mostly oil) phase. The aim of this operation is, as in Example 28, the narrowest possible droplet size distribution. It is normally characterized by the “Sauter mean diameter” d_{32} [125] or by the median d_{50} of the size distribution. Therefore d_{32} and d_{50} , respectively, represent the target quantity of this operation.

The *characteristic length* of the contacting chamber, e.g., the slot width between rotor and stator in teeth-rimed emulsifiers or the nozzle diameter in high pressure homogenizers (utilizing the shear of the high-speed liquid jet) will be denoted as d .

As *material parameters*, the densities and the viscosities of both phases as well as the interfacial tension σ must be listed. We incorporate the material parameters of the disperse phase ρ_d and μ_d in the relevance list and note separately the material numbers ρ/ρ_d and μ/μ_d . Additional (dimensionless) material parameters are the volume ratio of both phases, φ , and the mass portion c_i , of the emulsifying agent (surfactant) [e.g., given in ppm].

The *process parameters* have to be formulated as intensive quantities. In appliances which display *two* degrees of freedom, where the liquid throughput, q , of both liquids which have to be emulsified is adjusted independently of the power input P , the power per liquid volume, P/V , as well as the duration of it, namely the residence time of the throughput, $\tau = V/q$, must be considered:

$$(P/V) \tau = E/V \text{ [M L}^{-1} \text{ T}^{-2}] \quad (12.50)$$

In appliances with only *one* degree of freedom (e.g., high-pressure homogenizers) power is being introduced by the liquid throughput itself. Here, the relevant intensively formulated quantity is therefore jet power per liquid throughput, P/q . Due to the fact that in nozzles $P \propto \Delta p q$, this results in

$$P/q = (\Delta p q)/q = \Delta p \text{ [M L}^{-1} \text{ T}^{-2}] \quad (12.51)$$

Therefore, the volume-related energy input E/V and the throughput-related power input P/q ($\triangleq \Delta p$) represent homologous quantities of the same dimension. For the sake of simplicity, Δp will be introduced in the relevance list.

Now, this 6-parametric relevance list (the dimensionless parameters ρ/ρ_d , μ/μ_d , φ , c_i are anticipated) reads

$$\{d_{32}; d; \rho_d, \mu_d, \sigma; \Delta p\} \quad (12.52)$$

The corresponding dimensional matrix

	ρ_d	d	σ	Δp	μ_d	d_{32}
M	1	0	1	1	1	0
L	-3	1	0	-1	-1	1
T	0	0	-2	-2	-1	0
$M + T/2$	1	0	0	0	1/2	0
$3M + L + 3 T/2$	0	1	0	-1	1/2	1
$-T/2$	0	0	1	1	1/2	0

delivers the remaining three dimensionless numbers:

$$\Pi_1 \equiv \frac{\Delta p d}{\sigma} \equiv \text{Eu} \text{ We} \equiv \text{La} \text{ (Laplace number)}$$

$$\Pi_2 \equiv \frac{\mu_d}{(\rho_d d \sigma)^{1/2}} \equiv \frac{\text{We}^{1/2}}{\text{Re}} \equiv \text{Oh} \text{ (Ohnesorge number)}$$

$$\Pi_3 \equiv d_{32}/d$$

The complete pi-set is given as

$$\{d_{32}/d, \text{La}, \text{Oh}, \rho/\rho_d, \mu/\mu_d, \varphi, c_i\} \quad (12.53)$$

Assuming a quasi-uniform power distribution into the throughput or into the volume, a characteristic length of the contacting chamber becomes irrelevant. In the relevance list, eq. (12.52), the parameter d must be cancelled. The target number $\Pi_3 \equiv d_{32}/d$ has to be dropped and the dimensionless numbers La^* and Oh^* are formulated with d_{32} instead of d . At given constant material conditions ($\rho/\rho_d, \mu/\mu_d, \varphi, c_i = \text{const}$), the process characteristics will be represented in the following pi-space:

$$\text{Oh}^{*-2} = f(\text{La}^* \text{Oh}^{*2}) \rightarrow d_{32} \left(\frac{\rho_d \sigma}{\mu_d^2} \right) = f \left(\Delta p \frac{\mu_d^2}{\rho_d \sigma^2} \right) \quad (12.54)$$

Schneider and Roth [128] confirmed this dependence with two colloid mills using the scale $\mu = 1 : 2.2$, see Fig. 49. For the material system vegetable oil/water and $\varphi = 0.5$ they found the following process equation:

$$d_{32} = 4.64 \times 10^5 \Delta p^{-2/3} \quad d_{32} [\mu\text{m}]; \Delta p [\text{M}/(\text{L T}^2)] \quad (12.55)$$

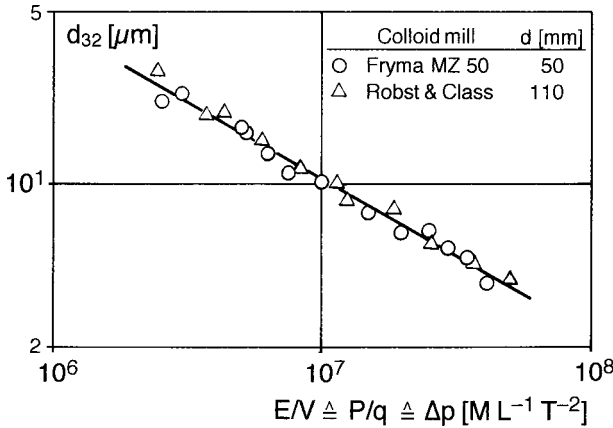


Fig. 49 Relationship $d_{32} = f(\Delta p)$ for two colloid mills of different size. Material system: vegetable oil/water and $\varphi = 0.5$; from [128].

H. Karbstein [58, 59] investigated two rotor-stator emulsifiers (teeth rings, ZKDM), two colloid mills (CM) and a high-pressure homogenizer (HPH). She also proved that the results (d_{32}) from both of the first mentioned emulsifiers *did not* depend on the chamber size and were only slightly dependent on the type of apparatus. Therefore, eq. (12.54) is confirmed. In contrast, in the case of HPH the nozzle diameter as well as the application of counter-pressure plays a role, see Fig. 50. (For details of the geometry of the contacting chamber see [58].)

The measurements were executed with the material system 30 % vegetable oil/water under the addition of a fast-absorbing emulsifying agent (Laurylthylenoxid LEO-10). The viscosity of the emulsion was 30 mPa s at $\dot{\gamma} = 1/\text{s}$. φ_d as well as μ/μ_d were proved to be irrelevant. In all measurements a weaker dependence $d_{32}(\Delta p)$ than in Fig. 49 was found:

$$d_{32} \propto \Delta p^{-1/2} \quad (12.56)$$

The serious disadvantage of dimensional representations $d_{32}(\Delta p)$, eq. (12.55–56), as compared to the dimensionless one, eq. (12.54), is that these relationships are solely valid for the material system used, for which the physical properties (σ , ρ_d , η_d) were not even specified. Therefore, a belated conversion in the pi-space, eq. (12.54), is not feasible.

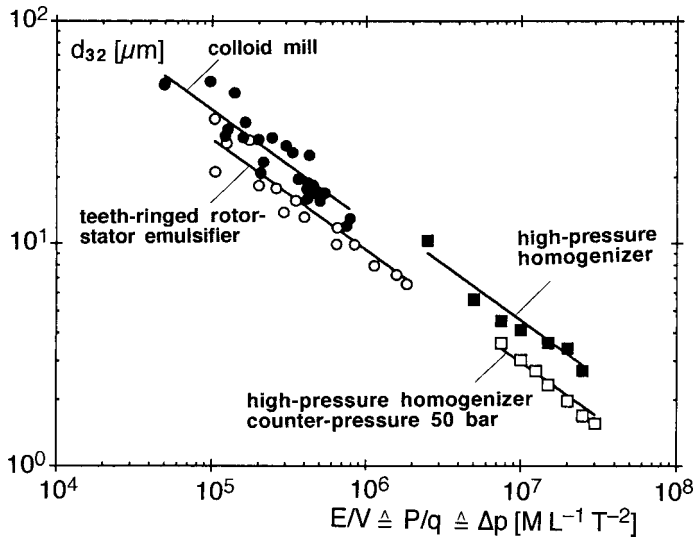


Fig. 50 Relationship $d_{32} = f(\Delta p)$ for different emulsifiers. ZKDM – teeth ringed rotor-stator emulsifier, CM – colloid mill, HPH – high-pressure homogenizer. Details on the material system in text; from [59].

The readiness to evaluate the emulsification tests in a dimensionless form according to eq. (12.54) obviously still does not exist. The insight is missing that a correct dimensional-analytical evaluation of measurements does not merely lead to their dimensionless formulation, but also represents the only chance to extend the test results as a binding scale-up basis to other material systems. This attitude is all the more surprising because all the material parameters (σ , ρ_d , η_d) necessary for it can be presupposed as available.

Jet emulsifiers belong to high pressure homogenizers, see Fig. 51. They display only one degree of freedom. For them, the same pi-space, eq. (12.53), applies. The coarse emulsion, produced by a nozzle, is conveyed under high pressure through the tiny bore-holes of the jet emulsifier. Extremely high shear forces disintegrate the primary droplets into finer ones and, in this manner, produce a stable, extremely fine emulsion.

Figure. 52 represents the correlation $d_{32}/d = f(La)$ produced by the emulsification of the material system paraffin oil/water ($\varphi = 0.5$; $\sigma = 0.7 \text{ mN/m}$; emulsifying

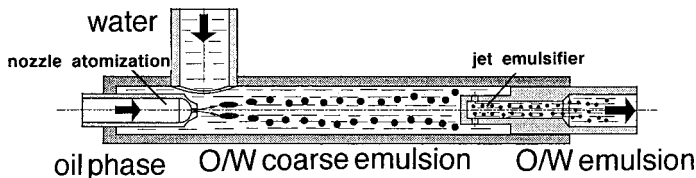


Fig. 51 Sketch of the jet emulsifier; from [64].

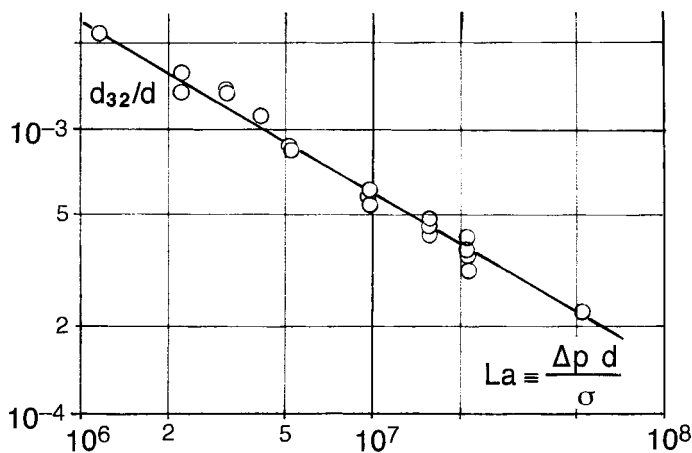


Fig. 52 Process equation in emulsification with jet emulsifier according to Fig. 51; from [64]. The fitting line corresponds to eq. (12.75).

agent Tween 80/Arlacel 80) in a jet emulsifier with bore-holes of $d = 0.75$ mm. (In these measurements, d was been varied!) The result reads [64]:

$$d_{32}/d = 9.15 \text{ La}^{-0.6} \quad \text{Oh}, \mu_d/\mu = \text{const} \quad (12.57)$$

In [143], the production of an extremely fine polyisocyanate emulsion in polyol with a jet emulsifier, is reported. The results obtained are evaluated in the pi-space (12.53). The process equation reads:

$$d_{32}/d = \text{const} \text{ La}^{-0.6} \text{ Oh}^{0.47} (\mu_d/\mu)^{0.025} \quad (12.58)$$

Here, the constant is only dependent on the type of apparatus. The remaining pi-numbers, see eq. (12.53), have presumably not been varied.

Example 31: Fine grinding of solids in stirred media mills

The fine grinding of solids in mills of different shape and mode of operation is used to produce very fine particles with a narrow particle size distribution. Therefore, as in the previous example, the target quantity is the median value d_{50} of the particle size distribution.

The characteristic *length* of a given mill type is d .

The *physical properties* are given by the particle density, ρ_p , the energy per fissure area, β , and the tensile strength, σ , of the material. Should there be additional relevant material parameters, these can be easily converted to dimensionless material numbers by the above mentioned ones.

As the process parameter, the energy input per unit mass, $E/\rho V$, must be taken into account. The relevance list reads:

$$\{d_{50}; d; \rho_p, \beta, \sigma; E/\rho V\} \quad (12.59)$$

	ρ_p	d	β	$E/\rho V$	σ	d_{50}
M	1	0	1	0	1	0
L	-3	1	0	2	-1	1
T	0	0	-2	-2	-2	0
M + T/2	1	0	0	-1	0	0
3 M + L + 3 T/2	0	1	0	-1	-1	1
-T/2	0	0	1	1	1	0

The pi-set reads

$$\{d_{50}/d, (E/\rho V)\rho_p d/\beta, \sigma d/\beta\} \quad (12.60)$$

Assuming a quasi-uniform energy input in the mill chamber, its characteristic diameter, d , will be irrelevant. Then the pi-set can be reduced to

$$\{(E/\rho V)\rho_p d_{50}/\beta, \sigma d_{50}/\beta\} \rightarrow d_{50} \frac{\sigma}{\beta} = f \left(\frac{E}{\rho} \frac{\rho}{V} \frac{\rho}{\sigma} = \frac{E}{V} \frac{\rho}{\sigma} \right) \quad (12.61)$$

In the case of unknown physical properties, σ and β , the above dependence is reduced to $d_{50} = f(E/\rho V)$ which can then be used for scale-up of a given type of mill and a given grinding material. The connection in eq. (12.61) suggests working with E/V instead of E/m .

For fine and very fine grinding of, e.g., limestone for paper and pottery manufacturing, *bead mills* are widely used. The beads of steel, glass or ceramic have diameters of 0.2–0.3 mm and occupy up to 90 % of the total mill volume ($\phi = 0.9$). They are kept in motion by perforated stirrer discs while the liquid/solid suspension is pumped through the mill chamber.

Mill types frequently in use are the stirred disk mill, centrifugal fluized bed mill, and annular gap mill.

H. Karbstein et al. [60] pursued the question of the smallest possible size of a laboratory bead mill which would still deliver reliable data for scale-up. In differently sized model appliances ($V = 0.25$ – 25 l), a sludge consisting of limestone ($d_{50} = 16 \mu\text{m}$) and a 10 % aqueous Luviscol solution (mass portion of solids $\varphi = 0.2$) was examined.

In **Fig. 53** the dependence $d_{50} = f(E/\rho V)$ is shown for four stirred disk mills of different sizes. The correlation

$$d_{50} \propto (E/\rho V)^{-0.43} \quad E/\rho V \leq 10^4 \quad (12.62)$$

also applies to the smallest mill shape ($V = 0.25$ l), but to obtain the same d_{50} values, a three-fold energy input is necessary.

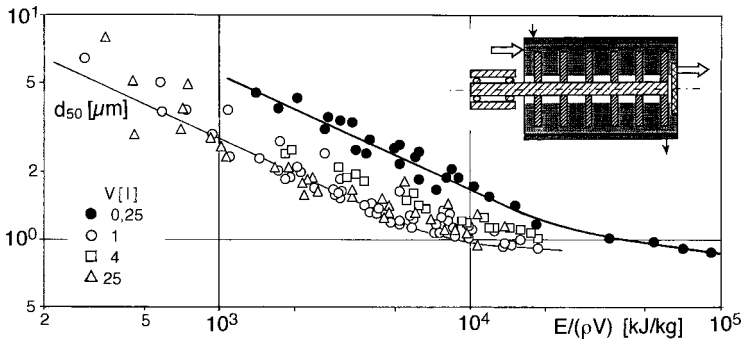


Fig. 53 Correlation $d_{50}(E/\rho V)$ for four differently shaped stirred disk mills; from [60].

Furthermore, it was found that the correlation (12.62) is valid only for $d_{50} \geq 1 \mu\text{m}$. To obtain even finer particles, a considerably higher energy input, according to (12.62), is necessary. A possible reason for this finding could be that, in this size range, the particle strength has a greater effect. It is also possible that the finest particles escape faster from the working zone between two beads [60].

The same result is also found in centrifugal fluidized bed mills, **Fig. 54**. Finer particles than $1 \mu\text{m}$ are not produced.

These facts and the scattering of results in differently sized mills made a systematic investigation of the grinding process necessary [73]. The grinding process

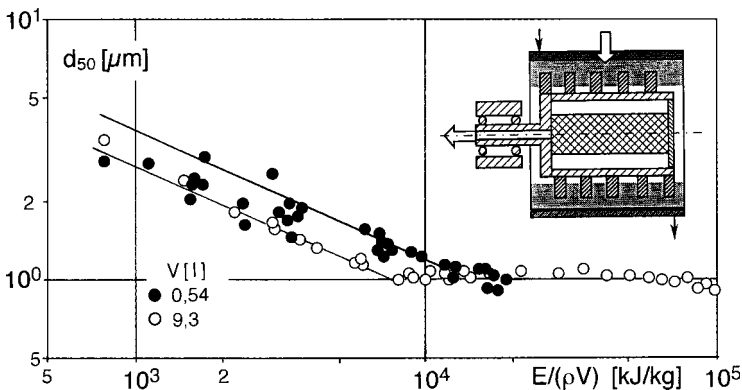


Fig. 54 The relationship $d_{50}(E/\rho V)$ for two centrifugal fluidized bed mills of different size, from [60].

in bead mills is determined by the frequency and the intensity of the collision between beads and grinding medium. According to this assumption, the grinding result remains constant, if both of these quantities are kept constant. The intensity of the collision between beads is essentially given by the kinetic energy of the beads:

$$E_{\text{kin}} \propto m_M u^2 \propto V_M \rho_M u^2 \propto d_M^3 \rho_M u^2 \quad (12.63)$$

(d_M , ρ_M – diameter and density of the beads; u – tip velocity of the stirrer). On the other hand, the collision frequency also depends on the size of the mill chamber and on the overall mass-related energy input. To achieve the same grinding result in differently sized bead mills, E_{kin} as well as $E/\rho V$ have to be kept idem.

The input of mechanical energy can be measured from the torque and the rotational speed of the perforated discs. The kinetic energy can be calculated from eq. (12.63).

The above assumption [73] was convincingly confirmed in a stirred media mill with a perforated stirrer disc of given size ($V = 5.54$ l) at a constant energy input per unit mass, $E/\rho V = 10^3$ kJ/kg. In batch-wise performed measurements, the kinetic energy of the beads has been varied by the tip speed of the perforated disk stirrers as well as by the density of the grinding media (glass, steel) and, in particular, by the bead diameter ($d_M = 97 - 4\,000$ μm).

With increasing E_{kin} the particle size diminishes at first, but afterwards increases from a certain E_{kin} value onwards. If one considers the implemented specific energy to be a *product* of the intensity and frequency of the collision between beads and grinding medium, it follows that, at $E/\rho V = \text{const}$ and with an increasing intensity of the collision, the frequency has to diminish and this results in a coarser product. This also explains why, in the previously discussed investigation [60], particles no finer than $d_{50} \approx 1$ μm were found, see Fig. 53 and 54.

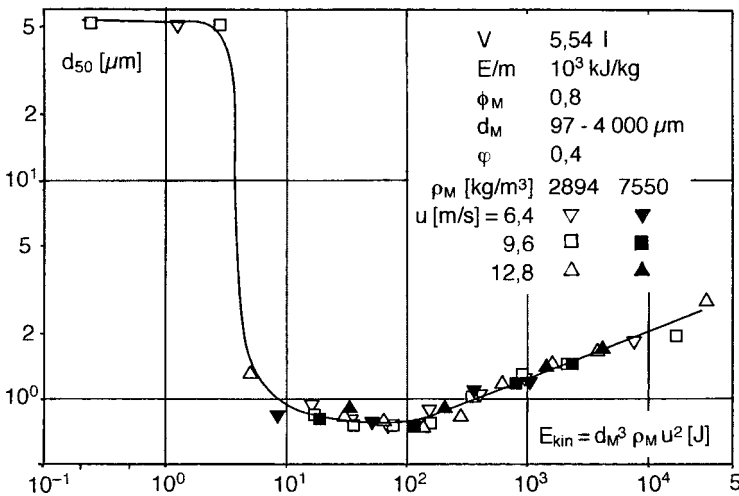


Fig. 55 The relationship d_{50} (E_{kin}) at $E/\rho V = 10^3$ kJ/kg in a stirred media mill with perforated stirrer disc ($V = 5.54$ l). Limestone/water ($\varphi = 0.4$; $\phi = 0.8$); from [73].

In Fig. 56, the results shown were obtained in three stirred media mills with a perforated stirrer disc of different sizes ($V [l] = 0.73; 5.54; 12.9$). The results are not satisfactory with respect to the scale-up rule. Here, too, it can be seen that small mills ($V < 1$ l), under otherwise identical conditions, produce a coarser product than larger ones. A satisfying correlation is achieved by plotting the median particle size, d_{50} , versus the mean stress intensity. This is defined as stress intensity of the grinding media multiplied by the term which takes into account the stress intensity distribution.

Due to the fact that [73] the energy input was split into mass-related ($E/\rho V$) and kinetic (E_{kin}) energies, an earlier paper concerning the emulsification process should also be referred to [69]. As emulsifier, a teeth-rimmed rotor-stator machine was used and the results (d_{50}) were correlated with both power per unit volume (P/V) and work per unit volume ($P\tau/m^3$). This paper is also of special interest because the evaluation is performed in a dimensional-analytical manner. The dimensionally formulated result reads:

$$d_{50} \propto (E/V)^{-0.3} (P/V)^{-0.1} \quad (12.64)$$

The behavior of suspensions having mean particle diameters of $d_p < 1 \mu m$ is essentially governed by the electrostatic interaction between particles. Here, the attractive *van der Waals's* forces are dominant and this leads, in particle collisions, to solid agglomerates and thus to a change in the rheological characteristics of suspensions. Each particle size distribution represents a steady state. It can, however, be influenced by an electrostatic stabilization. The newest investigations [82] in stirred media mills show that it is possible to achieve values of $d_{50} \approx 0.1 \mu m$ simply by influencing the chemical potential by a change in the pH.

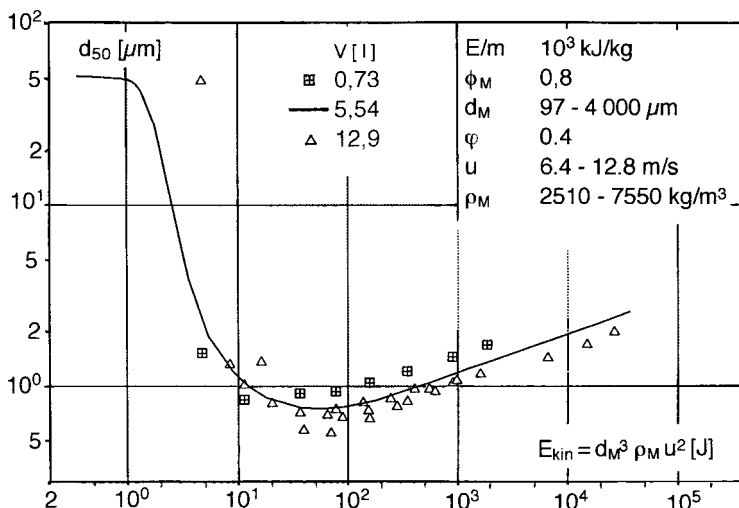


Fig. 56 The relationship $d_{50} (E_{kin})$ at $E/\rho V = 10^3$ kJ/kg for three differently sized stirred media mills with perforated stirrer discs; from [73].

Here we are primarily interested in the question of how the results obtained can be described in a dimensionless way by the appropriate physical properties and in this way to gain additional information on the comminution process.

In this context, the particle strength $\sigma \equiv F/A$ [$\text{Nm}^{-2} \triangleq \text{M L}^{-1} \text{T}^{-2}$] obviously represents an important material parameter. For limestone with particle diameters in the range of $d_p = 10^0 - 10^2 \mu\text{m}$ we can proceed, according to Fig. 11, from the correspondence:

$$\sigma = 1 \times d_p^{-1}; \quad \sigma \text{ in N mm}^{-2}; d_p \text{ in mm}$$

As a second material parameter, we nominate the specific cleavage area energy β [$\text{Jm}^{-2} \triangleq \text{M T}^{-2}$]. It is regarded as such by *H. Rumpf* in [123] and in [72] it is named as a typical material constant (for glass its values lies between 1 and 10 J/m^2 [123]; for china, 28 J/m^2 [72]; for Si_3N_4 , 20 – 40 J/m^2 ; and for SiC , 20 – 30 J/m^2 [83].) No information, however, is available on whether these values remain constant when the cleavage area is essentially diminished. For limestone we will estimate the value of β to be 10 $\text{J/m}^2 = 10 \text{ kg/s}^2$.

Using these values for limestone, with particles in the range of $d_p = 10^0 - 10^2 \mu\text{m}$, a constant value of the target number

$$d_{50} \frac{\sigma}{\beta} = 100$$

results, and the process number

$$\frac{E}{V \sigma}$$

is stretched with respect to $\sigma_B \propto d_p^{-1}$ (Fig. 11) and is directly proportional to the diminishing d_{50} . This means that, from one side, the process essentially depends on the particle strength of the comminuted material, whereas from the other side the energy should not be referred to the material mass, but to its volume. Therefore, it should be checked whether the results discussed could be better correlated if the energy input were formulated differently.

K. Schönert [10, 129] favors the specific energy of fracturing E_B as the material specific parameter in comminution. This is the energy supplied to the particle mass E_B/m_p [$\text{L}^2 \text{T}^{-2}$] or to the particle volume E_B/V_p [$\text{M L}^{-1} \text{T}^{-2}$] up to the breaking point. The introduction of this quantity, instead of the specific cleavage area energy β [M T^{-2}], would have no influence on the dimensional-analytical treatment of the comminution process – as can be seen from eq. (12.61). Only the process number would result in other numerical values.

Example 32: Scale-up of flotation cells for waste water purification

In this example, the development of a new flotation technique for waste water treatment is discussed. Dimensional analysis has been used since the onset of this work. It concerns the so-called Induced Air Flotation, IAF, which is an alternative to the well-known Dissolved Air Flotation, DAF, the latter having already been discussed in Example 7.

This example is divided into three parts. In (a), the development of a new, self-aspirating and radially discharging funnel-shaped nozzle is presented. In (b), a flotation cell with two spatially separated spaces is described. The inner chamber is used for contacting gas bubbles with flocks and the annular ring around it is needed for the tranquilization of liquid throughput. This facilitates the complete separation of the flocks from the biologically purified waste water. It is shown how the flotation kinetics can be determined in this continuously run cell and how this knowledge is used to scale-up a full-scale flotation plant. On the contrary, in (c), data from batch-wise performed experiments are used to evaluate the flotation kinetics and to scale-up continuously run full-scale flotation cells.

a) Development of a self-aspirating and radially discharging funnel-shaped nozzle

The funnel-shaped nozzle has been conceived as a means of gas/liquid contacting in a new class of flotation cells for waste water purification and for the removal of activated sludge from biologically purified waste water, see Fig. 57. It essentially consists of a cone with an angle of 90° which serves as the deflecting element for the liquid propulsion jet. It is surrounded by a housing which forms, with the cone, an annular channel. According to the *Bernoulli* principle, a pressure drop develops at this point and this is used to suck in the gas. The gas/liquid dispersion formed in this manner is discharged radially with a low clearance over the floor of the whole flotation cell. After losing the kinetic energy of the free jet, it disintegrates to a swarm of tiny gas bubbles which then slowly rise to the surface of the liquid.

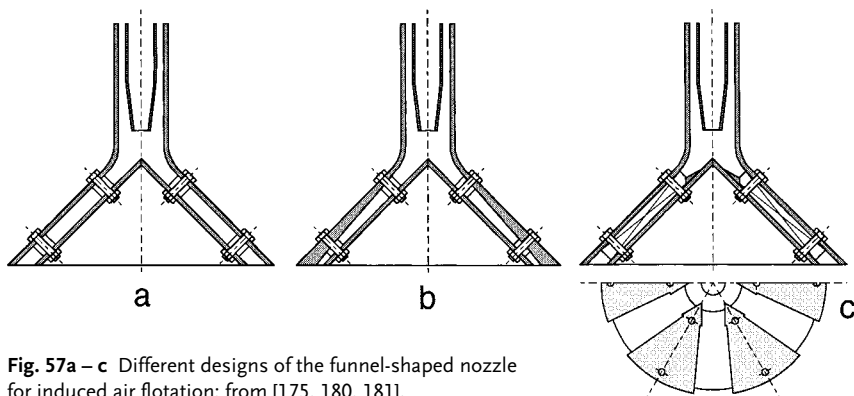


Fig. 57a – c Different designs of the funnel-shaped nozzle for induced air flotation; from [175, 180, 181].

The channel of the funnel-shaped nozzle, shown in **Fig. 57 a** [175], displays a constant hydraulic diameter, $D_h = D_a - D_i$. Therefore, the annular cross-sectional area steadily increases towards the cone base; a diffuser is formed.

In fact, this nozzle produces coarse bubbles which are not able to float very fine or strongly hydrophilic flocks. A nozzle which displays a channel with a constant hydraulic cross-sectional area, $S_h \approx D_a^2 - D_i^2$, see **Fig. 57 b**, serves this purpose better [180]. It also exerts a stronger suction and produces a larger gas throughput. This has to be throttled in order to prevent tiny gas bubbles from coalescing to bigger ones: The gas-side pressure drop, Δp_G , represents therefore an additional, freely eligible, parameter here.

However, this design has an inherent disadvantage, namely that the annular channel at the cone base has to be narrow. With cone base diameters of $D_c \gg 0.5$ m, a possible clogging can take place, especially if the nozzle is used for waste water treatment.

The ring channel can be enlarged to a sufficient extent, if one starts from a constant hydraulic diameter (**Fig. 57 a**) and divides it into segments ("star-shaped nozzle"); see **Fig. 57 c**. In addition, by the widening of the cone angle at the top of the cone, a trip edge [181] is formed, by which the impinging liquid jet is better spread out over the entire cross-sectional area of the annulus.

The relevance list for the suction characteristic of the "star-shaped nozzle":

Target quantity:	self aspirated gas throughput, q_G
Geometric parameters:	diameter of the propulsion jet nozzle, d cross-sectional area of the annular channel, s number of segments, z diameter of the cone base, D_c ($\triangle$ channel length $L_c = \sqrt{2} D_c/2$)
Material parameters:	liquid density, ρ
Process-related parameters:	liquid throughput, q_L liquid head above the nozzle, H' gravitational acceleration, g

This gives:

$$\{q_G, d, s, z, D_c, \rho, q_L, H', g\} \quad (12.65)$$

These nine dimensional parameters deliver the following six pi-numbers:

$$\{q_G/q_L, q_L^2/(d^5 g) \equiv Fr, H'/d, s^{0.5}/d, D_c/d, z\} \quad (12.66)$$

Self-aspirating devices (hollow stirrers, ejectors, funnel-shaped nozzles) have to overcome the hydrostatic pressure $\Delta p_{\text{hydr}} = \rho g H'$. This is taken into account by the combination $Fr' \equiv Fr (d/H')$. By the way, this extended Froude number, Fr' , represents the reciprocal value of the Euler number, Eu :

$$Fr' \equiv Fr(d/H') \equiv \frac{q_L^2}{d^5 g} \frac{d}{H'} = \frac{q_L^2}{d^4 H' g} = \frac{q_L^2 \rho}{d^4 H' g \rho} = \frac{q_L^2 \rho}{d^4 \Delta p_{hydr}} \equiv Eu^{-1} \quad (12.67)$$

It transpires [180] that the result (q_G/q_L) is independent of whether or not the nozzle *pushes* the gas against the hydrostatic pressure or *sucks* it from a space with lower pressure, Δp_G . Both pressures have to be taken as the sum: $\Sigma \Delta p = \rho g H' + \Delta p_G$. Therefore, the process number Fr' is expanded as follows:

$$Fr^* \equiv \frac{q_L^2 \rho}{d^4 (\rho g H' + \Delta p_G)} \quad (12.68)$$

Furthermore, the number of channels, z , produced by segmentation is combined with the cross-sectional area of the channel, s , to produce the total cross-sectional area, $S = z \times s$, of the “star-shaped nozzle”. Preliminary measurements showed that D_c/d does not influence the sucking performance, as long the channels are filled with the dispersion G/L. Therefore, the above 6-parametric pi-space (12.66) is reduced to only a 3-parametric one:

$$\{q_G/q_L, Fr^*, S/d^2\} \quad (12.69)$$

Experiments were carried out with three diameters of the propulsion jet nozzle, d , with a different number of channels, z , and with or without the trip edge. The result is represented in **Fig. 58**. It verifies that the trip edge essentially improves the sucking performance. The fitting line corresponds to the process equation:

$$q_G/q_L = 0.97 \ln (Fr^* d^2/S) + 0.06 \quad (12.70)$$

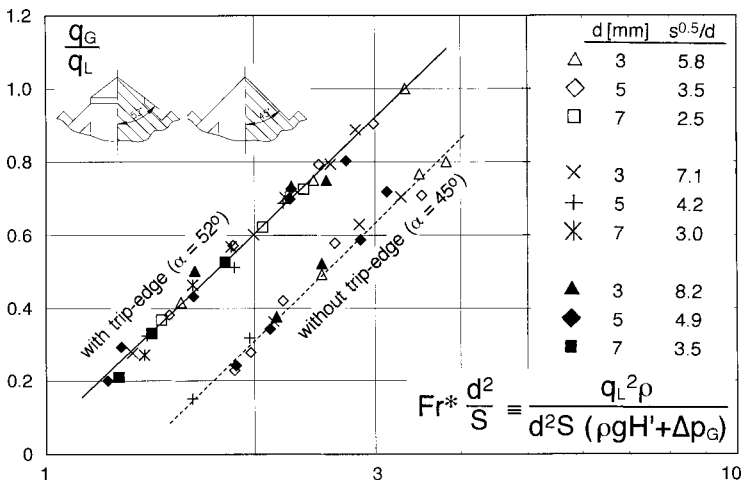


Fig. 58 Sorption characteristic of the “star-shaped nozzle” with and without a trip edge; after [127].

The parameter $S^{0.5}/d$ was varied within the limits 2.5–8.2. The best result was obtained at the lowest value.

b) **Scaling up a flotation cell with spatially separated aeration and tranquilization space for continuously carried out induced air flotation (IAF) in waste water treatment**

Following the biological waste water purification, an easy separation of activated sludge flocks can be achieved by flotation. To facilitate this separation, the flotation cell is subdivided into two parts, each having equal superficial areas, **Fig. 59**. The inner cylindrical vessel serves as the aeration chamber, this being equipped with the forementioned funnel-shaped nozzle. Here, particles are brought into intimate contact with gas bubbles, which cause them to float to the surface and are subsequently removed from it with a skimmer. The liquid throughput, already largely freed from solids, passes the adjacent tranquilising annular space from top to bottom (in a counter-current sense to the floating residual flocks). In the upper area of the annular space, the flocks form a filter which supports the separation.

If we assume that the current in the annulus is not back-mixed, then it has a residence time characteristic of an ideal plug flow. Due to the fact that flotation is a depletion process which obeys the first-order time law, the flotation kinetics is given by the correlation

$$-\ln \frac{\varphi_t - \varphi_\infty}{\varphi_0 - \varphi_\infty} = k_f \tau \quad 12.71$$

k_f [T^{-1}] is the flotation rate constant, τ [T] is the mean residence time, $\tau = V/q$, and φ is the mass portion of solids in the liquid. When liquid samples are taken along

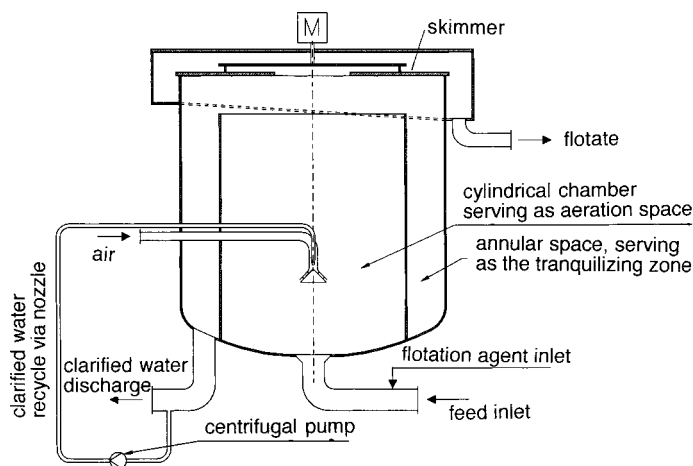


Fig. 59 Sketch of a flotation cell with spatially separated aeration and final solids separation space; from [182].

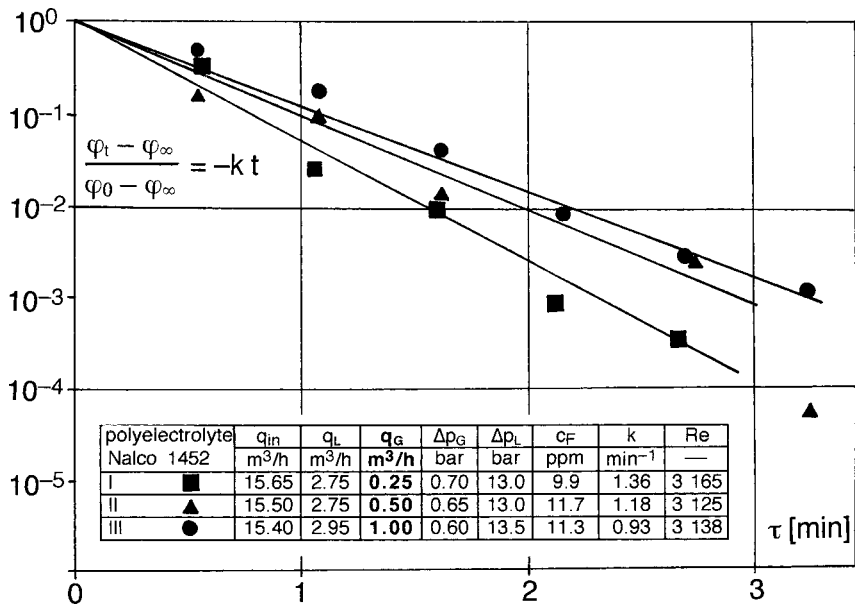


Fig. 60 Flotation kinetics in the material system activated sludge/biologically purified waste water; from [182]. Parameter: gas throughput, q_G .

the annulus and φ is plotted on a single-log-scale according to (12.71), a straight line with a slope of k_f must result if the above assumptions are fulfilled. In our case, this is fully confirmed, see **Fig. 60**. This graph verifies that the fastest flotation occurs at the lowest gas throughput, because here the bubble coalescence is least marked.

In scaling up of this type of flotation cell, the following must be considered. Both cell spaces should have a flow with a superficial velocity of $v \approx 10$ m/h (resulting in a total superficial velocity of $v \approx 5$ m/h). This guarantees a suitable separation of the flotat from the liquid throughput. In addition, the state of flow in the annular space must be laminar. To achieve $Re \approx 2\,000$, an insertion of a tranquilizing grid will possibly be necessary.

For a given liquid throughput, the superficial area of the flotation cell and its diameter can be calculated from the above details. The cell height, H , results from the measured $k_f \tau$ values according to the following reasoning:

$$\tau = V/q_L = S H/q_L; q_L = v S; \tau = H/v \rightarrow H = (k \tau) v/k_f \quad (12.72)$$

(S – superficial area). As a rule, the height of the flotation cell will be $H < 2$ m.

Example: $k_f = 1 \text{ min}^{-1}$; $v = 10 \text{ m h}^{-1}$; depletion $\varphi_t/\varphi_0 = 1.0 \times 10^{-4}$: $H = 1.53 \text{ m}$

c) **Scaling up of a continuously operating flotation cell on the basis of model experiments performed in batch experiments**

In a laboratory flotation cell (e.g., $\varnothing 200 \times 300$ mm) with only one space for aeration and flotation (full back-mixing), samples are taken during the flotation experiment and the mass portion of solids in liquid, φ , is determined. In the representation $\ln \varphi / \varphi_0 = f(t)$ a straight line with the slope k_f (flotation rate constant k_f) results. k_f depends on the material system, on the concentration and nature of flotation aids (flocclulants) as well on the process parameters q_L , q_G and g . Interestingly, in all investigated material systems (dye pigments, plastic particles, printer ink, film emulsions, i.e., Hg halogenides in gelatine) the same proportionality was found:

$$k_f \propto q_G q_L^2 \quad (12.73)$$

see Fig. 61. This expression (12.73) reads in a dimensionless form as:

$$k_F^* \propto \frac{q_G}{q_L} F_T^{3/2} \quad (12.74)$$

k_F^* indicates that $k_f [T^{-1}]$ can always be transformed into a pi-number with the aid of pertinent material parameters (this has been omitted here for simplicity). The

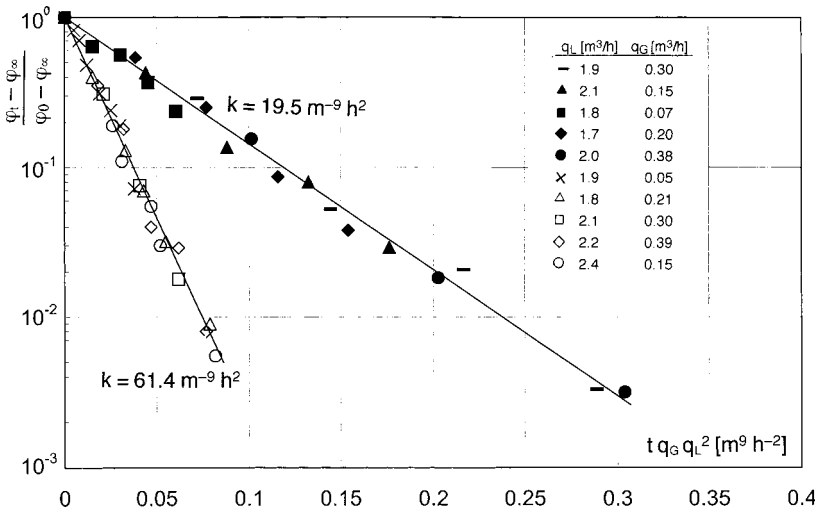


Fig. 61 Dependence of flotation kinetics on process conditions (q_G , q_L) in a batch process. *Full signs*: Cell ($0.5^\circ \times 0.6$ m) with a “star-shaped nozzle” according to Fig. 57 c, $D_c = 80$ mm; throttling of q_G . Material system: waste water from a printing works with 5–6 g TS/l, flocculant: 45 ppm Peratom 815 of Henkel/Düsseldorf; from [127].

Remaining signs: Cell ($0.2^\circ \times 0.2$ m) with a funnel-shaped nozzle according to Fig. 57 a, $D_c = 60$ mm; no throttling of q_G . Material system: Process waters from Novodur® pigment production with ca. 4 g TS/l, flocculant: 410 ppm RO + 15 ppm 417 S of Stockhausen/D-Krefeld; from [179].

inspection of this correlation in two geometrically similar flotation cells ($\mu = 1 : 2$) provided the proof that this approach is correct, see **Fig. 62**.

If the dependence (12.74) is known, a batch-wise operated full-scale flotation cell can be scaled up according to (12.74).

A continuously operated flotation cell with fully back-mixed contents – or a cells-in-series arrangement with N equally sized cells, respectively – is scaled up according to the same recipe as first-order chemical reactions:

$$N = 1 : \tau \equiv \frac{V}{q} = \frac{1}{k_f} \frac{\varphi_0 - \varphi}{\varphi} \quad (12.75)$$

$$N = N : \tau \equiv \frac{V}{q} = \frac{1}{k_f} \frac{\varphi_{N-1} - \varphi_N}{\varphi_N} \quad (12.76)$$

For the mass portion of solids, φ_N , in the outlet from a N cells-in-series it follows:

$$\varphi_N = \frac{\varphi_0}{(1 + k_f \tau)^N} \quad (12.77)$$

With the aid of correlations (12.75) and (12.76) and knowing the k_f value and the accepted φ value of the outlet (φ or φ_N), the mean residence time, τ , of the liquid throughput, q , and the liquid volume of the flotation cell, $V = \tau q$, can be calculated. To ascertain the same k_f value in all cells, $(q_G/q_L) Fr^{3/2} = \text{idem}$ must be kept.

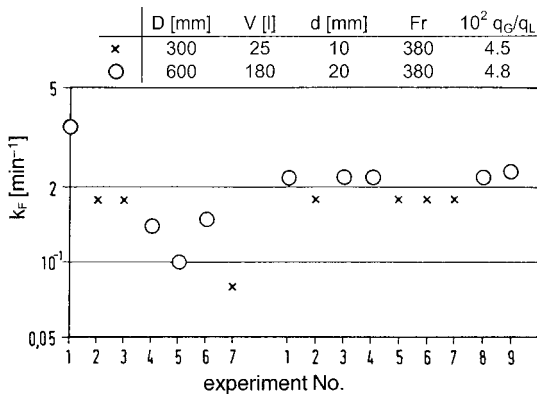


Fig. 62 Checking of the correlation $k_f \propto (q_G/q_L) Fr^{3/2}$ in two geometrically similar flotation cells ($\mu = 1 : 2$). Material system: Washing water from the production of the film emulsion (AGFA) with ca. 25 mg Ag/l; from [176].

Example 33: Description of the temporal course of spin drying in centrifugal filters

The centrifugal filter represents the most frequently used filter centrifuge operating on a horizontal axis. The operating cycle consists of loading, wet spin, cake wash, dry spin, and unloading (peel out). The dry spin requires the most time. It consists of the rapid draining of the mother liquor from the capillary spaces and the slow draining of the surface liquor. The dry spin, which governs the flow rate, has been completed when the equilibrium residual moisture, w_{∞} , is attained in the filter cake.

Before considering the dry spin process in a dimensional analysis, some terms have to be explained and defined.

1. Centrifugal acceleration, b [LT^{-2}], is expressed by the multiple (z) of the gravitational acceleration g : $b = z g$
2. The specific filter cake resistance, a [L^{-2}], is defined by the equation describing the pressure loss, Δp , of the liquid in the porous filter cake at laminar flow:

$$\Delta p = a v \mu h \quad (12.78)$$

$v \equiv q/S$ – liquid flow rate q related to the filter surface S ;
 h – cake height; μ – dynamic viscosity.

3. The porosity, ε [–], of the filter cake is defined as the ratio of pore volume to total volume.
4. The residual moisture, w [–], of the filter cake reflects the ratio of liquid mass to solid mass.
5. The degree of saturation, S [–], is defined as the ratio of the pore volume filled with liquid to the total pore volume:

$$S = w \frac{\rho_s}{\rho_w} \frac{(1 - \varepsilon)}{\varepsilon} = \frac{w}{w_{\max}} \quad (12.79)$$

where ρ_s and ρ_w are the densities of solid matter and water, respectively, and $w_{\max}$ is the cake moisture at saturation.

Equilibrium saturation of the cake, $S_{\infty} \equiv w_{\infty}/w_{\max}$, will initially depend on the physical properties of the filter cake. They are characterized by a , ε , Θ and K . Θ represents the contact angle (degree of wettability) and K any other grain parameters such as roughness and so on. Furthermore, the physical properties of the wash liquid (density, ρ , and surface tension, σ) and, finally, the centrifugal acceleration, b , as process parameter will be of importance:

$$\{S_{\infty}; a, \varepsilon, \Theta, K, \rho, \sigma; b\} \quad (12.80)$$

Four of these eight process-relevant variables are dimensionless, the other four form only one further pi-number:

$$\Pi_1 \equiv \frac{\sigma a}{\rho b}$$

From the relevance list (12.80) it follows that:

$$S_\infty \equiv w_\infty/w_{\max} = f(\Pi_1, \varepsilon, \Theta, K) \quad (12.81)$$

	ρ	a	b	σ
M	1	0	0	1
L	-3	-2	1	0
T	0	0	-2	-2
M	1	0	0	1
$-(3M + L + T/2)/2$	0	1	0	-1
$-T/2$	0	0	1	1

Tests [8] have shown that this relationship is described by the analytical expression

$$S_\infty = \Pi_1^{0.2} f(\varepsilon, \Theta, K) = \left(\frac{\sigma a}{\rho b} \right)^{0.2} f(\varepsilon, \Theta, K) \quad (12.82)$$

and hence by

$$S_\infty = \text{const } (1/z)^{0.2} \quad (12.83)$$

where $z = b/g$. Of course, the numerical values of the constants and the exponent are dependent on the material system under examination.

In order to track the time course of the dewatering process up to an average degree of saturation, $S_m \equiv w_m/w_{\max}$, the parameters time t , viscosity μ of the wash liquid and the geometric parameters of the cake (cake height, h , and cake residual height, h_0 , remaining after peel-out) must be added to the above relevance list. Since we are dealing with a creeping flow in the centrifugal field, ρ is only effective in combination with b : ρb ; compare the form of Π_1 . Apart from the obvious geometric numbers h/h_0 and ah^2 and the dimensionless parameters S_m , ε , Θ , K , two further numbers will be involved:

$$\Pi_1 \equiv \frac{\sigma a}{\rho b} \quad \Pi_2 \equiv \frac{\mu a^{1/2}}{\rho b t} \quad (12.84)$$

Π_1 is the same number as that formed before. The complete pi-set is now:

$$\{S_m, h/h_0, ah^2, \varepsilon, \Theta, K, \Pi_1, \Pi_2\} \quad (12.85)$$

	ρb	a	t	σ	μ
M	1	0	0	1	1
L	-2	-2	0	0	-1
T	-2	0	1	-2	-1
M	1	0	0	1	1
$-(2M + L)/2$	0	1	0	-1	-1/2
$2M + T$	0	0	1	0	1

The tests [8] were performed with small acryl glass spheres of $d_p = 20\text{--}50\text{ }\mu\text{m}$. The material numbers ε , Θ , K remained unchanged. However, h/h_0 , ah^2 , Π_1 and Π_2 were varied by changing b , t and h . It was found that the test results can be correlated in the pi-space $\{S_m, \Pi_2, ah^2\}$, i.e., neither Π_1 nor h/h_0 is significant. **Fig. 63** shows this result. The reciprocal value of Π_2 $(ah^2)^{0.5} \equiv [\rho b t / (\mu a h)]^{-1}$ is plotted on the abscissa. The process equation reads:

$$S_m = 0,26 \left(\frac{\rho b t}{\mu a h} \right)^{-2/3} \quad (12.86)$$

The irrelevance of h_0 is not surprising if the solid particles are neither damaged nor compressed in the peel-out process and if the capillary rise height is $\ll h_0$. The above pi-space should also apply for filter cakes which can be compressed to a greater extent [8]. The fact that Π_1 is not significant at total wetting, proves the irrelevance of σ during the dry-spin process.

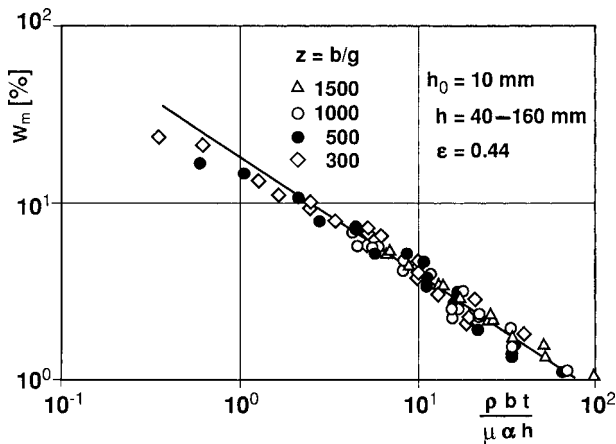


Fig. 63 Temporal course of the average residual moisture, w_m , as a function of $\rho b t / (\mu a h)$ for given geometric and material conditions at centrifugal accelerations of $b = 300\text{--}1500\text{ g}$; from [8].

Example 34: Description of particle separation by means of inertial forces

Let us consider the separation of aerosols (droplet size $d_p = 0.2 - 20 \mu\text{m}$) from a gas stream in a dust separator (e.g., wire filter, cyclone, etc.). The result, the “*fractional degree*” η_f , is characterized by $\eta_f = (\varphi_{\text{in}} - \varphi_{\text{out}})/\varphi_{\text{in}}$.

This target quantity is dependent on the following quantities:

Geometric parameters:	particle diameter, d_p and a characteristic length, D , of the separator
Physical properties:	particle density, ρ_p , density, ρ , and viscosity, μ , of the gas
Process parameters:	gas velocity, v , or the pressure drop $\Delta p \propto v^a$ ($a = 1-2$) because this variable is characteristic of the separating device.

This 7-parametric relevance list

$$\{\eta_f; d_p, D; \rho_p, \rho, \mu; \Delta p\} \quad (12.87)$$

can be streamlined after [17] by a closer examination of the problem and by preliminary tests:

1. For sufficiently small particles *Stokes* law applies. According to this, the *frictional force* is $F_{\text{fr}} = 6 \pi r \mu v = 3 \pi d_p \mu v$. At the same time, the *inertial force* is $F_{\text{mass}} = m b = (\pi/6) \rho_p d_p^3 b$. In the steady-state $F_{\text{fr}} = F_{\text{mass}}$. For the *settling velocity*, it follows that $v = \rho_p d_p^2 b / (18 \mu)$. From this we can deduce that $v \propto \rho_p d_p^2$.
2. The gas velocity, v , and the pressure drop, Δp , these being necessary for the separation of a specific limiting droplet size (e.g., d_{50}), decrease with d_p and are strongly dependent on the type of separator device: $\Delta p \propto d_p^{-3} \rightarrow \Delta p d_p^3 = \text{const}$ (geometry).

Since the particle size is bound by both $\rho_p d_p^2$ and $\Delta p d_p^3$, we can combine these two requirements to obtain:

$$\rho_p^3 d_p^6 \Delta p^2 \quad (12.88)$$

As a result of closer examination (1) and preliminary tests (2), the above 7-parametric relevance list can be reduced to the following 5-parametric one:

$$\{\eta_f; D; \rho, \mu; \rho_p^3 d_p^6 \Delta p^2\} \quad (12.89)$$

Since the target number η_f is dimensionless, we only have to form one single process number:

	ρ	D	μ	$\rho_p^3 d_p^6 \Delta p^2$
M	1	0	1	5
L	-3	1	-1	-5
T	0	0	-1	-4
M + T	1	0	0	1
3 M + L + 2 T	0	1	0	2
-T	0	0	1	4

$$A \equiv \frac{\rho_p^3 d_p^6 \Delta p^2}{\rho D^2 \mu^4} \quad (12.90)$$

We will first draw the square-root of this dimensionless group and then relate it to the well-known Euler, Reynolds and Stokes numbers:

$$Eu \equiv \Delta p / (\rho v^2); \quad Re \equiv v D \rho / \mu; \quad Sto \equiv \rho_p d_p^2 v / (D \mu).$$

It follows that

$$A^{1/2} \equiv \frac{d_p^3 \rho_p^{3/2} \Delta p}{\rho^{1/2} \mu^2 D} \equiv Eu Re^{1/2} Sto^{3/2} \quad (12.91)$$

Bürkholz [17] calls this combination of dimensionless numbers the “separation number” $\Psi_A^{3/2}$.

Furthermore, the dimensionless number $A^{1/2}$ indicates the existence of the relationship $\Delta p \propto D$. For each desired fractional degree of separation, the necessary pressure drop, Δp , is proportional to the characteristic linear dimension D . Different separator types can therefore be compared if D is selected sensibly, see [17].

The significance of this “separation number” is impressively demonstrated by the comparison in **Fig. 64**. To obtain the particle diameter d_p with the first power, the abscissa is transformed to $A^{1/6} \triangleq \Psi_A^{1/2}$.

H. Büttner [18] investigated the fractional degrees of several geometrically similar cyclones and plotted the results in the pi-space $\{\eta_F, Sto, Re, (d_e/d_a)\}$. In the range of the fractional degrees between $\eta_F = 10 - 90\%$ and $Re > 10^5$, the correspondence

$$\eta_F = f\{Sto^{1/2} Re^{1/3} (d_i/d_o)^{2/3}\}$$

was found, see **Fig. 65**.

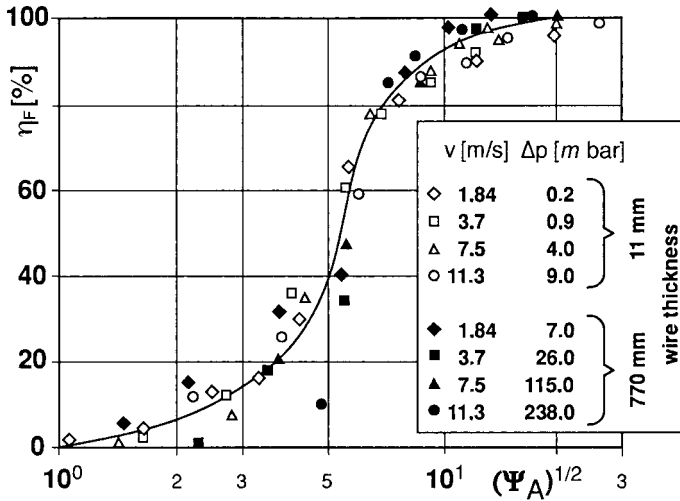


Fig. 64 Fractional degree of separation, η_F , of two different thicknesses of filter (11 and 770 mm) having the same wire thickness. Correlation of the measurements by means of the number $A^{1/6} \triangleq \Psi_A^{1/2}$; from [17].

Legend:

η_F fractional degree [–]

$\text{Sto} \equiv \frac{\text{Cu } \rho_p v d_p^2}{18 \eta d_e}$ Stokes number

$\text{Re} \equiv v d_e / \nu$ Reynolds number

Cu Cunningham correction factor

v entrance velocity of the gas

ρ_p and d_p density and diameter of the particles

η, ν dynamic and kinematic viscosity of the gas

d_i and d_o inlet diameter, defined by the equivalent cross-sectional area, and outlet diameter of the draught tube.

Measurements with cyclones in sub-atmospheric pressure (0.2 – 0.4 bar absolute pressure) for the production of submicron particles, showed that the results can be presented in the same space [96]. However, at the same entrance velocities at normal pressure a more favorable limiting diameter d_{50} is obtained than in sub-atmospheric operation.

W. Heikamp [45] reported the results of the aerosol ($d_p < 20 \mu\text{m}$) separation in a fixed bed of spheres ($d_s \approx 0.55 \text{ mm}$) in horizontal pipes (“coalescence knockout drum”). The disperse phase (organic as well as aqueous) had $\sigma = 2.6 - 50.7 \text{ mN/m}$ and $\mu = 0.41 - 150 \text{ mPa s}$, the density difference to that of air was $\Delta\rho = 47 - 316 \text{ kg/m}^3$. As the organic phase, cyclohexanon, heptane, octanol and paraffin were employed.

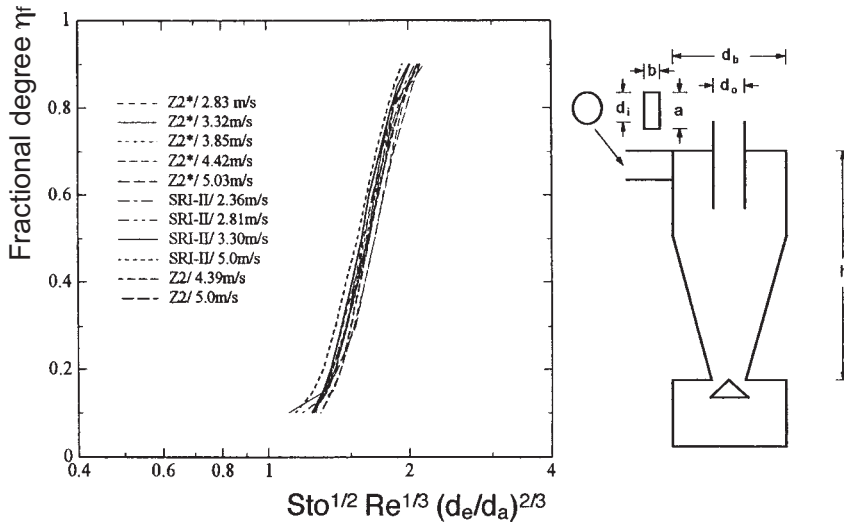


Fig. 65 Fractional degrees of cyclones of different construction (SRI-I, Z2, Z2*). The essential difference consists of the inlet design (round or slit shaped). The flow velocities given in the figure are mean velocities in the inlet. For the geometrical detail see [18].

The results were presented in the pi-space

$$\eta_F = f\left(\frac{1}{Ca} \frac{d_p}{d_K}\right)$$

η_F modified fractional degree [–]

Ca capillary number

d_p droplet diameter

d_s diameter of spheres in the bed.

The capillary number is defined as

$$Ca = \frac{\mu v}{\sigma}$$

where μ is the dynamic viscosity, σ the surface tension and v the velocity of the aerosol cloud passing the cavities of the fixed bed. The term “modified fractional degree” [45] indicates that the wetting degree of the spheres is taken into account.

Example 35: Gas hold-up in bubble columns

Bubble columns are important appliances for the absorption of gases in liquids and, consequently, for the execution of chemical reactions in gas/liquid system. In this context, the attainable interface ($\triangle$ sum of the surfaces of all gas bubbles) is of most interest because it affects the mass flow in a directly proportional manner. If a gas throughput q is introduced into a bubble column with the diameter D and the liquid height H , the liquid height rises by the amount occupied by the gas bubbles in the liquid. The gas fraction in the liquid, the so-called gas hold-up H^* can be determined from the liquid height H_b of the gassed and the non-gassed liquid (see sketch).

$$H^* \equiv \frac{V_G}{V_L} = \frac{H_b - H}{H} \quad (12.92)$$

In the course of extensive measurements on bubble columns with different dimensions [168], the gas hold-up H^* proved to be directly proportional to the volume-related mass transfer coefficient $k_L a$. For this reason, H^* will be the *target number* in the following considerations.

Bubble columns with a single-hole plate as gas distributor (see sketch) were used in these investigations. D , H and d (= hole diameter) therefore describe their *geometry* in full.

The densities (ρ and ρ') and the viscosities (ν and ν') of both phases (' gas) and the surface tension, σ , must be taken into account as *physical properties*.

The *process parameters* are the gas throughput q and, on account of the extreme differences in density, the gravity difference $g\Delta\rho = g(\rho - \rho')$.

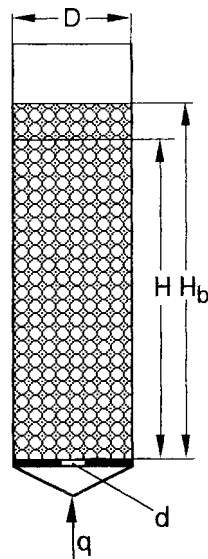
The complete relevance list is therefore:

$$\{H^*; D, H, d; \rho, \rho', \nu, \nu', \sigma; q, g\Delta\rho\} \quad (12.93)$$

If we exclude the target number H^* and the trivial numbers H/D , d/D , ρ'/ρ and ν'/ν , the remaining relevance list is

$$\{D; \rho, \nu, \sigma; q, g\Delta\rho\} \quad (12.94)$$

from which, via the dimensional matrix, the following three dimensionless numbers result:



	ρ	D	ν	σ	q	$g\Delta\rho$
M	1	0	0	1	0	1
L	-3	1	2	0	3	-2
T	0	0	-1	-2	-1	-2
M	1	0	0	1	0	1
3M + L + 2 T	0	1	0	-1	1	-3
-T	0	0	1	2	1	2

$$\Pi_1 \equiv \frac{\sigma D}{\rho \nu^2} \equiv \frac{\text{Re}^2}{\text{We}} \quad \Pi_2 \equiv \frac{q}{D \nu} \equiv \text{Re} \quad \Pi_3 \equiv \frac{g\Delta\rho D^3}{\rho \nu^2} \equiv \text{Ar} \equiv \frac{\text{Re}^2}{\text{Fr}} \quad (12.95)$$

This dimensional analysis produces two dimensionless numbers, Π_1 and Π_3 , which, apart from the column diameter, contain only physical properties, and the Reynolds number Π_2 as the process number (because it contains q).

However, this is unsatisfactory because we must expect the hydrodynamics of a bubble column to be substantially governed by $g\Delta\rho$. Consequently, the process number must contain $g\Delta\rho$. We must therefore combine Π_2 and Π_3 in order to obtain the modified Froude number Fr^* , which is probably the *true process number*.

$$\Pi_2^2 \Pi_3^{-1} \equiv \frac{q^2 \rho}{D^5 g\Delta\rho} \equiv \text{Fr}^* \quad (12.96)$$

The complete pi-set now becomes:

$$\{H^*; H/D, d/D; \rho'/\rho, \nu'/\nu, \text{Fr}^*, \text{Re}, \text{We}\}$$

Comprehensive measurements [168] were performed to verify this pi-space and to evaluate the following process characteristics:

- The relationship $H^* = f(\text{Fr}^*)$ was examined using a bubble column of given geometry; water was used as the liquid and the physical properties of the gas were varied over a wide range. Air, nitrogen and nitrogen/hydrogen mixtures were used. The result in **Fig. 66** demonstrates that Fr^* takes full account of the influence of both densities and that ν'/ν is obviously irrelevant. The process equation is:

$$H^* = 15.0 \text{Fr}^{*0.35} \quad (12.97)$$

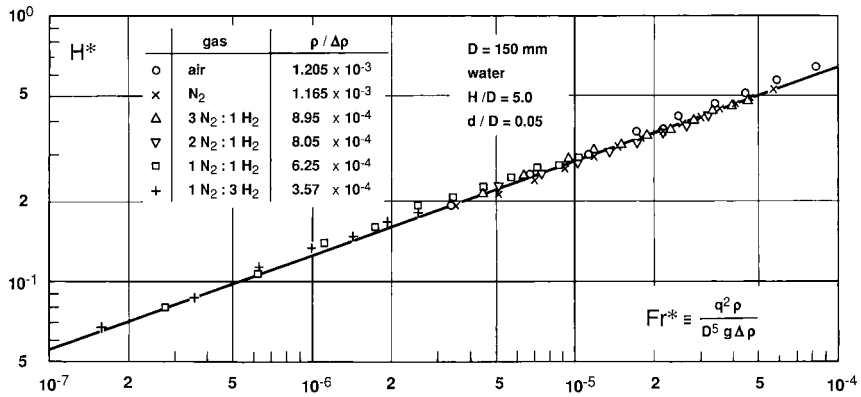


Fig. 66 The relationship $H^*(Fr^*)$ confirms the expectation that the gas hold-up H^* in a bubble column is dependent on the process number Fr^* ; from [168].

- b) Further measurements were carried out with one single material system (water/air) in bubble columns of different geometries, see **Fig. 67**. The extended process relationship now reads:

$$H^* = 16.7 Fr^{*0.35} (H/D)^{-0.25} (d/D)^{-0.125} \quad (12.98)$$

As far as the scale-up of a bubble column from a laboratory to an industrial scale is concerned, one will have to keep in mind that the scale-up rule is not $v = \text{idem}$, as is often stated in the chemical-engineering literature, but $Fr \equiv v^2/(d g) = \text{idem}$. This means:

$$v_T = v_M \sqrt{d_T/d_M} = v_M \mu^{0.5} \quad (12.99)$$

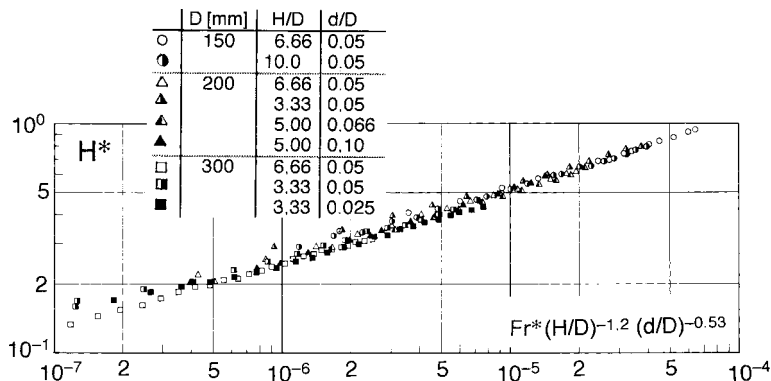


Fig. 67 Dependence of the gas hold-up H^* on the geometry of the column; from [89].

- c) The influence of the physical properties of the liquid phase was also investigated in one single model bubble column using water and, in addition, 12 different pure organic liquids, the physical characteristics of which varied substantially, see Table 2 in [168]. The result of these measurements is presented in Fig. 68. The dimensionless numbers had to be combined as follows in order to correlate the measured values:

$$B \equiv \frac{\rho/\rho' Fr^*}{ReWe^{0.5}} \equiv \frac{v(\sigma\rho)^{0.5}}{D^{2.5}g\Delta\rho} \quad (12.100)$$

The disadvantage of the resulting number combination B is that it is not a pure material number, but still contains the column diameter, D. For safe scale-up, the validity of this correlation will therefore have to be checked on columns of different diameters and with simultaneous alteration of the material system.

The correlation from Fig. 68 may give the impression that it will always be possible to depict an n-digit pi-space two-dimensionally using the analytical evaluation. This is not necessarily the case. The more complicated the physical facts, the more incomplete the analytical description and depiction will be. Indeed, it is quite easy to imagine situations where this will not even be possible. One example is the pressure drop characteristic of the straight, smooth pipe in Fig. 1, the analytical reproduction of which is likely to cause considerable problems!

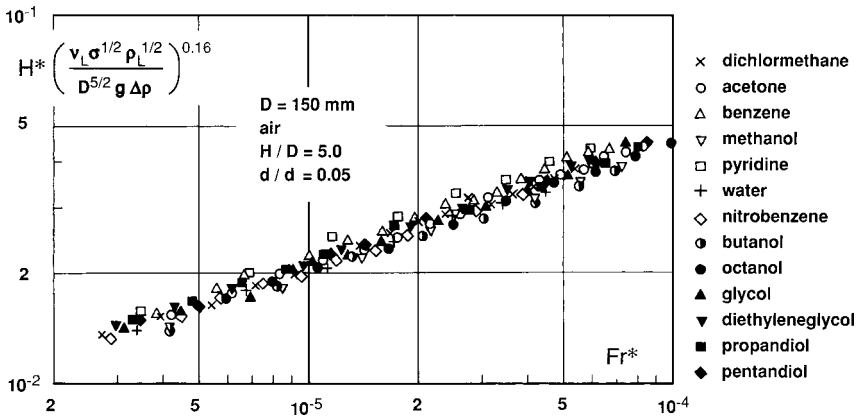


Fig. 68 Influence of the physical properties of both phases on the gas hold-up H^* in a bubble column of given geometry, from [168].

University research has always been carried out under laboratory conditions with a constant (small) column diameter. Therefore, it is not surprising that it has always been found as result that the superficial velocity v represents the only process parameter in bubble columns and that they therefore have to be dimensioned according to $v = \text{idem}$; see [160] and Section 10.1.1.

The presented example, in which the column diameter was varied by a scale factor of $\mu = 1 : 2$ and the extended Froude number was enlarged, by employing air as well as mixtures of N_2/H_2 , by a factor of 4, shows that the extended Froude number clearly represents the pertinent dimensioning criterion. This means that in using the same material system a four-fold column diameter ($\mu = 4$) entails a *doubled* superficial velocity v .

Example 36: Dimensional analysis of the tableting process

A rotating press operates in the same way as a revolver. The compressive force is generated between an upper and a lower pressure roll, so the tablet is compressed from both sides; from above as well as from below. The pairs of punches along with their dies are mounted in a rotating die platform. Each pair of punches produces one tablet per revolution. The capacities of the rotating presses depend on the number of punch pairs and the rate of rotation.

For very high-speed presses the persistence of the powder being fed into the dies can present problems. If the tablet press runs properly, then any perturbations of the process are limited to those related to the properties of the powder. The most important parameters, which influence the tableting process and the quality of the tablets, are the material properties of the powder, the moisture content and the compression time ("dwell time") $\tau^{(12)}$.

In introducing the dimensional analysis to the tableting process, the aim is not to lay the foundation for a scale-up of a tablet press, but to examine which process and material parameters govern the output of tablets of the same size and shape of a given appliance. What should be undertaken here to increase the capacity of the tablet press?

1. Relevance list

Target quantity

In the tableting process we are pursuing different quality features:

- a) the tensile strength (hardness) of the tablet $H [M L^{-1} T^{-2}]$,
- b) the stability of the tablet (mechanical disintegration time) $\theta_m [T]$,
- c) the bio-availability (dissolution time, release of the active ingredient) $\theta_b [T]$.

1 H. Leuenberger and B. D. Rohera: Fundamentals of Powder Compression. I. The Compactibility and Compressibility of Pharmaceutical Powders, Pharmaceutical Research 2 (1986) 1, 12 – 22

2 K. Sommer: Size Enlargement, Chapter on Tableting, in Ullmann's Encyclopedia of Industrial Chemistry, Volume B2, 7-31/35, VCH Weinheim/Germany, 1988

Each of these target quantities (quality features) entails a separate set of variables, which are not necessarily completely equal in each case. In the following we will pursue only the first of them, the tensile strength (or tensile stress) of the tablet H [$M L^{-1} T^{-2}$] with the dimension of pressure.

If the test method is designed in such a way that the tablet failure is a result of the application of tensile stress only, then the strength can be calculated from the relationship

$$H \equiv \frac{2 F}{\pi d h_t} \quad (12.100)$$

where H is the radial tensile strength, F is the force necessary to cleave the tablet and $\pi d h_t$ is the jacket surface of the tablet¹⁾.

(1) Geometric parameters

Given the cylindric shape of the tablets, the geometry of the die is described by only two geometric parameters:

the diameter of the die	d [L]
the depth of fill (loading depth)	H [L]

(2) Material parameters (physical properties of the powder mixture)

There is a wealth of material parameters which exert a big impact on the tableting process. They influence the flow behavior of the powder in the hopper (formation of bridges!) and of the feed to the dies. Too high a moisture content of the powder or too little lubrication leads to sticking, cracking, and coating. The interaction of the particle size distribution, the crystal shape (morphology) and other physical properties of the powder is the principal factor affecting the tensile strength, the disintegration time, and the release of the active ingredient^{1,2)}.

From the viewpoint of dimensional analysis it is not necessary to know and to list all these physical parameters, if we succeed in finding an intermediate quantity (a “lumped” parameter) which takes into consideration their influence on the process. As such we have chosen³⁾ the powder compressibility κ which is easily measurable and is defined as the relative change of the powder volume (or its height within the die) with the pressure:

$$\kappa \equiv \frac{1}{V_t} \frac{\Delta V}{\Delta p} = \frac{1}{h_t} \frac{H - h_t}{\Delta p} \quad [M^{-1} L T^2] \quad (12.101)$$

The subscript t relates to the tablet. κ has the inverse dimension of the pressure.

3 M. Levin and M. Zlokarnik: Dimensional Analysis of the Tableting Process, in *Phar-*

maceutical Process Scale-Up (ed. M. Levin), Marcel Dekker, Inc., New York 2002

(3) Process-related parameters

Besides the maximum applied pressure p [$\text{M L}^{-1} \text{T}^{-2}$] it is the compression rate v (linear velocity of the punch!), which affects the tableting and tablet properties the most¹⁾. It is defined as the ratio of the displacement of the punch from the first application of a detectable force up to the point of maximum force, to the time required for this displacement to occur¹⁾:

$$v \equiv \frac{H_t - h_t}{t_{p=\max} - t_0} [\text{m/s}]$$

The impact of the compression rate on the tableting is understandable because it causes the frictional effect of air, the rapidly escaping powder pressed between the punch and the die. Sliding and rearrangement processes of the particles are also hindered by turbulence in the powder due to rapid compression.

We have chosen the compression time ("dwell time") τ [s] instead³⁾. It is defined as that time when the flat portion of the punch head with the diameter of d_{ph} is in contact with the wheel. During this time the powder is submitted to the highest compression. (Contrary to this, the contact time is defined as the time during which the punch head is in contact with the wheel.)

MCC⁴⁾ employs the following formula for the determination of the dwell time:

$$\tau = \frac{d_{\text{ph}}}{\pi D n} [\text{s}] \quad (12.102)$$

d_{ph} diameter of the flat portion of the punch head [mm]⁵⁾

D circle diameter of the turret on which the dies are arranged [mm]

n rotational speed of the turret [s^{-1}]

The relevance list now reads as follows:

Target quantity

H [$\text{M L}^{-1} \text{T}^{-2}$] tensile strength of the tablet

Influencing parameters

geometric

d [L] diameter of the die

H [L] depth of fill (loading depth)

material

κ [$\text{M}^{-1} \text{L T}^2$] powder compressibility

process-related

p [$\text{M L}^{-1} \text{T}^{-2}$] max. applied pressure

τ [T^{-1}] dwell time

4 The measurements were performed at the Metropolitan Computing Corporation (MCC), East Hanover, New Jersey, USA

5 MCC puts $d_{\text{ph}} = 12.9 \text{ mm} = \text{const}$, because this happens to be a flat portion of the punch head for IBT B tooling.

$$\{H, d, H, \kappa, p, \tau\}$$

Of these six parameters three have the dimension of pressure, two of length and only one of time. The dimension of time cannot be eliminated. This situation shows that there must be one more influencing parameter having the dimension of time. This can only be the rotational speed of the turret, n . n and τ are primarily independent of each other, because the definition formula of dwell time, eq. (12.102) contains the diameter D of the turret.

We add the rotational speed of the turret to the relevance list and obtain

$$\{H, d, H, \kappa, p, \tau, n\} \quad (12.103)$$

2. Generation of pi-numbers

We obtain from the relevance list (12.103), $7 - 3 = 4$ dimensionless pi-numbers. They consist of one target, two process and one geometric number:

$$\{H/p, p \kappa, \tau n, H/d\} \quad (12.104)$$

A clearer insight into the dependence $H(p)$ may be gained by separating H and p :

$$\{H \kappa, p \kappa, \tau n, H/d\} \quad (12.105)$$

At this point it should be pointed out that the influence of the pure process number τn on the tableting process can be investigated only when different tablet presses are compared to each other. In working with only one tableting press this number remains constant – but it must be quantified and noted as such!

3. Carrying out and evaluation of measurements

The measurements were executed⁴⁾ on two tablet presses:

MANESTY Betapress	$D = 0.178 \text{ m}; \quad z = 16; \quad \tau n = 1.77 \times 10^{-2}$
FETTE, Model PT 2090 IC	$D = 0.30 \text{ m}; \quad z = 36; \quad \tau n = 9.86 \times 10^{-3}$

Both appliances had equal die diameter: $d = 9.52 \text{ mm}$

Two different powders were applied:

Avicel PH 101, with a density of	$\rho = 1.58 \text{ g/cm}^3$ and $\kappa = 2.45 \times 10^{-8} \text{ Pa}^{-1}$
Emcompress, with a density of	$\rho = 2.34 \text{ g/cm}^3$ and $\kappa = 7.58 \times 10^{-9} \text{ Pa}^{-1}$

In Fig. 69, 61 measurements on both powders and both tablet presses are presented in the form of the dependence, $H/p (n \tau)^a = f(p \kappa)$. It is striking that no influence of the type of tablet press is found. *All differences relate solely to different powder properties:*

- The ordinates (H/p) for both powders differ from one another by almost one decade.
- The type of powder determines the influence of the process numbers τn of the tablet press: The values of the exponent a in $(n \tau)^a$ are $a = 0.55$ for Avicel and $a = 0.70$ for Emcompress.
- In both cases the tensile strength H increases over-proportionally with the applied pressure, but in case of Avicel, only until $p \kappa = 2.0$ is reached.

The analytical expressions for fitting lines in Fig. 69 are:

Avicel:

$$[(H/p) (\tau n)^{0.55}]^{-1} = 1.46 \times 10^{-2} (p \kappa)^{-1.5} + 4.22 \times 10^{-1} (p \kappa) \quad 0.5 < (p \kappa) < 5.0$$

Emcompress:

$$(H/p) (\tau n)^{0.70} = 3.8 \times 10^{-4} (p \kappa)^{0.30} \quad 0.5 < (p \kappa) < 5.0$$

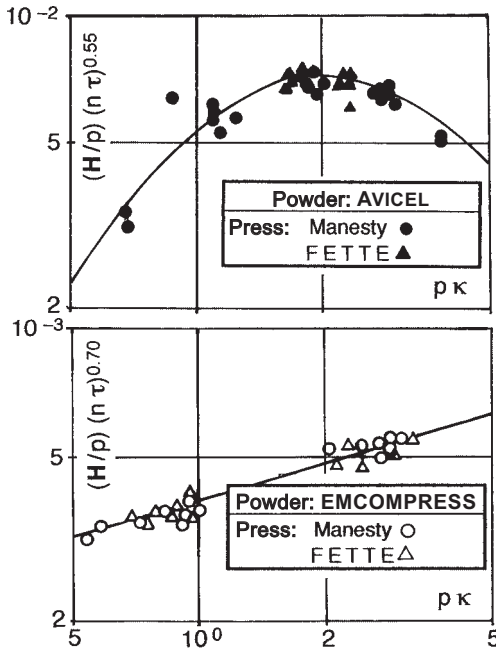


Fig. 69 Evaluation of the measurements in the pi-space $\{H/p, \tau n, p \kappa\}$.

It has been mentioned that a clearer insight into the dependence $H(p)$ might be gained by separating H and p . In **Fig. 70** the fitting lines of **Fig. 69** are presented in the corresponding dimensionless frame

$$\{H \kappa, p \kappa, \tau n, H/d\}$$

One can see that the dependence of H on p for both powders is not very different.

In these correlations an influence of the geometric parameter H/d was considered as insignificant and therefore neglected: H/d values varied depending on the type of press between $H/d = 0.73 - 0.96$.

In this examination the goal was neither to change the size of the tablet nor to change the tablet press, but to increase the tablet output by increasing the process number τn . Therefore different types of tablet presses will have to be examined in order to vary the process number more strongly.

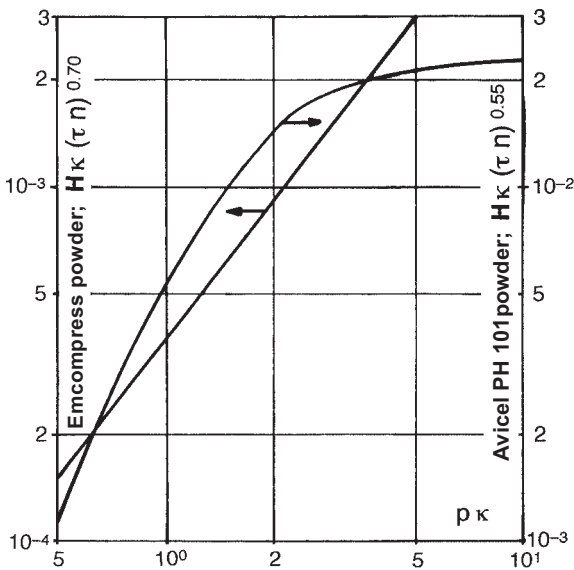


Fig. 70 Representation of fitting curves in **Fig. 69** in the π -space $\{H \kappa, \tau n, p \kappa\}$.

13

Selected Examples of the Dimensional-analytical Treatment of Processes in the Field of Thermal Unit Operations

13.1

Introductory Remarks

Besides fluid mechanics, thermal processes also include mass transfer processes (e.g. absorption or desorption of a gas in a liquid, extraction between two liquid phases, dissolution of solids in liquids) and/or heat transfer processes (energy uptake, cooling, heating, drying). In the case of thermal separation processes, such as distillation, rectification, extraction, and so on, mass transfer between the respective phases is subject to *thermodynamic laws* (phase equilibria) which are obviously not scale dependent. Therefore, one should not be surprised if there are no scale-up rules for the pure rectification process, unless the hydrodynamics of the mass transfer in plate and packed columns are under consideration. If a separation operation (e.g. drying of hygroscopic materials, electrophoresis, etc.) involves simultaneous mass and heat transfer, both of which are scale-dependent, the scale-up is particularly difficult because these two processes obey different laws.

Heat transfer processes are described by physical properties and process-related parameters, the dimensions of which not only include the base dimensions of Mass, Length and Time but also Temperature, Θ , as the fourth one. In the discussion of the heat transfer characteristic of a mixing vessel (Example 20) it was shown that, in the dimensional analysis of thermal problems, it is advantageous to expand the dimensional system to include the amount of heat, H [kcal], as the fifth base dimension. *Joule's* mechanical equivalent of heat, J , must then be introduced as the corresponding dimensional constant in the relevance list. Although this procedure does not change the pi-space, a dimensionless number is formed which contains J and, as such, frequently proves to be irrelevant. As a result, the pi-set is finally reduced by one dimensionless number.

This chapter contains seven examples. The first three deal with *heat transfer* in mixing vessels, in pipelines and in bubble columns. The additional three concern *mass transfer* in the Gas/Liquid (G/L) system in surface aeration in waste-water treatment ponds, in volume aeration (bulk aeration) in mixing vessels and in bubble columns, which use injectors as gas dispersers. Due to the fact that the gas bubble coalescence has an extreme impact on mass transfer in volume aeration,

this topic is treated separately. The seventh and last example deals with drying of films; this case treats a simultaneous mass and heat transfer.

Example 37: Steady-state heat transfer in mixing vessels

The pi-space of the heat transfer characteristics of a mixing vessel has already been treated in Example 21, page 97. The obtained dependence (9.10) reads:

$$\{Nu, Pr, Re, D/d, \gamma_0 \Delta T\} \tag{13.1}$$

It has already been pointed out that the technology of heat transfer utilizes the viscosity number $Vis \equiv \mu_w/\mu$ instead of $\gamma_0 \Delta T$. μ_w stands for the viscosity of the boundary layer at the wall, whereas μ represents the viscosity in the bulk of the liquid. The number Vis has been discussed in Example 15. Detailed examination of its influence in heat transfer in mixing vessels showed [166] that it is negligible and that this number can be omitted.

Figure 71 shows the dependence (13.1) for an anchor stirrer with two arms and a close wall clearance ($D/d = 1.02$). This figure consists of two parts representing the same measuring results in two different pi-spaces: above as $Nu = f(Re)$, below as $Nu = f(Re, Pr)$.

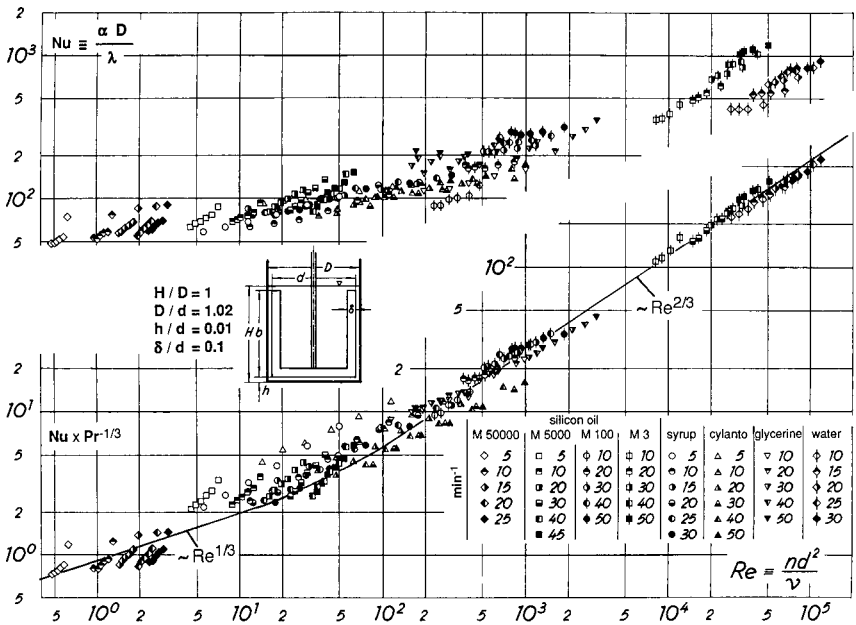


Fig. 71 Heat transfer characteristics of a mixing vessel with an anchor stirrer with two arms and $D/d = 1.02$; from [166].

A comparison of both representations verifies that the consideration of the Prandtl number only begins gaining weight increasingly from ca. $Re > 200$, in the range $Re < 200$ so its consideration is not helpful.

The variation of the measuring data in the range of $Re < 200$ is well-founded by the measuring technique applied here. The mean bulk temperature in the vessel in the laminar flow region could not be measured by a thermometer because around its wall an insulating border layer was formed. In this range the average temperature of the bulk liquid had to be calculated from the heat balance. In the range $Re > 200$ the correspondence between the calculated and measured values was approximately 10 % [166].

The fitting line in the lower figure verifies the following dependences:

$$Re > 200: Nu \propto Re^{2/3} Pr^{1/3} \quad (13.2)$$

$$Re < 200: Nu \propto Re^{1/3} Pr^{1/3} \propto Pe^{1/3} \quad (13.3)$$

This is compulsory. If in the creeping flow range ($Re < 200$) the influence of the Prandtl number is considered using $Pr^{1/3}$, then the Reynolds number must also exert the same exponent. Only then does a number (the Peclet number Pe) result, which does not contain the liquid viscosity and in which the liquid density ρ is combined with the *mass* related heat capacity c_p to produce the *volume* related one. In the creeping flow range, liquids behave like solids and then neither the viscosity nor the density affect the heat transfer:

$$Pe \equiv RePr \equiv n d^2 \rho c_p / k \equiv n d^2 / a \quad (13.4)$$

$a \equiv k/(\rho c_p)$ means the thermal conductivity; see also Example 3.

H. Judat [57] investigated heat transfer in a mixing vessel with an anchor stirrer having four arms with wipers on them ($D/d = 1$), **Fig. 72**. Using wipers, the liquid border layer at the wall is completely removed. As a result, the Nu values ($\triangleq a_i$ values) in the creeping flow area are increased by a full decade and, in addition, the creeping flow range expands up to $Re = 2 \times 10^3$ because the stirrer beams dip into the laminar border layer. Here, too, $Nu \propto Pe^{1/3}$ holds. For more detail see [57].

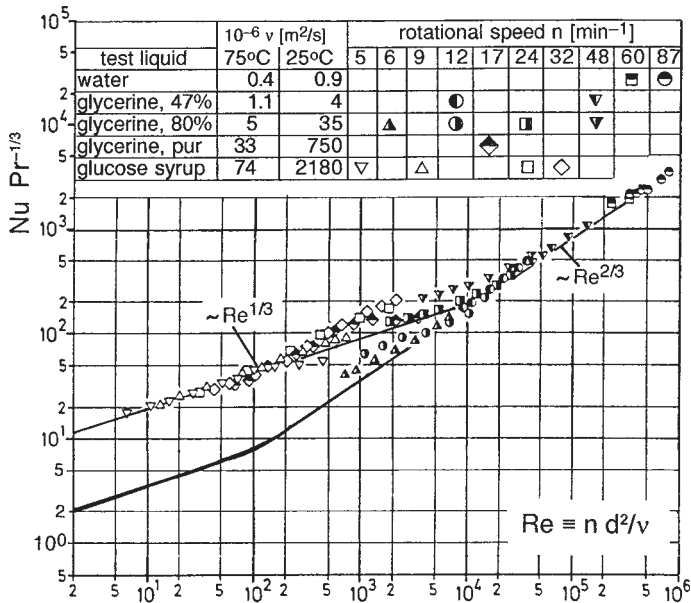


Fig. 72 Heat transfer characteristics of a mixing vessel with an anchor stirrer with four arms having wipers attached to them. ($D/d = 1.00$); from [57]. (For hysteresis in the range $Re = 10^3 - 10^4$ see [184], page 265.).

Example 38: Steady-state heat transfer in pipes

The pi-space for the heat transfer characteristics of a pipe corresponds to that of a mixing vessel:

$$\{Nu, Pr, Re, l/d, \mu_w/\mu\}, \quad (13.5)$$

only the diameter ratio D/d is exchanged by the length/diameter ratio l/d of the pipe. In creeping-laminar flow and $l/d < 400$ this ratio has a minor influence on the heat transfer due to the flow perturbances at the pipe entrance [40].

Regarding the viscosity ratio $Vis \equiv \mu_w/\mu$, it was explained in Example 15 that the assumption of a constant exponent $m = 0.14$ for the viscosity number, as found by *Sieder* and *Tate* [133], does not hold, see Fig. 17.

Figure 73 shows the renowned measurements of heat transfer in pipes by *Sieder* and *Tate* [133]. The similarity to the heat transfer characteristics of a mixing vessel is striking. In the creeping flow regime ($Re < 2.300$) the correlation

$$Re < 2.300: \quad Nu \propto Re^{1/3} Pr^{1/3} \propto Pe^{1/3} \quad (13.6)$$

holds, whereas for the range of the fully developed turbulent flow the process equation (acc. to Hausen, in [40]) follows:

$$\text{Re} > 10^4: \quad \text{Nu} \propto (\text{Re}^{2/3} - 125) \text{Pr}^{1/3} \quad (13.7)$$

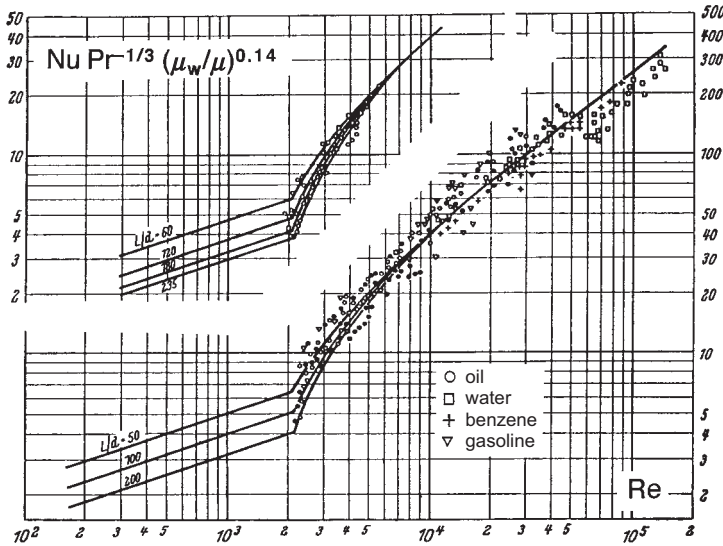


Fig. 73 Heat transfer characteristics of a pipe. Measurements of Sieder and Tate [133], taken from [40]. Open symbols: heating, full signs: cooling.

Example 39 Steady-state heat transfer in bubble columns

The pi-space of a heat transfer characteristic of a bubble column is different from that of a mixing vessel in that the pertinent process quantity is the gas throughput q . In addition, because of the extremely large density differences in the material system G/L , $g\Delta\rho$ will also play a decisive role:

Target quantity:	heat transfer coefficient, h
Geom. parameter:	column diameter, D
Physical properties:	density, ρ , and viscosity, μ , of the liquid heat capacity, C_p , and conductivity, k
Process parameters:	Gas throughput, q Gravity difference, $g\Delta\rho$

$$\{h; D; \rho, \mu, C_p, k; q, g\Delta\rho\} \quad (13.8)$$

This 8-parametric set delivers the following four pi-numbers:

$$\text{Nu} \equiv h D/k \quad \text{Nusselt number}$$

$$\text{Pr} \equiv C_p \mu/k \quad \text{Prandtl number}$$

$$\text{Re} \equiv q \rho/(D \mu) \quad \text{Reynolds number}$$

$$\text{Fr}^* \equiv q^2 \rho/(D^5 g \Delta \rho) \quad \text{Froude number}$$

$$\{\text{Nu}, \text{Pr}, \text{Re}, \text{Fr}^*\} \quad (13.9)$$

W. Kast [61] found that in bubble columns the intensity variable superficial velocity, $v \propto q/D^2$, is also decisive for heat transfer. (The same was found for mass transfer in bubble columns, see Example 10). From the dimensional analysis point of view, v is an intermediate variable, the introduction of which reduces the above 4-parametric pi-space to a 3-parametric one. For this purpose, first the two q -containing numbers have to be formulated with v instead of q :

$$\text{Re} \equiv v D \rho/\mu \quad \text{Reynolds number}$$

$$\text{Fr}^* \equiv v^2 \rho/(D g \Delta \rho) \quad \text{Froude number}$$

By the combination of the three D -containing numbers (Nu , Re , Fr^*), D can be eliminated, which leads to the following two pi-numbers:

$$\text{St} \equiv \frac{\text{Nu}}{\text{Re} \text{Pr}} = \frac{h}{v \rho C_p} \quad \text{Stanton-Kennzahl}$$

$$\text{ReFr}^* \equiv \frac{v^3}{v g} \frac{\rho}{\Delta \rho}$$

This leads to the following pertinent pi-space:

$$\{\text{St}, \text{Pr}, \text{ReFr}^*\} \quad (13.10)$$

We would have obtained this 3-parametric pi-set if we had correspondingly rearranged the relevance list:

$$\{h; \rho, \mu, C_p, k; v, g \Delta \rho\} \quad (13.11)$$

	ρ	μ	$g\Delta\rho$	C_p	h	k	v
M	1	1	1	0	1	1	0
L	-3	-1	-2	2	0	1	1
T	0	-1	-2	-2	-3	-3	-1
Θ	0	0	0	-1	-1	-1	0
Z_1	1	0	0	0	1/3	0	-2/3
Z_2	0	1	0	0	1/3	1	1/3
Z_3	0	0	1	0	1/3	0	1/3
$-\Theta$	0	0	0	1	1	1	0

$$Z_1 = M + T + A - 4/3 \Theta \quad Z_2 = 3M + L + T + A + 2/3 \Theta$$

$$Z_3 = A + 2/3 \Theta \quad A = -1/3 (3M + L + 2T)$$

$$\Pi_1 = \frac{h}{(\rho \mu g \Delta \rho)^{1/3} C_p} \quad \Pi_2 = \frac{k}{\mu C_p} \quad \Pi_3 = \frac{v \rho^{2/3}}{(\mu g \Delta \rho)^{1/3}}$$

$$\Pi_1 \Pi_3^{-1} = \frac{Nu}{Re Pr} \equiv St \quad \Pi_2 \equiv Pr^{-1} \quad \Pi_3^3 \equiv Re Fr^*$$

This dimensional analysis is only briefly presented here, because it leads to pi-expressions with broken exponents.

The evaluation of test results [61] in **Fig. 74** shows that the 3-dimensional pi-space can be further reduced to a 2-dimensional one through the product combination of $Re Fr Pr^2$. Apart from his own measurements, *Kast* also included extensive experimental material by *Kölbel* et al. [68]. The correlation

$$St \propto \left[\sqrt[3]{(Re Fr^*) Pr^2} \right]^{2/3}$$

leads to the dependence $h \propto v^{1/3}$ when the same material system is used.

Zlokarnik [163/2] determined the heat transfer characteristic of a mixing vessel, equipped with a self-aspirating hollow stirrer, using the material system water/air. This enables a direct comparison between the heat transfer behavior in a stirring vessel at gassing and a bubble column. **Figure 75** shows that in gas/liquid contacting in a mixing vessel, approximately twice as much heat can be transferred through the wall as in a bubble column. Besides this, $h \propto v^{1/5}$ applies here. This also confirms that the hydrodynamics in a gassed stirred vessel is more strongly influenced by the stirrer than by the gas throughput.

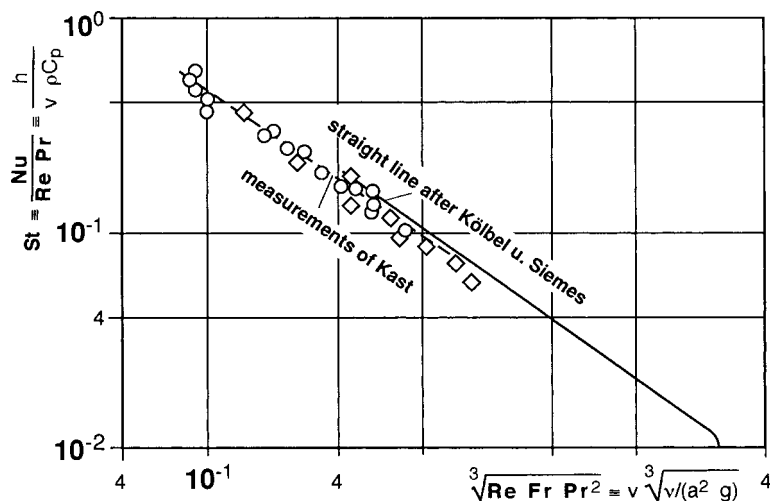


Fig. 74 Heat transfer characteristic of a bubble column, from [61]. $a \equiv k/(\rho C_p)$ – thermal diffusivity

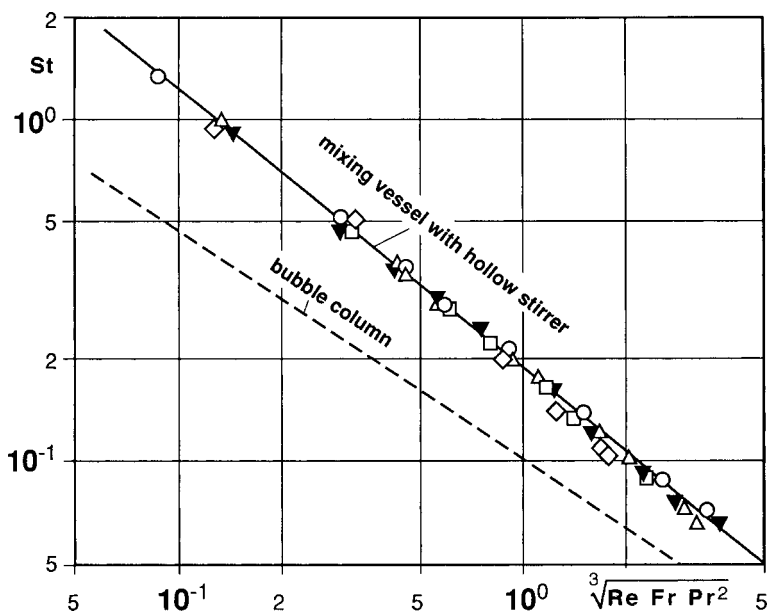


Fig. 75 Comparison of the heat transfer behaviour of a mixing vessel with a self-aspirating hollow stirrer [163/2] and a bubble column [61].

This example has been chosen to demonstrate how a set of dimensionless numbers can be transformed if one of the influencing quantities is to be eliminated. In the present case the column diameter D was eliminated under the premise that the intermediate quantity, the superficial velocity $v \propto q/D^2$, which replaces

gas throughput q as well as the column diameter D , fully describes the flow regime in a bubble column.

It should be repeated here, what has already been stated in the introduction to the chapter on intermediate quantities. It is not likely that the superficial velocity v would represent a genuine intermediate quantity, which eliminates the diameter of the device. The state of flow is extremely scale-dependent and the characteristic length scale is given here by the column diameter. The heat transfer characteristics of bubble columns, measured in laboratory devices where $D \approx \text{constant}$, have in fact been correlated by the superficial velocity, but this must necessarily be verified by changing the scales.

13.2

Foundations of the Mass Transfer in a Gas/Liquid (G/L) System

A short introduction to Examples 40, 41 and 42

This introduction is necessary because the mass transfer between two fluid phases is in many aspects different to the steady-state heat transfer from a solid wall to the liquid. In heat transfer, the driving force consists in leveling out the temperature levels, thus in the equalization of the energy contents of the systems. In contrast to this, in mass transfer in the G/L system, the driving force consists in the removal of an equilibrium disturbance and this concerns a thermodynamic quantity, namely the chemical potential, μ .

It is often presumed that leveling out of concentration differences between two phases is the cause for mass transfer. This is not the case and this can easily be demonstrated by a group of examples. It should suffice to remark that a material system consisting of granular cooking salt in its saturated aqueous solution is in equilibrium, the chemical potential in both phases is equal, but the concentration of NaCl in granular cooking salt is 100 %, whereas in the saturated aqueous solution it is only 26.4 % (at 20 °C).

The bridge between the chemical potential, which can be determined only with difficulty, and the concentration difference, which is easily determinable, was made by Lewis and Whiteman in 1923/24 who postulated in their “Two Film Theory” (with regard to the system G/L) the following:

- (a) On both sides of the border area between phases, laminar boundary layers exist which the gas can pass only by *diffusion*.
- (b) In the phase border area the thermodynamic equilibrium occurs *immediately*.

In a G/L mass transfer, the transport from the gas phase to the phase border can be neglected because it is at least 50 times faster than the transport from the phase border through the laminar boundary layer to the bulk of a liquid. If – as postulated – thermodynamic equilibrium always exists in the phase border, then

the saturation concentration of the gas is also always present in it: $c^* = c_s$. Under these conditions the driving force of the mass transfer is in fact given by the concentration difference:

$$\Delta c = c_s - c. \quad (13.12)$$

Thus the general mass transfer equation is given by the following relationship:

$$G = k_L A \Delta c \quad (13.13)$$

Legend:

G [kg gas/s]	mass transfer rate [kg s ⁻¹] through the phase boundary
A [m ²]	interfacial area (sum of the surfaces of all gas bubbles)
k_L [m s ⁻¹]	liquid-side mass transfer coefficient
Δc [kg m ⁻³]	characteristic concentration difference of the dissolved gas between the phase border and the liquid bulk

k_L can physically be comprehended as $k_L \sim D/x_L$, where D is the diffusion coefficient of the gas in the liquid-side laminar boundary layer of thickness x_L . Thus k_L represents a definition quantity which is as difficult to measure as the interfacial area A . Both of them are influenced by the kinematics of the process and by the material conditions. In surface aeration they are combined to form the only quantity which can easily be determined by the constituents of the overall mass transfer equation (13.13):

$$k_L A \equiv \frac{G}{\Delta c} \quad (13.14)$$

$k_L A$ represents the target quantity of the mass transfer in *surface aeration*.

In volume (bulk) aeration it is assumed that the process takes place in a liquid which is turbulently mixed and thus in each volume element an equal number of gas bubbles of an equal size exists. Under these circumstances it is advisable to formulate the overall mass transfer equation (13.13) as a volume-related one:

$$G/V = k_L (A/V) \Delta c \quad (13.15)$$

(This contains an additional advantage, because Δc is also a volume-related quantity per definition.) A/V is abbreviated to $a \equiv A/V$ and – seeing that it is not measurable – combined with k_L to the “overall liquid-side mass transfer coefficient $k_L a$ ”

$$k_L a \equiv \frac{G}{V \Delta c} \quad (13.16)$$

$k_L a$ is easily measurable by equation (13.16) and represents the target quantity of the mass transfer in *bulk aeration*.

From the viewpoint of dimensional analysis a large distinction exists between the two target quantities $k_L A$ and $k_L a$.

$k_L A$ is an extensively defined quantity which therefore necessarily has to depend on extensive parameters like the stirrer diameter d and the stirrer speed n .

$k_L a$, in contrast, is a volume-related, intensive quantity, which therefore necessarily has to depend on intensively formulated process parameters like power per liquid volume P/V and gas throughput q per aerated cross-sectional area S , the so-called superficial velocity, $v \propto q/S$.

Example 40: Mass transfer in surface aeration

The supply of atmospheric oxygen to municipal sewage purification plants has been performed for decades by turbine surface aerators installed in the liquid surface of shallow waste-water treatment ponds ($H \leq 4$ m). The stirrers work as water pumps: they suck in waste-water, spray it over the liquid surface and then whirl it up. In this way the mass transfer G/L is effected [172].

As already shown, the target quantity of this process is $k_L A$, available from eq. (13.14). As an extensive target quantity it depends on the following influencing quantities:

- Geometric parameter: diameter of the surface aerator, d
- Material parameters: density and kin. viscosity of the liquid, ρ, ν
- Process parameters: rotational speed of the surface aerator, n
acceleration due to gravity, g

The acceleration due to gravity, g , is essential in this process because it influences the parabolic throw of the drawn in and ejected liquid. We will renounce two further possible parameters, the diffusion coefficient D and the surface tension σ , because in waste-water treatment the only liquid of interest is water. Thus the constricted relevance list reads:

$$\{k_L A; d; \rho, \nu, n, g\} \quad (13.17)$$

This 6 parametric set delivers following three dimensionless numbers:

$$(k_L A)^* \equiv \frac{G}{d^3 \Delta c} \left(\frac{\nu}{g^2} \right)^{1/3} \quad \text{sorption number}$$

$$\text{Re} \equiv \frac{n d^2}{\nu} \quad \text{Reynolds number}$$

$$\text{Fr} \equiv \frac{n^2 d}{g} \quad \text{Froude number}$$

To achieve the situation where the only variable process parameter, n , does not appear in both process numbers (Fr and Re), we first decide which of the both process numbers we regard as essential, and then free the other one by combining it with the first one from n .

The process takes place in water, so the state of flow will be a turbulent one. From this, Re cannot play an important role. The process will be governed essentially by the Froude number. We will combine Re with Fr and we obtain

$$Ga \equiv \frac{Re^2}{Fr} \equiv \frac{d^3 g}{\nu^2} \quad \text{Galilei number}$$

Thus, the pi-space reads:

$$\{(k_L A)^*, Fr, Ga\} \quad (13.18)$$

Due to the circumstance that the second essential parameter, the acceleration due to gravity, g , cannot be varied on Earth, the measurements must be performed by a change of scale, which means in differently scaled measuring devices. (This is the only way to decouple Fr and Re ; see 7.4 c, page 54.)

Figure 76 represents the measuring data of three investigators. A turbine stirrer (Rushton turbine) has been used, its disk was installed in the liquid surface. While data of *Zlokarnik* and of *Roustan* scatter too much to try a correlation by a Galilei number, the data of *Schmidtke* and *Horvath* show an unambiguous dependence on stirrer diameter and can clearly be correlated by Ga . The fitting curve for the data of *Schmidtke* and *Horvath* reads [172, 174]:

$$(k_L A)^* = 1.41 \times 10^{-4} Fr^{1.205} Ga^{0.115} \quad Fr = 0.02 - 0.34; Ga = 1.5 \times 10^9 - 2 \times 10^{11}$$

In Section 10.3.2 both possible dependences $\{(k_L A)^*, Fr, Ga\}$ and $\{(k_L A)^*, Fr\}$ have been discussed with regard to the efficiency of mass transfer in surface aeration

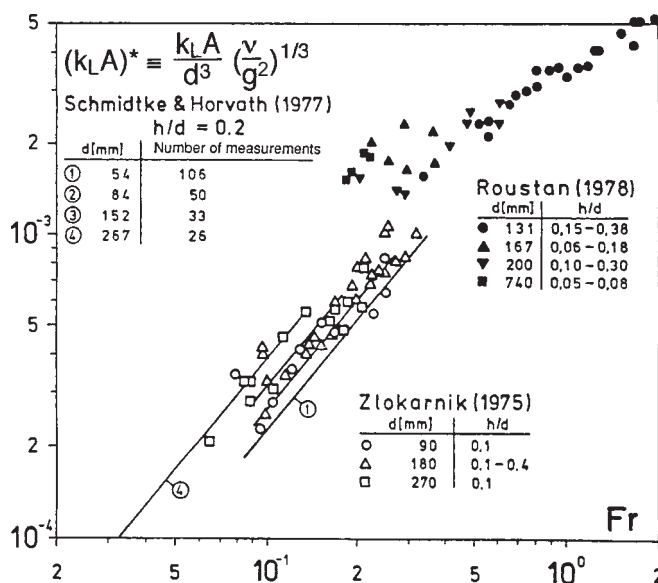


Fig. 76 Mass transport characteristics of the turbine stirrer in surface aeration; from [172, 174].

by turbine stirrers. In fact the distinction is enormous. One should not be discouraged by the power of 0.115 of the Galilei number, because the diameter d enters into the Galilei number with the 3rd power!

Example 41: Mass transfer in volume aeration in mixing vessels

As already discussed in the introductory Chapter 13.2: “Foundation of the mass transfer in the material system Gas/Liquid (G/L)”, the target quantity of the volume (bulk) aeration is the “overall liquid-side mass transfer coefficient $k_L a$ ”. The selection of the volume-related and, therefore, intensively formulated variable $k_L a$ implies the following consequences:

1. Since a quasi-uniform material system is assumed, $k_L a$ should not depend on *geometric* parameters.
2. Since $k_G \gg k_L$, $k_L a$ must be independent of the *physical* properties of the gas phase.
3. Since the target quantity $k_L a$ is an intensity variable, the *process* parameters must also be formulated intensively.

According to these premises, the relevance list must be formed with the following parameters: *Target quantity*: $k_L a$; *physical properties*: density ρ , viscosity μ , diffusivity D and the coalescence parameters S_i of the liquid phase. Despite extensive research, coalescence phenomena have still not been clarified to such an extent as to permit explicit formulation of the coalescence parameters; see section 13.3: Coalescence. In the following they will be taken into consideration together as S_i .

Process parameters: the volume-related mixing power P/V , the superficial velocity v of the gas and the gravitational acceleration g . (The decision in favor of P/V and v instead of P/q and q/V was based on extensive research results obtained during the last three decades, see Section 7.4, example d on page 55.)

We start from the following relevance list:

$$\{k_L a, \rho, \mu, D, S_i, P/V, v, g\} \quad (13.19)$$

	ρ	μ	g	$k_L a$	P/V	v	D
M	1	1	0	0	1	0	0
L	-3	-1	1	0	-1	1	2
T	0	-1	-2	-1	-3	-1	-1
Z_1	1	0	0	1/3	2/3	-1/3	-1
Z_2	0	1	0	-1/3	1/3	1/3	1
Z_3	0	0	1	2/3	4/3	1/3	0

$$Z_1 = M + T + 2A \quad Z_2 = 3M + T + A \quad Z_3 = A = -1/3 (3M + L + 2T)$$

$$\Pi_1 = (k_L a)^* \equiv k_L a \left(\frac{\mu}{\rho g^2} \right)^{1/3} = k_L a \left(\frac{\nu}{g^2} \right)^{1/3}$$

$$\Pi_2 = (P/V)^* \equiv \frac{P/V}{(\rho^2 \mu g^4)^{1/3}} = \frac{P/V}{\rho(\nu g^4)^{1/3}}$$

$$\Pi_3 = \nu^* \equiv \frac{\nu \rho}{(\mu g)^{1/3}} = \frac{\nu}{(\nu g)^{1/3}} \quad \Pi_4 \equiv Sc^{-1} \equiv \frac{D\rho}{\mu} = \frac{D}{\nu}$$

The following pi-set resulted here:

$$\{(k_L a)^*, (P/V)^*, \nu^*, Sc, S_i^*\} \quad (13.20)$$

Figure 77 shows a correlation of the mass transfer measurements in this pi-space. The measurements were performed under unsteady-state conditions, by several authors, in the water/air system which is a coalescent one, using the turbine stirrer (see sketch in Fig. 35) as a mixing device. The measurements cover an extreme experimental scale of $\mu \approx 1$ –80. The geometric parameters were broadly varied: $d = 0.05$ –3.1 m; $D = 0.15$ –12.2 m; $H = 0.15$ –6.1 m. Between B and ν^* the following correlation exists: $B = (\pi/4) \nu^*$.

In contrast, Fig. 78 shows the results of mass transfer in the system: aqueous 1-n sodium sulphite solution/air. These measurements were carried out under steady-state conditions in vessels with hollow stirrers on the scale $\mu = 1 : 5$

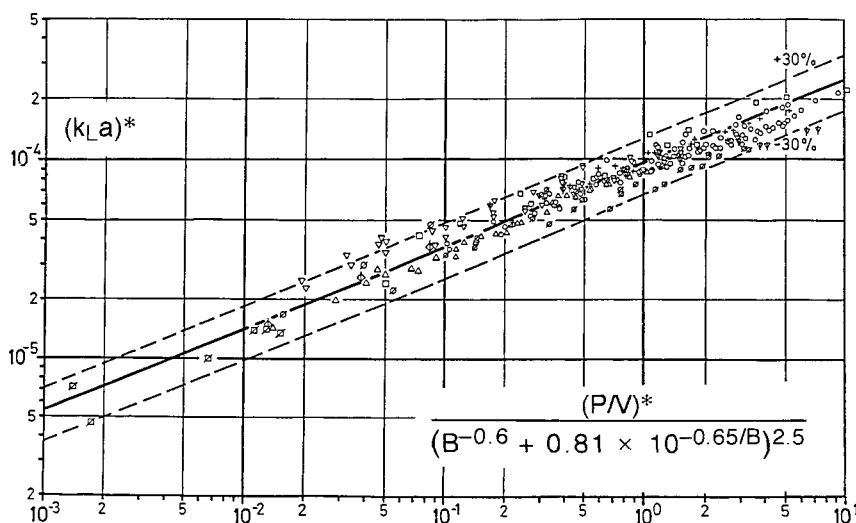


Fig. 77 Sorption characteristic of a mixing vessel with turbine stirrer for a coalescing material system (water/air), from [56]. $B = (\pi/4) \nu^*$.

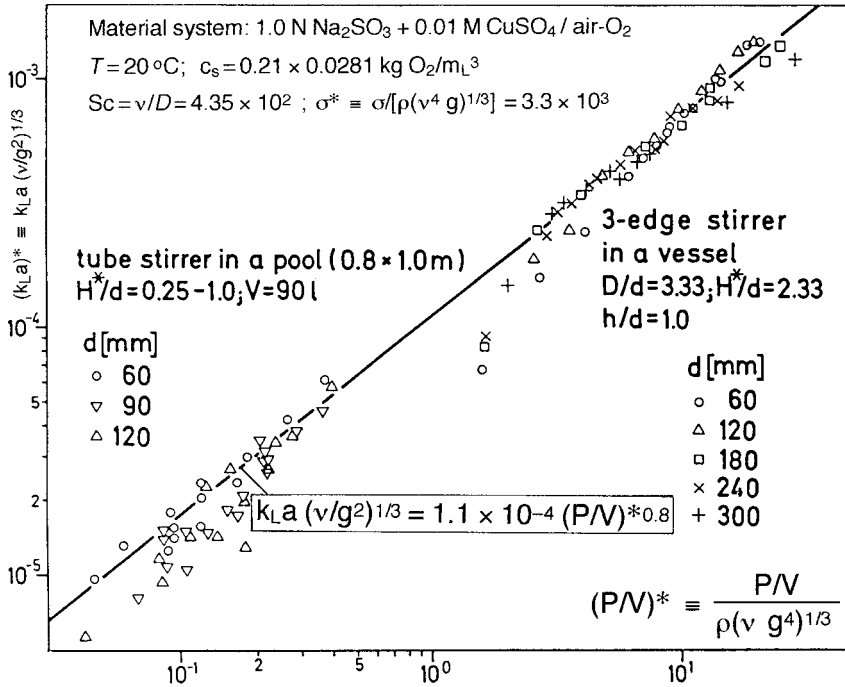


Fig. 78 Sorption characteristic of a mixing vessel with a self-aspirating hollow stirrer in a material system (70 g Na₂SO₃/l) with fully suppressed coalescence; taken from [163/2, 170].

[163/2, 170]. In this material system, the high salt concentration (70 g/l) fully suppresses bubble coalescence. In the case of the self-aspirating hollow stirrer (see Fig. 29), the stirrer power and gas throughput were coupled via the stirrer speed and were therefore dependent on each other. Consequently, v^* does not occur explicitly in the representation in Fig. 78, because it is a function of $(P/V)^*$.

These results can be summarized as follows. In the case of a coalescent material system, for example in pure liquids of low viscosity (e.g. water), the absorption rate depends to an equal extent on P/V and on $v \propto q/D^2$:

$$(k_L a)^* \propto (P/V)^{0.4} v^{0.5} \quad (13.21)$$

In completely coalescence-suppressed systems, for example, in many highly concentrated aqueous salt solutions – see [184], Section 4.10 – in contrast to the above, the following applies:

$$(k_L a)^* \propto (P/V)^{0.7} v^{0.2} \quad (13.22)$$

Only with self-aspirating hollow stirrers, where v^* is not an independent process parameter, was the following correlation found:

$$(k_L a)^* \propto (P/V)^{0.8}. \quad (13.23)$$

This is impressively demonstrated by Fig. 78.

From the above, the following conclusion can be drawn. High power inputs (P/V) are justified only in coalescence-inhibited material systems. In other words, the generation of very fine primary gas bubbles in coalescent-prone systems is not economically justified.

Example 42: Mass transfer in the G/L system in bubble columns with injectors as gas distributors. Optimization of the process conditions with respect to the efficiency of the oxygen uptake $E \equiv G/\Sigma P$ [kg O₂/kWh]

Injectors are two-component nozzles which utilize the kinetic energy of the liquid propulsion jet to disperse the gas continuum into very fine gas bubbles and to distribute them into the liquid. (In contrast, with *ejectors*, the kinetic energy is utilized to produce *suction*.) Their *advantage* over stirrers is that the liquid jet causes gas dispersion directly while the stirrer has to set the entire contents of the vessel in motion in order to generate the necessary shear rate in the liquid. Their *disadvantage* is the predominance of severe coalescence on account of the high gas bubble density in the free jet of the G/L dispersion. However, in contrast to stirrers, the injector cannot cause redispersion of the large gas bubbles.

Design data for the so-called slot injector – see the sketch in Fig. 80 – are presented in the following. The shape of its mixing chamber performs two different functions:

- a) Due to the converging walls of the casing, the shear rate of the free jet increases along the mixing chamber. However, because the cross-sectional area of the mixing chamber remains unchanged, this does not result in an additional pressure drop.
- b) The free jet of the G/L dispersion leaves the slot-shaped mouthpiece in the form of a ribbon which mixes more quickly into the surrounding liquid than a jet with a circular cross-section. This counteracts bubble coalescence.

For optimum design of injectors for G/L contact, their pressure drop and sorption characteristics must be known. The first is needed to dimension the conveying devices (pumps and blowers), the latter to establish the necessary gas and liquid throughputs.

- a) The *pressure drop characteristics* of an injector are based on the following relevance lists:

$$\text{for the gas throughput: } \{\Delta p; d_M; \rho; v_L; q; q_L\} \quad (13.24)$$

$$\text{for the liquid throughput: } \{\Delta p_L; d; \rho_L; v_L; q; q_L\} \quad (13.25)$$

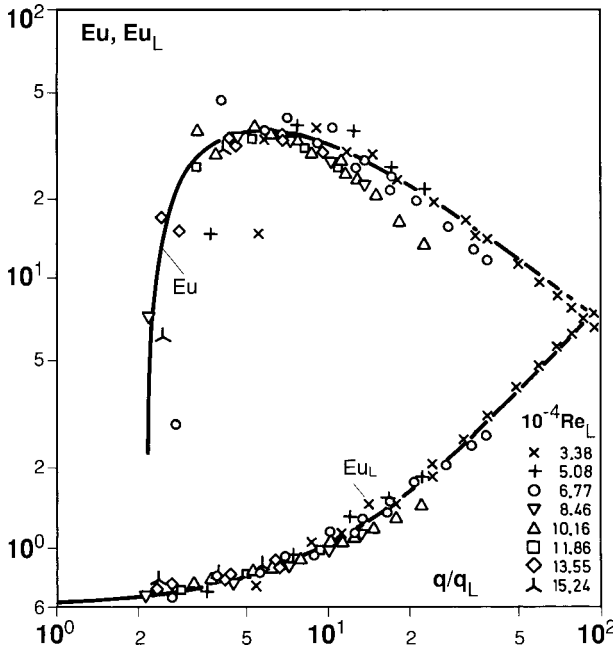


Fig. 79 Pressure drop characteristics of an injector; q is related to standard conditions (20°C, 1 bar)

Δp – pressure drop of the respective medium in the propulsion jet nozzle of diameter, d , in the mixing chamber of diameter, d_M ; q – throughputs; ρ and ν densities and kinematic viscosities of the respective medium. (Gas: without subscript; Liquid: subscript L). The following pi-sets result:

$$\text{for the gas throughput: } \left\{ Eu \equiv \frac{\Delta p d_M^4}{\rho q^2}, \frac{q}{q_L}, Re \equiv \frac{q_L}{\nu_L d_M} \right\} \quad (13.26)$$

$$\text{for the liquid throughput: } \left\{ Eu_L \equiv \frac{\Delta p_L d^4}{\rho_L q_L^2}, \frac{q}{q_L}, Re \equiv \frac{q_L}{\nu_L d} \right\} \quad (13.27)$$

Measurements have shown that Re is irrelevant in the range of $Re > 10^4$. Therefore, in both cases, q/q_L is the only process number effecting Eu ; see Fig. 79.

- b) The *sorption characteristic* of an injector is formed with intensively formulated process parameters. The gas throughput, q , will be replaced by superficial velocity, $v \approx q/D^2$, since it has proved suitable for the correlation of $k_L a$ values in laboratory bubble columns: $k_L a/v = \text{const.}$ (see Example 10, bubble column). Following the sorption characteristics of a mixing vessel, instead of the liquid throughput, q_L , we will use the power of the liquid jet, $P_L = \Delta p_L q_L$, per gas throughput,

q : P_L/q [171]. Including the physical parameters, the following relevance list results:

$$\{k_L a/v, \rho, \nu, D, S_i, P_L/q, g\} \quad (13.28)$$

This leads to the following pi-set:

$$\{(k_L a/v)^*, (P_L/q)^*, Sc, S_i^*\} \quad (13.29)$$

pi-numbers indicated by * have the following meaning:

$$(k_L a/v)^* \equiv \frac{k_L a}{\nu} \left(\frac{\nu^2}{g} \right)^{1/3} \equiv Y - \text{sorption number}$$

$$(P_L/q)^* \equiv \frac{P_L/q}{\rho(\nu g)^{2/3}} \equiv X - \text{dispersion number}$$

The sorption characteristics of an industrial-sized slot injector were measured in a bubble column of technical size ($3^\circ \times 8$ m), as dependent on the common salt (NaCl) content, this influencing the coalescence behavior of the material system, see **Fig. 80**. The results demonstrate that already small amounts of cooking salt (5 g/l $\triangleq$ 0.5 %) suffice to increase the absorption rate by ca. 30 %. The following correlations were found:

g NaCl/l	
0	$Y = 2.4 \times 10^{-6} X^{0.33}$
3	$Y = 2.2 \times 10^{-6} X^{0.37}$
5	$Y = 2.0 \times 10^{-6} X^{0.39}$
10	$Y = 1.5 \times 10^{-6} X^{0.43}$

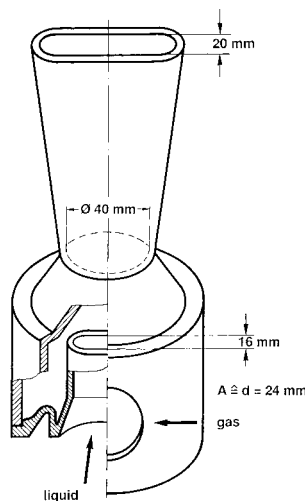
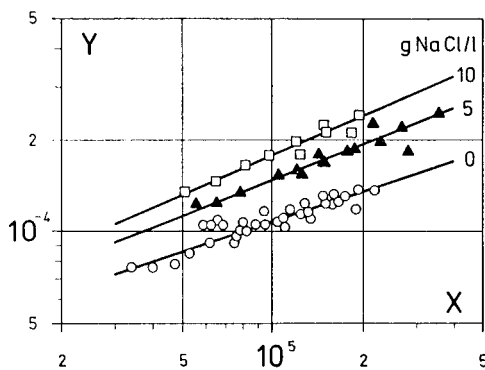


Fig. 80 Sorption characteristics of a slot injector of industrial-size (sketch) in dependence upon the coalescence degree of the system; from [171].

When the coalescence of the primary gas bubbles is more strongly suppressed, the power input of the propulsion jet (P/q) is more efficiently utilized. The above table shows how, with increasing salt concentration (increasing suppression of coalescence), the power of X also increases.

In the following, it will be explained why these characteristics can be used solely as design data for optimizing the running conditions of this *particular size* of injector and why they are definitely not suitable for a scale-up of this device.

In fact, the concept of the quasi-homogeneous gas/liquid mixture, on which also the formulation of the target pi-number $Y \equiv (k_{La}/v)^*$ with intensity quantities is based, and which was fully verified in bubble columns with perforated plates as gas distributors, proves to be *totally inappropriate* when injectors are used as gas dispersers. The explanation for this fact is that, in the case of injectors, the coalescence takes place both in the free jet of the G/L dispersion and at its disintegration into a bubble swarm, while in the case of gas distribution with perforated plates this process has already been completed just above the perforated plate.

This is verified by the measuring data obtained in a column of 1.6 m $\varnothing$, in which a slot injector was installed with a bottom clearance of 1 m and an angle of 25° towards the bottom. The liquid head H above the injector was varied in the range $H = 1\text{--}7$ m. It is shown that the influence of the bubble coalescence in the G/L free jet on the mass transfer – which occurs a short distance from the nozzle orifice – is equalized only after $H = 3$ m; see Fig. 81. This finding proves that the pi-set, eq. (13.19), is not complete but has to be widened by a pi-number which essentially contains liquid at height H . It can be formulated by $H^* \equiv H (g/v^2)^{1/3}$.

Furthermore, it is clear that the scale-up of an injector inherently lessens its efficiency. This is caused by the fact that the dispersing effect of the liquid propulsion jet is restricted to its circumference which, in the case of geometrically similar scale-up, increases only linearly ($u = \pi d$) while its cross-sectional area increases quadratically ($S = \pi d^2/4$). This means that, with increasing diameter of the device, an increasingly smaller fraction of the liquid throughput is dispersed. The dispersion efficiency of injectors inherently diminishes with increasing scale.

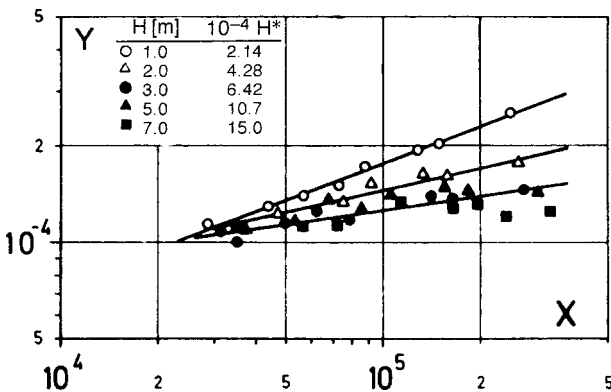


Fig. 81 Dependence of the sorption characteristic $Y(X)$ on the liquid head above the injector [171].

This is confirmed by mass transfer measurements conducted with three differently shaped slot injectors, see Fig. 82. Two of them have been geometrically similarly scaled up (scale $\mu = 1 : 2$). The smaller one ($d = 2 \text{ cm } \varnothing$; $\circ$ signs) was installed in a bubble column of $D = 1.6 \text{ m}$, the larger one ($d = 4 \text{ cm } \varnothing$; $\triangle$ signs) in a bubble column of $D = 2.8 \text{ m}$. In both cases, the liquid head above the injectors was equal ($H = 7 \text{ m}$). Both injectors were attached to the vessel wall with a bottom clearance of 1 m and inclined at an angle of 25° and 35° , respectively, towards the bottom. In this manner it was ensured that the free jet desintegrated into a bubble swarm just above the bottom. The result of these measurements is represented as $Y(X)$ in Fig. 82. It proves that the larger injector has an efficiency which is approximately 30 % lower than the smaller version thereof. A correlation of both of these straight lines can be obtained by the representation: $Y [d (g/v^2)^{1/3} v^{2/3}] = f(X)$.

A solution to this problem was to avoid scaling-up under geometrically similar conditions and to increase only the diameters and not the lengths of the mixing chamber by a factor of 2. As a result, all angles are doubled: the shear rates are enhanced and the free jet fans out more, which further suppresses coalescence. The result ($d = 4 \text{ cm } \varnothing$; $\bullet$ signs) shows that this injector is, at least from $X = 2 \times 10^5$ on, equal to the small one. The positive aspects of this scale-up approach counteract the negative ones which were discussed before.

An enlargement of an injector is necessary if it is to be implemented in wastewater treatment plants. In this case the propulsion jet nozzle must be prevented from being clogged by solid particles contained in the waste water.

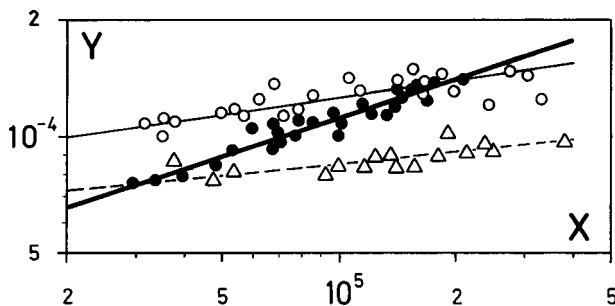


Fig. 82 Sorption characteristics of three slot injectors of different shape and size (explanation in text); from [171].

Last but not least, the influence of bubble coalescence in the treatment space should be addressed. The pi-set for the sorption characteristic only takes account of gas dispersion and not of bubble coalescence in the treatment space. According to the laws governing the free jet, the free jet of the G/L dispersion sucks in the liquid which surrounds it and then it loses its kinetic energy and decomposes into a gas bubble swarm. This process is extremely scale dependent and would need to be considered in a separate relevance list in which all the relevant geometric parameters are included.

To give an example of the dramatic influence which the geometric parameters can have on coalescence behavior, **Fig. 83** shows $Y(X)$ correlations for the industrial-size slot injector which were obtained in a vessel of $3^{\circ} \times 8$ m water height. The injector was positioned 1 m above the bottom at the vessel wall in such a way that its axis formed an angle of 0° , $+35^{\circ}$ resp. -35° with the horizontal. Only in the last case, the free jet was pointed towards the floor and decomposed into the bubble swarm just above it. Near the floor, the suction of the free jet is weakest on account of bottom friction. Furthermore, the bubble swarm which has formed does not exert a “chimney effect” there. Consequently, liquid entrainment into the free jet is suppressed at exactly that point at which it would be particularly supportive of coalescence on account of the weakened kinetic energy of the free jet.

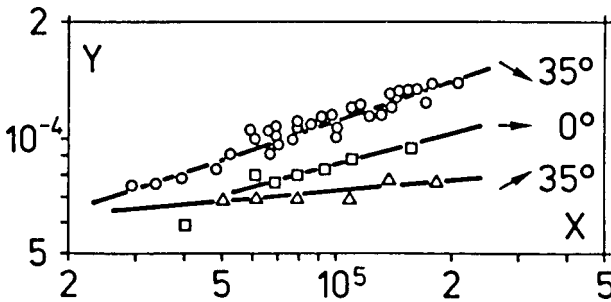


Fig. 83 Influence of the inclination angle of the free jet on bubble coalescence and, consequently, on $Y(X)$; from [171].

Therefore, the sorption characteristics $Y(X)$ presented in **Fig. 80–83** can only be used as design data for the injector type measured in the respective case. They can not be used for scale-up!

In view of the fact that injectors exhibit favorable characteristics for G/L contact, which makes them superior to stirrers with respect to power consumption, systematic research in this area would be most desirable. However, it has been pointed out that it is necessary to perform the measurements, at least in the final stage, in full-scaled plants in order to obtain reliable information on the bubble coalescence in the respective treatment space.

The data presented in **Fig. 83** deal with flow states, these being far more complicated than those encountered in the ventilation technique. It would certainly be of interest to examine which tips and statements can be delivered here by Computational Fluid Dynamics (CFD)!

The presented pressure drop characteristics $Eu_L (q_L/q_G)$ and $Eu_G (q_L/q_G)$, respectively, see **Fig. 79**, as well as the sorption characteristics $Y = f(X) \equiv (k_L a/v)^* = f(P_L/q_G)^*$, see **Fig. 80**, allow the determination of the required pair of $\{q_L, q_G\}$ for any desired O_2 uptake G (e.g., 5 kg O_2 /h per injector) under specific frame conditions (liquid height H , concentration of dissolved O_2 to be maintained, e.g., 1 ppm). However, there exists only one pair of these process parameters, which

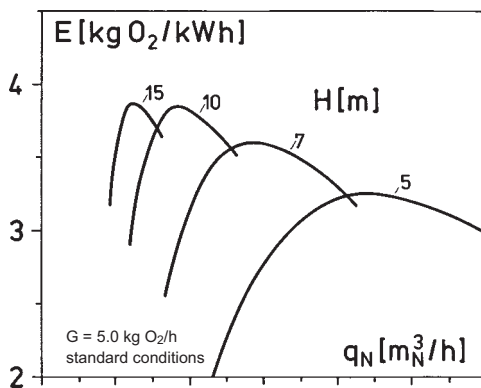


Fig. 84 Mathematical evaluation of the facts presented in Figures 79 and 80 with respect to the optimization of the efficiency of the O_2 uptake $E \equiv G/\Sigma P$ [kg O_2 /kWh]. The efficiency of the blower ($\mu = 0.60$) and of the water pump ($\mu = 0.75$) is taken into account; from [171].

delivers the desired O_2 uptake with a minimum of power consumption ($\Sigma P = P_G + P_L$) and by this with a maximum efficiency $E \equiv G/\Sigma P$ [kg O_2 /kWh].

This is shown in **Fig. 84**. With increasing liquid height the efficiency E increases, because a higher liquid height makes possible a longer residence time for the gas bubbles and thus they can deliver more oxygen to the liquid. The highest achievable $E_{\max}$ value is obtained at an increasingly lower gas throughput.

If $E_{\max}$ values are plotted against liquid height H , **Fig. 85** follows. This representation is the foundation of the so-called “BAYER Tower Biology”, which has been implemented in many countries in the world. It represents the first waste-water

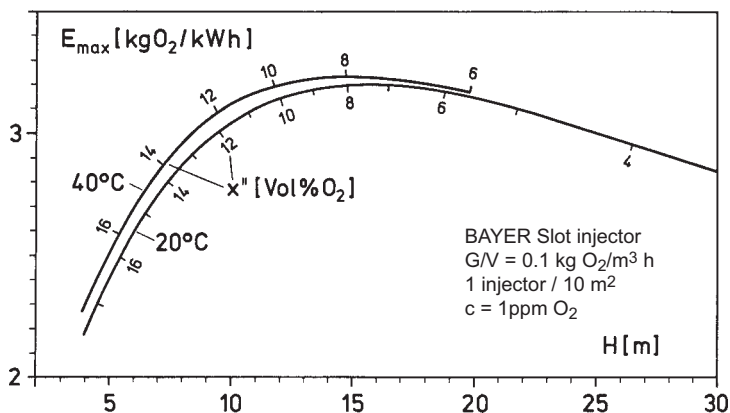


Fig. 85 Foundations of the “BAYER Tower Biology”. Correspondence between the height of the liquid column and the maximum efficiency of the O_2 uptake is taken from **Fig. 84**. x'' is the partial O_2 concentration in the off-gas. Further details are given in the text; from [171].

treatment technology which is optimized with regard to the efficiency of the O_2 uptake.

The fact that 40°C is superior to 20°C with regard to the mass transfer can be explained by the circumstance that, in this temperature interval, an increase of $Sc \equiv \nu/D$ – and thus of k_L – is not counteracted by the simultaneous decrease in Δc – due to the $c_s(T)$ dependence.

13.3

Coalescence in the Gas/Liquid System

This topic concerns the treatment of variable material properties in model experiments. Here, the aeration of liquids is treated and the answer to the question, posed in section 8.1.e, will be answered, i.e., whether or not a waste-water treatment plant may be designed on the basis of laboratory measurements with pure water.

In processes in which one fluid phase is dispersed in the other one (G/L, L/L), under appropriate process conditions, a bubble/droplet size distribution results, which reflects the steady-state between the fluid dispersion and its simultaneous coalescence. This state determines the mass transfer rate. It is extremely dependent on the nature of the continuous phase.

Relatively few publications deal comprehensively with this topic. For the material system L/L the publication [75] and for the material system G/L [186] should be mentioned.

In this chapter, only the mass transfer in the G/L system is discussed, therefore only the coalescence in this material system will be briefly presented.

The coalescence of mechanically produced primary gas bubbles ($d_b \approx 0.3 - 0.5 \text{ mm}$) occurs because, in approaching neighboring gas bubbles, the separating liquid lamella thins and finally bursts. The resistance of the liquid lamella to bursting depends on the nature of the liquid, which is based on the structure of water in aqueous solutions (cluster building of water molecules). Often extremely small amounts of certain inorganic or organic additives to water can exert a large impact on the structure of water and, through it, on the coalescence behavior.

With regard to the effect of the additives, one distinguishes between structure builders and structure destroyers. Only structure builders make the liquid lamella resistant to bursting. These can be certain inorganic salts – in particular electrolytes with small anions and large charge – but also certain organic compounds like normal aliphatic alcohols. This is demonstrated in **Fig. 86** and **Fig. 87**.

In these figures the accelerating factor for physical absorption, m , is plotted against the molar concentration of the additives. In this way the accelerating factor reflects the positive influence of the additives against pure water under otherwise equal experimental conditions.

Depending on the type of additive and its concentration, the absorption rate can be increased by a factor of 6 – 7. Figures 86 and 87 confirm that it is not the surface tension of the solution that matters (inorganic salts increase it, organic sol-

vents decrease it) but the difference in the surface tensions between the liquid boundary layer around the bubbles and the liquid bulk (Gibbs's absorption isotherm). Details are given in [186].

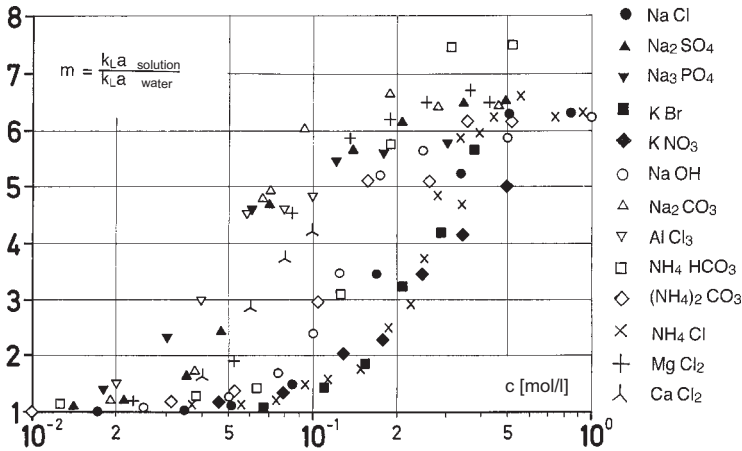


Fig. 86 Accelerating factor for physical absorption, m , in dependence on the type and concentration of some of the inorganic salts which belong to "structure builders". Details are in [173, 186].

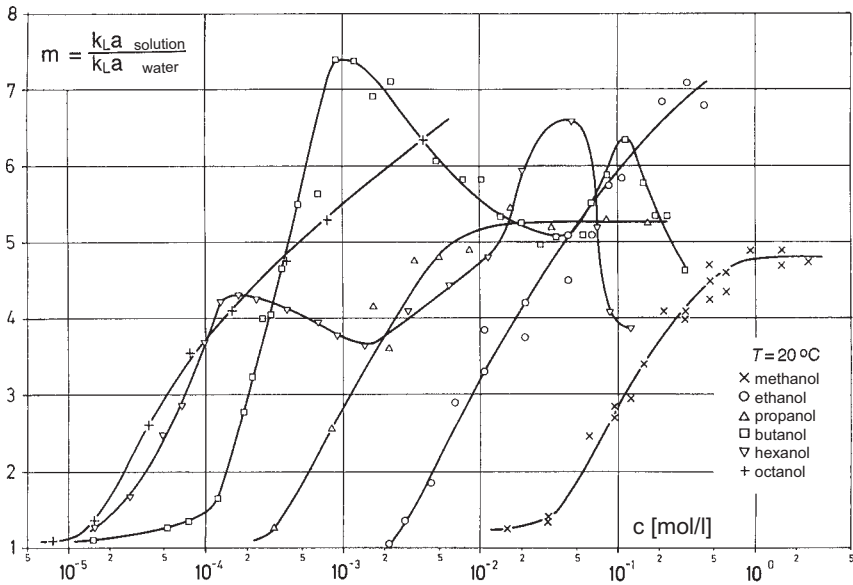


Fig. 87 Accelerating factor for physical absorption, m , in dependence on the concentration of normal aliphatic alcohols belonging to "structure builders". Details are in [173, 186].

The question posed in section 8.1.e, of whether the devices for aeration of a waste-water treatment plant may be designed on the basis of the laboratory measurements in pure water, can now easily be answered. One has to determine by appropriate laboratory experiments [186] the degree of coalescence of the waste water to be treated and then make tests with a model material system exerting the same degree of coalescence.

Example 43: Scaling up of dryers

Drying belongs to those thermal unit operations in which a simultaneous heat and mass transfer occurs. Therefore, it is not surprising to learn that this complicated process has never been treated by dimensional analysis. Instead, intuition and practical experience are used to construct industrial devices. At the same time, mathematical models existed which treat single process steps (e.g., mass transfer from solid particles into the air stream, heat transfer to the solid particles, product flow through the dryer, moisture mass balances, etc.) present in the course of single drying periods. Today, these aspects can be treated by modern computers using numerical fluid simulation (CFD).

In 1994, two issues of the magazine *Drying Technology* were devoted to the dimensioning of dryers [126]. Their content verified the above conclusions.

In his introductory communication, *Kerkhof* [63] referred to the fact that the scaling up of dryers is made difficult by the non-linearity of the occurring physical processes. In addition, he recalled the simultaneous changes of essential physical properties due to a permanent change in drying conditions during the course of drying. In a slow drying process, the heat uptake and the moisture removal are balanced. A flat moisture profile exists within the particle and the process is governed “externally”. In a fast drying process, the moisture diffusion to the surface of the particle is not as fast as its removal. Within the particle a sharp moisture profile exists and the process is governed “internally”.

As a measure of the degree with which these states govern the drying process, an extended Sherwood number Sh^* is introduced (called a “modified Biot number” in [63]):

$$Sh^* \equiv \frac{k_G d_p}{D_{eff}} \frac{\rho_{wG}}{\rho_s} \frac{1}{F_{crit}^*} \quad (13.32)$$

k_G	gas-side mass transfer coefficient
d_p	particle diameter
D_{eff}	effective diffusion coefficient
ρ_{wG}, ρ_s	densities of the saturated water steam and of the solids
F_{crit}^*	mass portion of the moisture in the solid particle

As a rough estimate, the border line between both states will lie at $Sh^* = 1$.

In a spray drying process, the determining dimension is the particle diameter, d_p . With $d_p = 0.1$ mm and assuming conditions usually prevailing in spray dryers, $Sh^* \approx 70$. Therefore, this process will be diffusion-limited, that is “internally” governed.

Genskow [34] pointed out that a correct scale-up is of higher importance for a drying process than in many other unit operations. This is because this process is determining a variety of different product qualities, such as density, particle size distribution, wettability, flow ability, composition, taste, color, and so on. He also concluded that dimensional analysis should be introduced as a compulsory topic in the study of process engineering.

After a review of the customary calculation methods [140] for both essential classes of dryers (convective and contact dryers), the dimensioning methods for spray dryers [78, 95], fluidized and spouted bed dryers [4, 100, 136], cascading rotary dryers [99], pneumatic conveying dryers [62], conductive-heating agitated dryers [97] and layer dryers [88] were presented. They all confirmed the initial conclusion that the scaling up of dryers is still made today without dimensional analysis and the model theory based thereon.

Drying of solvent-moist films and coatings is an important task in the photochemical industry. If the film surface is sufficiently moist, evaporation from a free surface takes place. Consequently, the drying rate only depends on the mass and heat transfer at the surface (“surface evaporation”, 1st drying period). After the critical humidity has been achieved, heat transfer and diffusion within the goods interior govern the drying process (“diffusion-limited”, 2nd drying period).

Y. Sano [124] described the influence of the film thickness, δ , on the drying course of water-moist polyimide films. In thick films ($\delta \approx 1$ mm), the liquid-side diffusion plays an important role from the very beginning. The surface concentration quickly drops off to an equilibrium value and the temperature at the film surface increases to the drying air temperature, without reaching a constant steady-state goods temperature. A period of constant drying rate does not appear.

The curves of the drying course for thicker polyimide films correlate, if the evaporated amount of the solvent is plotted against the Fourier number, Fo . This pi-number is defined as follows:

$$Fo \equiv \frac{x a}{d^2 v} = \frac{\text{momentary position on the belt} \times \text{thermal diffusivity}}{(\text{layer thickness at the beginning})^2 \times \text{belt velocity}} \quad (13.33)$$

From this relationship, the belt velocity of the dryer, necessary to achieve a certain degree of moisture, can be predicted.

In contrast, thin films ($\delta \approx 50 \mu\text{m}$) display, at the beginning of the drying process, a period of constant drying rate at which the evaporation takes place from the free liquid surface. Only when the film is impoverished on solvent, is drying governed by diffusion through the polymer matrix.

H. Jordan [55] experimentally investigated the drying course of a tetrahydrofuran/cyclohexane-moist film. The following assumptions were made:

1. The mass transfer rate is essentially dependent upon the degree of moisture $F^* \equiv F/F_0$. A diffusive mass transfer of

solvent exists within the polymer film, whereby the diffusion coefficient depends on the degree of moisture, $D(F)$. (Later on, this situation will not be considered.)

2. The film base is impenetrable for the solvent, mass transfer occurs solely into the gas space.
3. Because the solvent transfer within the polymer film is rate determining, the temperature field within the film remains practically constant and equal to the temperature on the film surface.
4. The incoming drying gas is solvent-free. Outside the laminar boundary layer the solvent concentration is practically zero.

The course of drying is described by the following parameters:

Target quantity: degree of moisture $F^* \equiv F/F_0$

Influencing parameters:

geometric: film thickness of dried film, δ
film length, L

material: densities of gas, ρ , and of solvent, ρ_L
kinematic viscosity of gas, ν
heat capacities of gas, C_p , and of solvent, C_{pL}
thermal diffusivity of gas, a
diffusion coefficient of solvent in drying gas, D
mass transfer coefficient of solvent, k_L
vapour pressure of solvent, p_L
evaporation enthalpy of solvent per unit mass, ΔH

process related: gas throughput, q
pressure in gas space, p
gas temperature, T
duration of drying, t

The relevance list therefore consists of 17 parameters:

$$\{F^*; \delta, L; \rho, \rho_L, \nu, C_p, C_{pL}, a, D, k_L, p_L, \Delta H; p, q, T, t\} \quad (13.34)$$

In connection with the dimensional system $[M, L, T, \Theta]$, 13 pi-numbers will be produced. To facilitate the dimensional analysis, six of them can be anticipated as trivial pi-numbers:

$$\{F^*; \delta/L, \rho_L/\rho, C_{pL}/C_p, p_L/p, \nu/D\} \quad (13.35)$$

$\nu/D \equiv Sc$ – Schmidt number. The reduced relevance list only contains eleven parameters:

$$\{\delta; \rho, \nu, k_L, C_p, a, \Delta H, p, q, T, t\} \quad (13.36)$$

	ρ	δ	t	T	ν	k_L	C_p	a	ΔH	p	q
M	1	0	0	0	0	0	0	0	0	1	0
L	-3	1	0	0	2	1	2	2	2	-1	3
T	0	0	1	0	-1	-1	-2	-1	-2	-2	-1
Θ	0	0	0	1	0	0	-1	0	0	0	0
M	1	0	0	0	0	0	0	0	0	1	0
3M + L	0	1	0	0	2	1	2	2	2	2	3
T	0	0	1	0	-1	-1	-2	-1	-2	-2	-1
Θ	0	0	0	1	0	0	-1	0	0	0	0

From this $11 - 4 = 7$ pi-numbers result which can be combined to form known or practical pi-numbers (right-hand side):

$$\Pi_1 \equiv \frac{\nu t}{\delta^2} \quad \Pi_1 a / \nu = \frac{a t}{\delta^2} \equiv \text{Fo (Fourier)}$$

$$\Pi_2 \equiv \frac{k_L t}{\delta} \quad \Pi_2 \Pi_4 a / D = \frac{k_L \delta}{D} \equiv \text{Sh (Sherwood number)}$$

$$\Pi_3 \equiv \frac{C_p t^2 T}{\delta^2} \quad \Pi_3^{-1} \Pi_2 \Pi_5^{-1} = \frac{k_L^2}{\Delta H}$$

$$\Pi_4 \equiv \frac{a t}{\delta^2} \quad \Pi_4^{-1} \Pi_1 = \frac{\nu}{a} \equiv \text{Pr (Prandtl number)}$$

$$\Pi_5 \equiv \frac{\Delta H t^2}{\delta^2} \quad \Pi_5 \Pi_3^{-1} = \frac{\Delta H}{C_p T}$$

$$\Pi_6 \equiv \frac{p t^2}{\rho \delta^2} \quad \Pi_6 \Pi_5^{-1} = \frac{p}{\rho \Delta H}$$

$$\Pi_7 \equiv \frac{q t}{\delta^3} \quad \Pi_7 \Pi_1^{-1} \delta / L = \frac{q}{\nu L} \equiv \text{Re (Reynolds number)}$$

The complete set of 13 pi-numbers therefore consists of:

Target number:	F^*
Geometric pi-number:	δ/L
Material pi-numbers:	ρ_L/ρ , $C_{p,L}/C_p$, p_L/p , Sc , Sh , $k_L^2/\Delta H$, Pr
Process related pi-numbers:	Fo , $\Delta H/C_p T$, $p/\rho \Delta H$, Re

In the diploma thesis conducted by *Jordan* [109], concerning the drying of tetrahydrofuran/cyclohexane-wetted polyvinylbutyral films, the influence of only a few of the given parameters, namely δ/L , Re , $\Delta H/c_p T$, on the dependence $F^*(Fo)$ could be experimentally examined. Their influence was then taken into account by simple power products, **Fig. 88**.

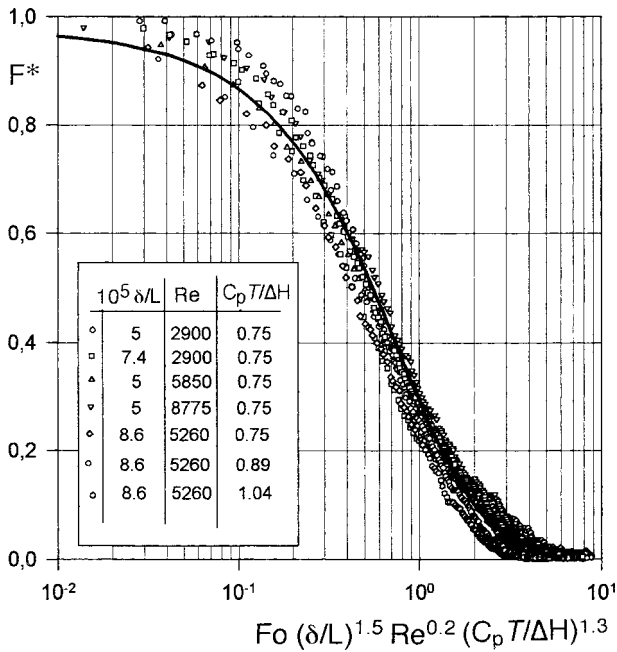


Fig. 88 The correlation $\{F^*, Fo, \delta/L, Re, \Delta H/C_p T\}$ for the discussed material system.

14

Selected Examples for the Dimensional-analytical Treatment of Processes in the Field of Chemical Unit Operations

Introductory Remark

Chemical reactions obey the rules of chemical thermodynamics and chemical reaction kinetics. One can represent them easily in a dimensionless space. However, if they take place slowly and without significant heat of reaction in the homogeneous system ("micro-kinetics") they are not subject to any scale-up rules.

Nonetheless, such a reaction course occurs only very infrequently in chemical reaction engineering. Most chemical reactions take place in heterogeneous material systems (G/L, L/L, G/S, L/S, G/L/S) and generate considerable amounts of reaction heat. Consequently, the genuine chemical action is accompanied by mass and heat transfer processes ("macro-kinetics") which are scale-dependent. The course of such chemical reactions will be similar on a small and large scale, if the mass and heat transfer processes are similar and the "chemistry" remains the same.

In a continuous reaction process, the actual residence time of the reaction partners in the reactor plays a major role. It is governed by the residence time distribution of the reactor which gives information on back-mixing (macro-mixing) of the throughput. This emphasizes the interaction between chemical reaction and fluid dynamics.

Chemical reactions exist in which mass and/or heat transfer represent the rate-controlling step. If both transfer processes occur *simultaneously*, special scale-up problems may arise because they obey different laws; see the reaction in a catalytic packed-bed reactor, Example 44/2.

From the point of view of dimensional analysis, a chemical engineering problem presents itself with the appearance of chemical parameters containing an additional base dimension, namely the amount of substance N in their respective dimensions (base unit: mole). Consequently, we then have to deal with a 5-parametric dimensional system $[M, L, T, \Theta, N]$.

Example 44: Continuous chemical reaction process in a tubular reactor

Historical credit goes to *Gerhard Damköhler* (1908–1944) who was the first to use the theory of similarity [21] to investigate a chemical process in conjunction with mass and heat transfer. In a purely theoretical way he examined the conditions under which scale-up would be possible in the case of (inevitably) partial similarity and he checked the consequences which would result from such a procedure. However, before analysing his method, we shall discuss this problem from the point of view of dimensional analysis.

1. Homogeneous irreversible 1st order reaction

A homogeneous, constant-volume chemical reaction taking place in a tubular reactor is influenced by mass and heat transfer processes. The flow condition is described by

$$\{v, d, L, \rho, \mu\} \quad (14.1)$$

(v – flowrate, d , L – diameter and length of the tubular reactor, ρ , μ – fluid density and viscosity). All physical properties are related to the known inlet temperature T_0 . In contrast to the continuous reaction in the catalytic packed-bed reactor – see section 2 – it is assumed here that c and T differences in radial direction are negligible (plug flow).

The chemical reaction and its conversion are characterized by the *inlet* and *outlet* concentrations c_{in} and c_{out} as well as by the effective reaction rate constants k_{eff} . It should be noted that the reaction order – here 1st order – governs the dimension of k_0 ! At the temperature field actually predominating in the reactor, the effective reaction rate constants k_{eff} in the reactor will adjust themselves in accordance with Arrhenius' law:

$$k_{eff} = k_0 \exp(E/RT) \quad (14.2)$$

The mass and heat transfer is described by

$$\{D, C_p, k, c_{in}\Delta H_R, T_0, \Delta T\} \quad (14.3)$$

(D – diffusion coefficient, C_p – heat capacity, k – thermal conductivity, $c_{in}\Delta H_R$ – heat of reaction per unit time and volume, T_0 – inlet temperature, ΔT – temperature difference between fluid and tube wall). The complete relevance list is therefore:

$$\{v, d, L, \rho, \mu, c_{in}, c_{out}, k_0, E/R, D, C_p, k, c_{in}\Delta H_R, T_0, \Delta T\} \quad (14.4)$$

Only nine numbers are formed from these 15 dimensional parameters if the amount of heat, H , is added to the five primary quantities $[M, L, T, Q, N]$ as the sixth base dimension: $15-6=9$. If L/d , c_{out}/c_{in} , E/RT_0 , and $\Delta T/T_0$ are anticipated as trivial numbers, the other five pi-numbers can be obtained using the following simple dimensional matrix:

	ρ	L	k_0	T_0	$c_{in}\Delta H_R$	v	μ	D	C_p	k
M	1	0	0	0	0	0	0	0	-1	0
L	-3	1	0	0	-3	1	-1	2	0	-1
T	0	0	-1	0	0	-1	-1	-1	0	-1
Θ	0	0	0	1	0	0	0	0	-1	-1
H	0	0	0	0	1	0	0	0	1	1
Z_1	1	0	0	0	0	0	1	0	-1	0
Z_2	0	1	0	0	0	1	2	2	0	2
Z_3	0	0	1	0	0	1	1	1	0	1
Z_4	0	0	0	1	0	0	0	0	-1	-1
Z_5	0	0	0	0	1	0	0	0	1	1

$$Z_1 = M; \quad Z_2 = 3M + L + 3H; \quad Z_3 = -T; \quad Z_4 = \Theta; \quad Z_5 = H$$

The following dimensionless numbers result:

$$\Pi_1 = \frac{v}{L k_0} = (k_0 \tau)^{-1} \text{ (mean residence time } \tau \equiv L/v \text{ at pipe flow!)}$$

$$\Pi_2 = \frac{\mu}{\rho d^2 k_0} = (k_0 \tau \text{ Re } L/d)^{-1}$$

$$\Pi_3 = \frac{D}{L^2 k_0} = (k_0 \tau \text{ Re } \text{Sc } L/d)^{-1}$$

$$\Pi_4 = \frac{\rho C_p T_0}{c_{in} \Delta H_R} = \text{Da}^{-1} \text{ (Da – Damköhler number)}$$

$$\Pi_5 = \frac{k T_0}{c_{in} \Delta H_R d^2 k_0} = (k_0 \tau \text{ Re } \text{Pr } \text{Da } L/d)^{-1}$$

The power products formed with the aid of the dimensional matrix can be traced back to known, mostly named, numbers (Re, Pr, Sc). The only new dimensionless

number here is the Damköhler number, Da , which will be discussed later. The numbers obtained, together with the four anticipated trivial numbers, give the following dimensional-analytical framework:

$$\{L/d, c_{out}/c_{in}, E/RT_0, \Delta T/T_0, k_0\tau, Re, Sc, Pr, Da\} \quad (14.5)$$

The pi-space under consideration is described completely by this pi-set.

It is obvious that when scaling up chemical reactors it is generally unacceptable to change the reaction temperature T_0 because this would impede the reaction course (and hence k_0) or at least the selectivity of the reaction. For the same reason, it is not possible to vary the physical or chemical properties of the reaction partners.

If scale-up of the tubular reactor of the given geometry ($L/d = idem$) is performed at T_0 and $\Delta T/T_0 = idem$, taking account of these restrictions, the kinetic and material numbers E/RT_0 , Da , Sc , Pr remain unchanged. Therefore, to attain a specified degree of conversion $c_{out}/c_{in} = idem$, it is only necessary to ensure that the other two numbers $Re = v d \rho/\mu$ and $k_0\tau \equiv k_0 L/v$ are adjusted in such a way that they remain *idem*. However, it is immediately clear that this is an impossibility in the case of $L/d = idem$ because

$$v d \rightarrow v L = idem \text{ and } L/v = idem$$

cannot be fulfilled simultaneously in the tubular reactor! (In a mixing vessel this problem does not exist.)

In the scale-up of a tubular reactor, the problem is to increase the flowrate $q \propto v d^2$ by a factor n (not to be confused with the scale μ !) while retaining the chemical efficiency (yield, conversion, selectivity, etc.):

$$q_T = n q_M \rightarrow v_T d_T^2 = n v_M d_M^2$$

How does this demand react with the conditions $Re = idem$ and $k_0\tau = idem$?

$$Re \propto v d = idem \rightarrow d_T = n d_M; \quad v_T = v_M/n$$

It follows that

$$V_T = n^3 V_M \text{ and hence } \tau_T = n^2 \tau_M \quad (\tau = V/q)$$

The volume of the full-scale facility is n^3 times larger than that of the model. However, the flow through it is only $n q_M$, consequently the residence time τ is n^2 times larger. From $k_0\tau = idem$ it follows that the reaction rate constant k_0 in the prototype would be smaller by a factor of n^2 and this contradicts the precondition stated above!

At this point, one might ask whether or not the demand for $Re = idem$ is justified. In the last instance we are dealing with a fast reaction if we have selected a

tubular reactor. Consequently, the flow through this pipe reactor will certainly be turbulent. It is well known that Re only has a slight influence in the turbulent flow regime!

2. Heterogeneous catalytic 1st order reaction

Let us now consider a “catalytic packed bed reactor”, i.e., a tubular reactor filled with a grained catalyst through which the gas mixture flows. With the particle diameter of the catalyst, d_p , an additional dimensionless number d_p/d is added to the pi-space; the Reynolds number is now expediently formed with d_p . The reaction rate is related to the unit of the bulk volume and characterized by an effective reaction rate constant $k_{0,eff} \equiv k^*$. The thermal conductivity (k) also has to be valid for the gas/bulk solids system and diffusion can be considered as being negligible (Sc is irrelevant). The complete pi-space is therefore:

$$\{c_{out}/c_{in}, L/d, d_p/d, E/RT_0, \Delta T/T_0, k^*\tau, Re, Pr, Da\} \quad (14.6)$$

Since the diameter of the catalyst grain has a considerable influence on the reaction rate, its variation will not be permitted during scale-up; this means that the geometrical similarity will *inevitably* be violated by $d_p/d \neq idem$. Therefore, scale-up of the tubular reactor filled with catalyst is, at best, possible through adherence to partial similarity whereby it is necessary to check whether violation of the geometric similarity alone is enough to guarantee scale-up.

The scale-up problem under discussion is completely covered by the given pi-space. However, it can be considered in greater depth by compiling fundamental differential equations which mathematically formulate the conditions for preservation of mass, impulse and energy (c.f. statement in Fig. 6).

G. Damköhler [21] used this possibility to develop *Navier-Stokes* differential equations of the mass and heat transfer for the case of an adiabatic reaction. Analytical solution of these differential equations is not possible. However, if they are made dimensionless, it becomes apparent that the pi-space is formed by the five dimensionless numbers listed below:

$$Re \equiv \frac{vL\rho}{\mu}$$

$$I \equiv \frac{k^*L}{v} \rightarrow k^*\tau \text{ (for the pipe flow)}$$

$$II \equiv \frac{k^*L^2}{D} = \frac{k^*L}{v} \frac{vL}{v} \frac{v}{D} = k^*\tau Re Sc$$

$$III \equiv \frac{k^*c_{in}\Delta H_R L}{\rho C_p T_0 v} = \frac{c_{in}\Delta H_R}{\rho C_p T_0} \frac{k^*L}{v} = Da k^*\tau$$

$$IV \equiv \frac{k^* c_{in} \Delta H_R d^2}{k T_0} = \frac{c_{in} \Delta H_R}{\rho C_p T_0} \frac{k^* L}{v} \frac{v L \rho}{\mu} \frac{C_p \mu}{k} = Da \, k^* \tau \, Re \, Pr$$

Although *Damköhler* traced numbers I to IV back to the above combinations of named dimensionless numbers known at that time, numbers I to IV have come to be known as the four *Damköhler* numbers Da_I to Da_{IV} in chemical literature. We will *not* identify them in this way, instead we will only refer to the new, genuine reaction kinetic pi-number

$$\frac{c_{in} \Delta H_R}{\rho C_p T_0} \equiv Da \quad (14.7)$$

as the *Damköhler* number Da .

In fact, the advantage of these combinations of numbers obtained by making differential equations dimensionless, over those combinations delivered by dimensional analysis, is that they characterize certain types of mass and heat transfer, respectively. For example, III represents the ratio of the reaction heat to heat removal by convection, while IV expresses the ratio of the reaction heat to heat removal by conduction.

G. *Damköhler* bases his analysis of the scale-up problem relating to the catalytic tubular reactor on the following pi-set (D and hence number combination II are irrelevant):

$$\left\{ \frac{L}{d}, \frac{d_p}{d}, \frac{v d_p \rho}{\mu}, \frac{k^* L}{v}, \frac{k^* c_{in} \Delta H_R L}{\rho C_p T_0 v}, \frac{k^* c_{in} \Delta H_R d^2}{k T_0} \right\} \quad (14.8)$$

Re I III IV

He knows that he may not vary the temperature T_0 and d_p if he does not want to risk influencing the chemical course of the reaction. Consequently, as already mentioned, geometric similarity is inevitably violated during scale-up on account of $d_p/d \neq \text{idem}$. *Damköhler* is therefore prepared to waive adherence to $L/d = \text{idem}$ as well. However, he points out that this will necessarily lead to consequences for heat transfer behavior. In this case, he uses the *hypothesis* that thermal similarity is guaranteed if the ratio of IV to III (heat conduction through the tube wall to heat removal by convection) is kept equal:

$$\frac{IV}{III} = \frac{k^* c_{in} \Delta H_R d^2}{k T_0} \frac{\rho C_p T_0 v}{k^* c_{in} \Delta H_R L} = \frac{\rho C_p v d}{k L} \equiv Pe \frac{d}{L} = \text{idem}$$

Scale-up must therefore be effected in the pi-space $\{k^* \tau, Re, Pe\}$. It then follows that:

$$Re = \text{idem} \rightarrow v = \text{idem}$$

$$k^* \tau = k^* L/v = \text{idem} \rightarrow L = \text{idem}$$

$$Pe = \text{idem} \rightarrow d = \text{idem}$$

The requirement $d = \text{idem}$ means $\mu = \text{idem}$ and this makes a change in scale impossible.

Result: Abandoning geometric similarity is not enough to guarantee chemical similarity which requires that T_0 and hence k^* be idem.

Damköhler now proposes to abandon not only geometric, but also fluid dynamic similarity ($Re = \text{irrelevant}$). The scale-up should depend exclusively on thermal and reaction similarity. This means that, apart from $k^*\tau$, only III and IV must be kept constant. The pi-space is then:

$$\left\{ \frac{k^* L}{v}, \frac{k^* c_0 \Delta H_R L}{\rho C_p T_0 v}, \frac{k^* c_0 \Delta H_R d^2}{k T_0} \right\} \quad (14.9)$$

Since, according to *Damköhler*, the heat conductivity is approximately proportional to the flowrate ($k_{\text{eff}} \propto v$) in the turbulent flow regime and, from a certain small d_p/d ratio, independent of it, the following scale-up rules result from the above pi-space:

$$Da_I \equiv k^* \tau \rightarrow L/v = \text{idem}$$

$$Da_{III} \rightarrow L/v = \text{idem}$$

$$Da_{IV} \rightarrow d^2/k \propto d^2/v = \text{idem}$$

In conjunction with the flow rate equation and the enlargement factor for the liquid throughput $n \equiv q_T/q_M$, it follows that

$$Da_{IV} = \text{idem} \rightarrow d_T = n^{1/4} d_M \text{ as well as } v_T = n^{1/2} v_M$$

$$Da_I = \text{idem} \text{ as well as } Da_{III} = \text{idem} \rightarrow L_T = n^{1/2} L_M$$

Consequently,

$$(L/d)_T = n^{1/4} (L/d)_M \quad (14.10)$$

and, in conjunction with $\Delta p \propto L v^2$, the following is valid

$$\Delta p_T = n^{3/2} \Delta p_M$$

If one assumes that both hypotheses (irrelevance of both geometric and fluid dynamic similarity) are applicable, this result shows that the industrial tubular reactor would be influenced by a pressure increase. This would not only incur costs but could also have an unknown effect on the course of reaction. Therefore, tube bundle reactors are more economic.

Scale-up at partial similarity was discussed in Section 6.2. It was pointed out that various strategies exist and Froude's method which is based on *dividing the process into parts* that can be investigated separately, was presented.

The present example illustrates another method for dealing with partial similarity. It is based on *deliberately abandoning certain similarity criteria* and theoretically and/or practically checking the effects on the entire process. Damköhler's example convincingly demonstrates that valuable information concerning the scale-up of a complex chemical process can be deduced from theoretical considerations alone if the principles relating to the theory of similarity are used consistently.

However, Damköhler seems to have been very disappointed with the result of his study. His conclusions, in [21], as quoted below, cannot be interpreted any other way:

"Although it is basically possible to apply the theory of similarity to chemical processes and to scale up one of these processes in such a way that geometric, fluid dynamic, thermal and reaction-kinetic similarity is retained to a greater or lesser extent, these transformation processes are only of limited importance. They may be quite useful for increasing equipment performance two to five-fold but hardly to much larger amounts. This circumstance is of importance since it is more or less equivalent to practical failure of the theory of similarity. This, however, was not to be expected from the beginning, especially in view of the fact that the theory of similarity proved itself brilliantly in the solution of other heat transfer problems where no additional chemical conditions had to be fulfilled".

The results of his studies and the efficiency of methods based on the theory of similarity have been assessed differently by posterity. If the method shows that scale-up is not possible, this by no means points to failure of the method but rather is a valuable indication of the given facts!

Example 45: Description of the mass and heat transfer in solid-catalyzed gas reactions by dimensional analysis

In this example, the composition of the catalyst surface is responsible for its activity. Therefore, catalysts are placed on porous supporting material (pellets) which have specific surface areas of some hundred m^2/g pellet. Because the pellet core has the largest surface area, the reaction predominantly takes place here.

In solid-catalyzed gas reactions, the rate equations have to be extended by the mass and heat transfer terms. The following have to be considered:

1. Diffusion resistance for educts in the gas film and in the catalyst pores.
2. Sorption processes and reaction on the catalyst surface.
3. Diffusion resistance for products in the gas film and in the catalyst pores.

If the reaction proceeds without a change in molar volume, steps 1 and 3 can be considered as countercurrent diffusion.

In this field, intensive research was carried out about 40–60 years ago, see, e.g., [2, 11, 114, 139, 155]. The results of these studies are contained in textbooks on chemical engineering, see, e.g., [76]. Only those results will be presented and discussed here, which are directly associated with dimensional analysis.

1 Outer transfer processes

The isothermal and the non-isothermal reaction with the diffusion resistance in the gas film will be treated in succession.

1.1 Surface reaction with diffusion resistance in the gas film

Here, we look at the conversion of a gaseous component A on a non-porous catalyst under isothermal conditions. In the steady-state, the volume-related rates of the gas-side mass transfer and the surface reaction are equal to each other:

$$k_G a (c_G - c_S) = k c_S \quad (14.12)$$

k_G – gas-side mass transfer coefficient; a – surface area per unit volume; k – reaction rate constant; subscripts: G – gas phase; S – solid phase.

Using this equation, the unknown gas concentration prevailing on the catalyst surface, c_S , can be expressed as

$$c_S = \frac{k_G a}{k + k_G a} c_G = \frac{1}{1 + k/k_G a} c_G \quad (14.13)$$

and the effective reaction rate $r_{\text{eff}} = k c_S$ formulated:

$$r_{\text{eff}} = \frac{k}{1 + k/k_G a} c_G = k_{\text{eff}} c_G \quad (14.14)$$

At $k/k_G a \ll 1$, the chemical reaction is much slower than the mass transfer rate. In this case, a concentration difference in the gas boundary layer does not exist. Therefore, $k/k_G a$ can be interpreted as the ratio of the reaction rate without transport limitation ($c_S = c_G$) to the reaction rate at transport limitation ($c_S \rightarrow 0$).

$k/k_G a$ is also named the second Damköhler number Da_{II} , because in $Da_{II} \equiv k_1 L^2/D$, the expression $(D/L)L^{-1}$ can be interpreted as $k_G a$: $D/L k_G$; $A/V \equiv a$ and k_1 is the reaction rate of a first-order reaction.

The quotient from the effective reaction rate (r_{eff}) and that (r) without diffusion control ($c_G = c_S$) defines the outer catalyst effectiveness factor, η_{ext} :

$$\eta_{\text{ext}} \equiv \frac{r_{\text{eff}}}{r} = \frac{k_{\text{eff}} c_G}{k c_G} = k \frac{1}{1 + Da_{II}} c_G \times \frac{1}{k c_G} = \frac{1}{1 + Da_{II}} \quad (14.15)$$

Whether r_{eff} is diffusion controlled or not, i.e., whether $\eta_{\text{ext}} = 1$ or $\eta_{\text{ext}} < 1$, r_{eff} can be ascertained if the measured k value is first multiplied by the (solely known) c_G

$$r_{\text{eff}} = r \eta_{\text{ext}} = \eta_{\text{ext}} k c_G \quad (14.16)$$

and then divided by the maximum attainable mass transfer

$k_G a (c_G - c_s)$ at $c_s = 0$, i.e. with $k_G a c_G$

$$\frac{r_{\text{eff}}}{k_G a c_G} = \eta_{\text{ext}} \frac{k^* c_G}{k_G a c_G} = \eta_{\text{ext}} \text{Da}_{\text{II}} \quad (14.17)$$

For this purpose the $k_G a$ value has to be determined separately: a can be measured and k_G can be calculated. Then η_{ext} can be determined from the correlation $\eta_{\text{ext}}(\eta_{\text{ext}} \text{Da}_{\text{II}})$ in Fig. 89.

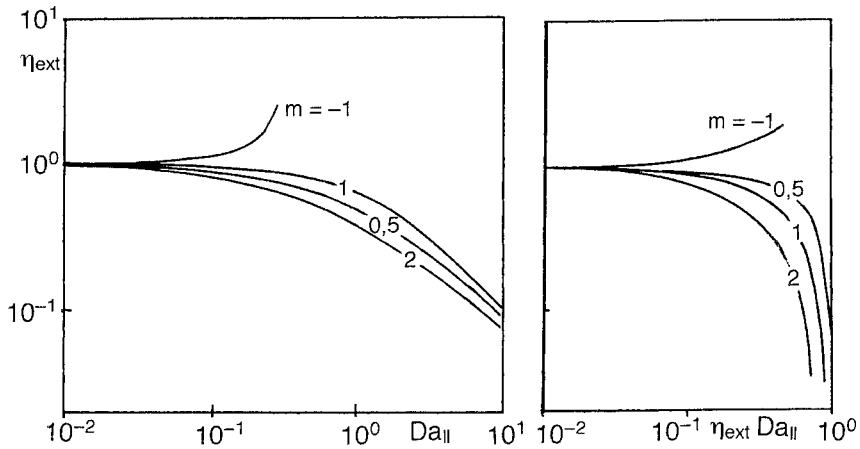


Fig. 89 External catalyst effectiveness factor η_{ext} as a function of the Damköhler-II number $\text{Da}_{\text{II}} \equiv k/k_G a$ and the reaction order m ; from [11].

1.2 Surface reaction with diffusion and heat transfer resistance

In fast exothermic reactions, in addition to $\text{grad } c$, $\text{grad } T$ ($T_G \neq T_s$) is also present in the boundary layer between the gas bulk phase and the catalyst surface. For the outer effectiveness factor η_{ext} this means that

$$\eta_{\text{ext}} = \frac{r_{\text{eff}}}{r} = \frac{k(T_s) c_s}{k(T_G) c_G} \quad (14.18)$$

The quotient c_s/c_G corresponds to the expression

$$\frac{c_s}{c_G} = \frac{k_G a - \eta_{\text{ext}} k(T_G)}{k_G a} = 1 - \eta_{\text{ext}} \frac{k(T_G)}{k_G a} = 1 - \eta_{\text{ext}} \text{Da}_{\text{II}} \quad (14.19)$$

Coupling of (14.19) and (14.18) gives the sought after correlation between $\text{grad } T$ and η_{ext} :

$$\eta_{\text{ext}} = \frac{k(T_s)}{k(T_G)} (1 - \eta_{\text{ext}} \text{Da}_{\text{II}}) \quad (14.20)$$

The introduction of the Arrhenius term for $k(T)$ and the pi-number $\text{Arr} \equiv E/RT$ gives:

$$\eta_{\text{ext}} = \exp \left[\text{Arr}_G \left(1 - \frac{T_G}{T_s} \right) \right] (1 - \eta_{\text{ext}} \text{Da}_{\text{II}}) \quad (14.21)$$

For the determination of η_{ext} , not only the accessible quantity $\eta_{\text{ext}} \text{Da}_{\text{II}}$ but also T_G/T_s must be known. This can be obtained by the following reasoning. In the steady-state, heat removal equals the heat production:

$$Q/V = h a (T_s - T_G) = r_{\text{eff}} (-\Delta H_R) \quad (a \equiv A/V) \quad (14.22)$$

Resolved after T_s/T_G , the equation reads

$$\frac{T_s}{T_G} = 1 + \frac{r_{\text{eff}} (-\Delta H_R)}{T_G h a} \quad (14.23)$$

h can be expressed via an analogy between the mass and heat transfer according to the correlation $\text{Sh} = \text{Nu} (\text{Sc}/\text{Pr})^n$ as follows:

$$h = k_G (k/D) \text{Le}^{-n} \quad (\text{Lewis number } \text{Le} \equiv \text{Sc}/\text{Pr})$$

Now, the reciprocal value of T_s/T_G is formed, see eq. (14.23), and h is expressed by the, just presented, correlation:

$$\frac{T_G}{T_s} = \frac{1}{1 + \frac{r_{\text{eff}} (-\Delta H_R)}{T_G a k_G (k/D) \text{Le}^{-n}}} \quad (14.24)$$

A multiplication of the fraction in the denominator by c_G finally gives:

$$\begin{aligned} \frac{T_G}{T_s} &= \frac{1}{1 + \frac{r_{\text{eff}} (-\Delta H_R)}{T_G a k_G (k/D) \text{Le}^{-n}} \times \frac{c_G}{c_G}} = \frac{1}{1 + \boxed{\frac{(-\Delta H_R) c_G}{(k/D) \text{Le}^{-n} T_G}} \times \frac{r_{\text{eff}}}{k_G a c_G}} \\ &= \frac{1}{1 + \beta \times \eta_{\text{ext}} \text{Da}_{\text{II}}} \end{aligned} \quad (14.25)$$

The fraction surrounded by a box in eq. (14.25) is named the Prater number β . It is formed with the gas-side values (c_G , T_G). The fraction on the right-hand side of the box is the already known pi-number combination $\eta_{\text{ext}} \text{Da}_{\text{II}}$. When this correlation is introduced into the equation for η_{ext} , then it follows that:

$$\eta_{\text{ext}} = \exp \left[-\text{Arr}_G \left(\frac{1}{1 + \beta \eta_{\text{ext}} \text{Da}_{\text{II}}} - 1 \right) \right] (1 - \eta_{\text{ext}} \text{Da}_{\text{II}}) \quad (14.26)$$

The Prater number β represents a combination of pi-numbers which can be traced back to the already known ones:

$$\beta \equiv \frac{\text{Da}_{\text{IV}}}{\text{Da}_{\text{II}} \text{Le}} \times \text{Le}^{-n} \rightarrow \beta \equiv \text{Da} \times \text{Le}^{-(1+n)} \quad (14.27)$$

For Da see the definition, eq. (14.7). From this it follows that the name “Prater number” is superfluous.

Equation (14.26) is represented in **Fig. 90** for two different Arr values. It can be seen that the outer effectiveness factor η_{ext} can attain values of $\gg 1$. In such cases the reaction takes place on the surface of the catalyst at $T_s \gg T_G$. The difference in T can reach 10–30 K and more.

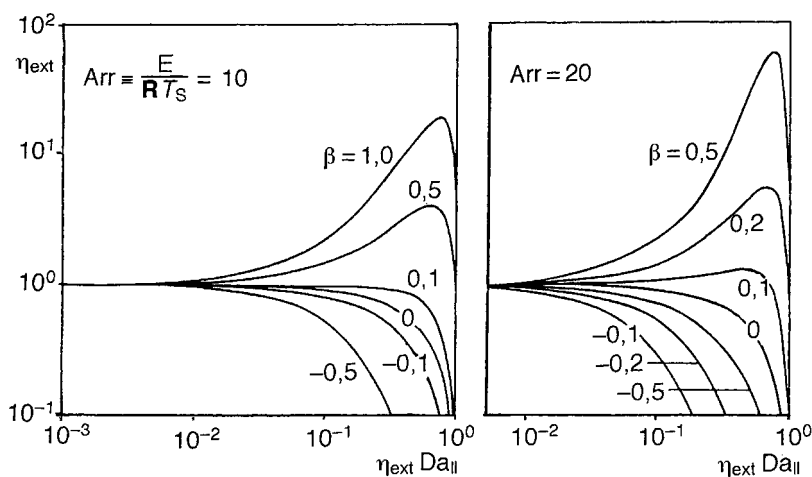


Fig. 90 Outer catalyst effectiveness factor, η_{ext} , as a function of the measurable quantity $\eta_{\text{ext}} \text{Da}_{\text{II}}$ for two values of the Arrhenius number, Arr, and for different values of the Prater number $\beta \triangleq \text{Da} \times \text{Le}^{-(1+n)}$.

2 Inner transfer processes

In analogy to the above, the isothermal and the non-isothermal reaction will be treated in succession with respect to diffusion resistance in the catalyst pore.

2.1 Isothermal reaction with the diffusion resistance in the pore

Solving the differential equation for the mass balance at steady-state (output – input + disappearance by reaction = 0), see, e.g., [76], in a volume element of the

pore delivers an equation which describes the change of concentration in the pore as a function of its length, L :

$$\frac{c}{c_s} = \frac{\cosh m(L-x)}{\cosh mL} = \frac{\cosh[\Phi(1-x/L)]}{\cosh \Phi} \quad (14.28)$$

where $\Phi \equiv mL \equiv L \sqrt{k_1/D}$, a dimensionless number named the Thiele modulus, Φ , was introduced. This naming is also completely superfluous, it could be replaced by $\sqrt{Da_{II}}$.

The effectiveness factor of the pore, η_p , is defined similarly to η_{ext} and is the ratio of r_{eff} to r without any transport limitation. For a 1st order reaction, a simple correlation between η_p and Φ exists, because here $r \propto c$ holds:

$$\eta_p = \frac{c}{c_s} = \frac{\tanh \Phi}{\Phi} = f(\Phi) \quad (14.29)$$

(c_s means c at the pore entrance) This correlation is valid for a straight cylindrical pore. To be valid also for a porous pellet, the molecular diffusivity D must be replaced by D_{eff} in the porous pellet and the influence of the pellet shape taken into consideration by a characteristic length $L_c \equiv (V/S)_p$. Consequently, the modified Thiele modulus, Ψ , is given by

$$\Psi \equiv L_c \sqrt{k_1/D_{eff}} \quad (14.30)$$

The previously made statements are only valid for irreversible reactions of the 1st order. To obtain an expression which is also valid for optional reaction orders, m , a modification of the Thiele modulus has to be made:

$$\Psi \equiv L_c \sqrt{[(m+1)/2](k_1 c_s^{m-1}/D)} \quad (14.31)$$

If kinetic measurements have been obtained with a certain catalyst, the question arises as to whether or not these data have been influenced by the pore resistance. To answer this question, one has first to choose an optional reaction order and to suppose that the film resistance can be neglected. For a 1st order reaction:

$$-r = k_1 c_s \eta_p \quad (14.32)$$

and eliminating the unknown rate constant k_1 using the Thiele modulus Φ we obtain:

$$\Phi \equiv L \sqrt{k_1/D_{eff}} \rightarrow k_1 = D_{eff} \Phi^2 / L^2,$$

Therefore, it follows from (14.32) that

$$\frac{-rL^2}{D_{eff}c_s} = \eta_p \Phi^2 \equiv \Psi' \quad \Psi' - \text{Weisz-Modul} \quad (14.33)$$

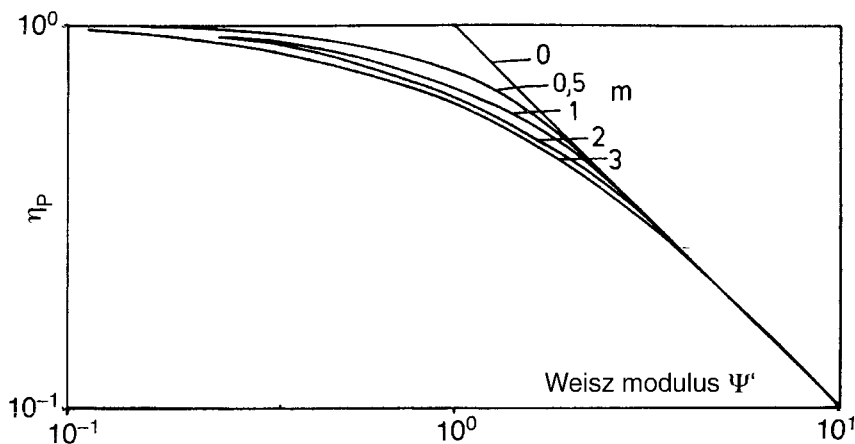


Fig. 91 Catalyst effectiveness factor η_p as a function of the Weisz modulus, $\Psi' \triangleq \eta_p Da_{II}$, and the reaction order, m .

Keeping in mind that the Thiele modulus, $\Phi \equiv L \sqrt{k_1/D}$, can be replaced by $\sqrt{Da_{II}}$, this means that the naming of the Weisz modulus Ψ' was also superfluous, because $\Psi' \equiv \eta_p Da_{II}$.

The correlation between η_p and Ψ' as being dependent on the reaction order, m , is represented in **Fig. 91**.

2.2 Non-isothermal reaction with the diffusion resistance in the pore

In an exothermal reaction, temperature gradients will arise within the pellet and, consequently, the temperature of the pellet will be elevated compared with its surroundings. As a result, the reaction will be faster than the isothermal counterpart. ΔT in the film as well in the pellet can be theoretically predicted and, consequently, the maximum ΔT between the outer surface (T_s) and the inside of the catalyst (T_{in}) calculated, when $c = 0$:

$$\frac{(T_{in} - T_s)_{\max}}{T_s} = \frac{(-\Delta H_R)c_s}{(k/D_{eff}T_s)} \equiv \beta \quad (14.34)$$

The Prater number β – in contrast to eq. (14.25) is related to T_s and not to T_G – and the Arrhenius number both have a major influence on the development of the T and c profiles. The pore utilization factor η_p is therefore dependent upon Arr , β and the Thiele modulus Φ . The correlation between these four pi-numbers is represented in **Fig. 92**. For η_p and Φ the following definitions apply:

$$\eta_K \equiv \frac{r_{\text{eff}}}{r(T_s, c_s)} \quad \text{and} \quad \Phi^2 \equiv r_p^2 \frac{k_s c_s^{m-1}}{D_{eff}} \exp(-Arr) \quad (14.35)$$

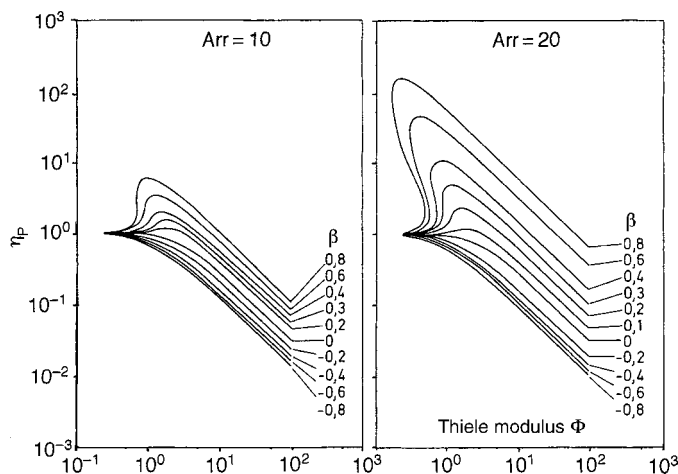


Fig. 92 Catalyst effectiveness factor (pore utilization factor), η_p , for two Arrhenius numbers and different Prater numbers β as a function of the Thiele modulus, Φ .

However, a comparison of the numerical values of Φ , β and Arr , established in full-scale reaction plants for exothermal catalytic gas reactions, has revealed that due to mostly very low β values ($\beta \approx 0.01$ – 0.1), values of $\eta_p > 1$ do not appear. This means that, within the pellets, larger T gradients are not to be expected.

Finally, a few remarks should be made with respect to the naming of characteristic pi-numbers in the macro-kinetics. Pi-numbers are associated with researchers names in order to point clearly and easily to a particular dimensionless formulated context (e.g., the Reynolds number). In doing so, it was primarily not intended to highlight the respective researcher in some manner. Seen from this point of view, the posthumous naming of the four Damköhler numbers seems unnecessary, particularly because even Damköhler himself referred to the fact that pi-numbers $k L/v \triangleq k \tau$ and $k L^2/D$ had already been introduced before him. Because these two pi-numbers are the most important pi-numbers in the field of chemical engineering, their convenient naming after *Damköhler* as Da_I and Da_{II} represents no disadvantage. Whether one can say the same for the Prater number and the Thiele and Weisz modulus is a question which everybody should answer for themselves. (For comparison, see the listing of the important named pi-numbers in the appendix.)

Example 46: Scale-up of reactors for catalytic processes in the petrochemical industry

Today, solid-catalyzed gas reactions are executed to the extent of several billions of tonnes per annum. To name only a few important examples: Steam reforming of methane-rich natural gas to “synthesis gas” ($\text{H}_2 + \text{CO}$), conversion of CO with H_2O to $\text{CO}_2 + \text{H}_2$, conversion of CO with H_2 to methane, ammonia synthesis, oxidation of SO_2 to SO_3 (sulphuric acid production), oxidation of NH_3 to NO (nitric acid production). The world production of ammonia in 1995 amounted to 90 m tonnes and of sulphuric acid to 140 m tonnes. Alone BAYER AG, D–Leverkusen, produced 1000 tonnes H_2SO_4 per day in the early 1990s.

In this specific field, in particular, the petrochemical industry is subjected to gigantism, due to the fact that the investment costs for petrochemical plants do not increase proportionally but to the 0.7 power of their capacity [141]. It is therefore profitable to build the production units as large as possible – market permitting.

In the development of these processes and their transference into an industrial-scale, dimensional analysis and scale-up based on it, play only a subordinate role. This is reasonable, because one is often forced to perform experiments in a demonstration plant which copes in its scope with a small production plant (“mock-up” plant, ca. 1/10th of the industrial scale). Experiments in such plants are costly and often time-consuming, but they are often indispensable for the layout of a technical plant. This is because the experiments performed in them deliver a valuable information about the scale-dependent hydrodynamic behavior (circulation of liquids and of dispersed solids, residence time distributions). As model substances hydrocarbons as the liquid phase and nitrogen or air as the gas phase are used. The operation conditions are ambient temperature and atmospheric pressure (“cold-flow model”). As a rule, the experiments are evaluated according to dimensional analysis.

P. Trambouze [28, 141] from Institut Français du Pétrole (IFP) demonstrated on the basis of three petrochemical processes how “mock-up” plants enabled the acquisition of pertinent data for a reliable scale-up. This work will be presented here in greater detail because it shows that, for a reliable scale-up of industrial plants in the petrochemical industry – whose investment costs per plant often amount to a three digit million US\$ sum – measurements on a large scale are often indispensable, leaving hardly any place for a classical model scale-up. Seen from this point of view, the quotation in [141] is perfectly understandable:

“In pilot plants, scale-up does not correspond to a change in size that is achieved by multiplying characteristic dimensions by a factor greater than one.”

1 Hydrotreating petroleum cuts

Catalytic hydrogenation of organic compounds containing S, N and O, into H_2S , NH_3 and H_2O , respectively, is necessary today for environmental reasons as well as to protect exhaust gas catalysts in automobiles. It is performed in catalytic fixed-bed reactors through which the gaseous and liquid phases flow simultaneously in a co-current downflow. The flow of the liquid phase corresponds to the plug flow. Back-mixing is negligible for catalyst bed heights of $> 1\text{ m}$. Problems are caused by a poor liquid distribution that leads to preferential paths. Certain portions of the catalyst bed may not even be wetted by the liquid. Reactions may take place in the dry zones involving the gas phase only, these are faster and liable to give rise to hot spots. Another problem is caused by the broader residence time distribution, which may have an effect when an extremely high conversion degree is required in order to remove traces of certain products.

In a mock-up plant the catalyst bed consists of the industrially used catalyst. As the liquid phase hydrocarbons and as the gas-phase nitrogen are used. A geometric similarity cannot be kept because this would lead to a pilot reactor with the height of the industrial plant. The capacity of the pilot plant has to be reduced. In order to do this, two alternatives exist. The first is to perform an operation with a cocurrent upflow, this makes sure that the catalyst bed is wetted by the liquid. The second possibility is to have a downflow operation. In this case the catalyst must be diluted with inert particles having a much smaller particle size distribution. This significantly increases the retention time of the liquid phase and, accordingly, the wetting of the catalyst.

The first alternative was examined. Measurements confirmed that the hydrodynamic conditions were compatible with the assumptions made for this model.

Recent studies (made public in 1988) revealed that the fixed bed technology was no longer the best, given the rapid deactivation of the catalyst that resulted from the deposition of metals contained in the feedstock. The moving bed alternative was therefore considered and countercurrent flow appeared to offer the optimal technology.

A feasibility study, carried out with mock-ups of various sizes, ensured that the catalyst bed could flow in countercurrent to the fluids (gas and liquid) and that the flow of the particulate solids was sufficiently close to plug flow. A moving bed reactor ($40^\circ\text{ cm} \times 20\text{ m}$ high) was successfully operated as a demonstration plant for several months, whereby different heavy residues were treated.

2 Regenerative catalytic reforming

This serves to increase the fuel octane number. It is performed in catalyst fixed beds in the gas phase (ca. 500°C , $p = 10\text{--}50\text{ bar}$). Thermal effects are fairly pronounced during the first phase of the conversion, which essentially corresponds to the dehydrogenation of the cycloparaffins.

Conventional technology, which has been employed for over 25 years, uses three or four fixed bed reactors in series, these operating under adiabatic conditions. They are preceded by heating furnaces that compensate for the overall endothermicity of the reaction. Catalyst performance was investigated separately in a pilot plant under isothermal conditions, employing ca. 300–400 g of catalyst.

This process could cope with gasoline demand by only steadily improving the catalyst stability. Nevertheless, this process would reach its limits if process technology would remain unchanged. Consequently, moving bed technology, which allows continuous or semicontinuous regeneration of the catalyst, was developed.

Before building an industrial plant using this new technology, a number of investigations were obviously necessary. They had to be performed in mock-ups of different sizes in order to ascertain that a largely plug flow motion of the catalyst could be achieved.

After regeneration, the catalyst enters the first reactor at the top. It passes through the reactor by gravity and is then transferred from the bottom of each reactor to the top of the next by a gas-lift. To minimize the pressure drop across the bed, a cross-flow technology was adopted: The catalyst flows downwards from the top of the reactor between two concentric cylinders made up of grids, this allowing the radial passage of the gas phase.

Therefore, it was possible to choose the direction of fluid flow. Firstly, either from the inside towards the outside or, secondly, from the outside towards the inside of the reactor. The interactions between the gas phase and the granular solid phase were liable to create irregularities in the solid downflow. The main risk was that the catalyst would be plastered against the grid and would no longer be able to move.

This question was also investigated in differently sized mock-ups, the largest having the geometry and dimensions of the planned industrial reactor. To reduce the catalyst and gas throughputs, only a cut-off section of the cylinder was used. The radial cross-sectional area was made of Plexiglas which permitted the observation of the solid flow by means of a tracer technique (layer of colored spheres). This mock-up contained a solid volume of 2.5 m^3 .

As solids, only spheres were used and these served as a support for the catalyst. The gas phase was air. The operation conditions were ambient temperature and atmospheric pressure.

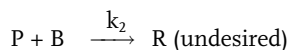
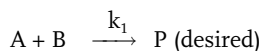
3 Catalytic cracking

This is to cleavage of C–C and C–H bonds in high boiling crude oil fractions to transform long chain hydrocarbons into short chain ones and, consequently, to increase the fuel yield. In this field, Institut Français du Pétrole (IFP) together with “TOTAL France” developed a new technique (R2R process) which utilizes a riser and two regenerators. Because the hydrodynamics of the three phase, gas-liquid-solid flow inside the riser could not be investigated on a small scale, a large mock-up had to be build. In addition, an injector was also developed. This intro-

duced the cracking feed at the bottom of the riser and mixed it as rapidly and uniformly as possible with the catalyst.

Example 47: Dimensioning of a tubular reactor, equipped with a mixing nozzle, designed for carrying out competitive-consecutive reactions

This example deals with the dimensioning data for a chemical reactor intended to perform a homogeneous, competitive-consecutive chemical reaction. The reaction course is given by:



This type of reaction is by no means rare. Many chlorinations, phosgenations, and so on, belong to this type. If a high selectivity with respect to P, S_P , is demanded, P may not come into contact with B. Therefore, a stirring vessel would be completely unsuitable for this task! It is recommended to perform this reaction in a continuous mode and to use a tubular reactor because it exhibits ideal plug-flow characteristics with respect to macro-mixing (i.e., no back-mixing). Fast micro-mixing of the reaction partners is taken care of by a propulsion jet nozzle installed at the inlet of the pipe, this impinges the smaller volume flow of B into the larger volume flow of A (stoichiometrically surplus component). A annular inset is installed at a short distance after the propulsion jet nozzle in order to prevent a possible back-mixing eddy which would mix the reaction mixture with the free jet. (This happens if the free jet is able to suck more liquid from its surroundings than is supplied with it at the inlet.)

In experiments [125], a well known azo reaction (conversion of 1-naphthol with diazo sulphanilic acid into a simple coupled – desired product – and twice coupled product) is carried out. It allows the quantitative determination of micro-mixing on a “molecular scale”. Its selectivity is determined following the proposal of J.R. Bourne [5] with respect to the undesired product, R. S_R is defined by P and R, which only can be detected analytically, according to the relationship

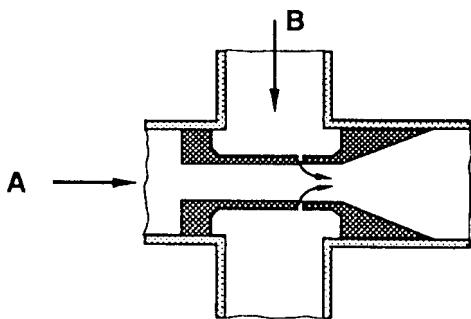


Fig. 93 Sketch of the propulsion jet nozzle of a tubular reactor for carrying out competitive reactions.

$$S_R \equiv \frac{2R}{2R + P} = \frac{\text{mols B consumed for the formation of R}}{\text{total moles of B consumed}} \quad (14.36)$$

The reaction engineering task will be to *minimize* the quantity S_R .

We will start with the following relevance list:

Target quantity:	selectivity, S_R
Geom. parameter:	tube diameter, d (as the characteristic length)
Physical properties:	$c_A, c_B; \rho_A, \rho_B; \nu_A, \nu_B; D$
Process parameters:	$k_1, k_2; q_A, q_B$

c_i are the concentrations, ρ_i and ν_i are the densities and kinematic viscosities of the educts, D is the diffusion coefficient of A in B, q_i are the liquid throughputs and k_i [$L^3 T^{-1} \text{ Mol}^{-1}$] the reaction rate constants of 2nd order isothermal reactions.

From this relevance list, first the trivial pi-numbers can be anticipated:

$$S_R; \frac{c_A}{c_B}, \frac{\rho_A}{\rho_B}, \frac{\nu_A}{\nu_B}, \frac{D}{\nu_B}, \frac{k_1}{k_2}, \frac{q_A}{q_B}$$

In the remaining relevance list

$$\{d, c_B, \rho_B, D, k_2, q_B\}$$

only ρ_B contains the base dimension [M] and therefore has to be cancelled. Only c_B and k_2 contain the base dimension [N]; this is eliminated in the combination $c_B k_2$. The residual relevance list is then

$$\{d, D, c_B k_2, q_B\}$$

The dimensional matrix with the rank 2 supplies two pi-numbers. They are the Reynolds number and the Damköhler-II number for a 2nd order reaction:

$$\text{Re} \equiv \frac{q_B}{\nu d} \quad \text{and} \quad \text{Da}_{II} \equiv \frac{d^2 c_B k_2}{D}$$

The complete pi-set comprises nine numbers

$$\left\{ S_R, \frac{c_A}{c_B}, \frac{\rho_A}{\rho_B}, \frac{\nu_A}{\nu_B}, \frac{D}{\nu_B}, \frac{k_1}{k_2}, \frac{q_A}{q_B}, \text{Re}, \text{Da}_{II} \right\} \quad (14.38)$$

For the given reaction in the given material system, four pi-numbers display constant numerical values

$$\frac{\rho_A}{\rho_B}, \frac{\nu_A}{\nu_B}, \frac{D}{\nu_B}, \frac{k_1}{k_2} = \text{const}$$

Therefore, only a dependence between five pi-numbers has to be investigated:

$$S_R = f\left(\frac{c_A}{c_B}, \frac{q_A}{q_B}, \text{Re}, \text{Da}_{II}\right) \quad (14.39)$$

This pi-space can be further reduced if the liquid throughput, q_A , forming the jet in the propulsion jet nozzle, is replaced by an intermediate quantity, jet power per unit throughput, P/q_A . Due to the relationship $P = q_A \Delta p$, P/q_A corresponds to the pressure drop, Δp , in the propulsion jet nozzle. The reduction of the pi-space is now possible because by the introduction of Δp , not only is q_A replaced, but also – due to the intensive character of this variable – d becomes irrelevant and must be deleted.

In order to replace q_A by Δp , we combine the Euler and Reynolds numbers accordingly

$$\text{EuRe}^2 \equiv \frac{\Delta p d^4}{\rho q^2} \frac{q^2}{v^2 d^2} \equiv \frac{\Delta p d^2}{\rho v^2}$$

and eliminate d from the resulting pi-number using Da_{II} :

$$\text{EuRe}^2 \text{Da}_{II}^{-1} \text{Sc} \equiv \frac{\Delta p d^2}{\rho v^2} \frac{D}{d^2 c_B k_2} \frac{v}{D} \equiv \frac{\Delta p}{\mu c_B k_2} \equiv \Psi$$

The required dependence for the selectivity, S_R , of a fast chemical reaction carried in a tubular reactor with a propulsion jet nozzle as a mixing device, is now represented by only a four-parametric pi-space:

$$S_R = f\left(\frac{c_A}{c_B}, \frac{\rho_A}{\rho_B}, \Psi\right) \quad (14.40)$$

Laboratory tests [69] were performed in glass tubes ($d = 3\text{--}25$ mm) with converging propulsion jet nozzles manufactured by the Schlick company ($d' = 0.3\text{--}1.6$ mm). q_B formed the propulsion jet. A temperature of 20°C , $\text{pH} = 10$ and mole ratio of $\frac{c_A}{c_B} \frac{q_A}{q_B} = 1.05$ were kept constant, so that only a further reduced relationship of

$$S_R = f\left(\frac{\rho_A}{\rho_B}, \Psi\right) \quad (14.41)$$

could be investigated. Test conditions which permitted the formation of a back-mixing eddy in the pipe reactor and hence back-mixing of R and B were also adjusted.

The correlation between the three pi-numbers in (14.41) is presented in **Fig. 94**. The analytical expressions for the two groups of curves are:

$$\text{without back-mixing:} \quad S_R = 1.3 (\Psi \times c_B/c_A)^{-0.5} (q_A/q_B)^{1.25} + 0.001$$

$$\text{with back-mixing:} \quad S_R = 5.0 (\Psi \times c_B/c_A)^{-0.5} (q_A/q_B)^{1.25} + 0.005$$

First, **Fig. 94** shows which P/q_A value is necessary to obtain a high selectivity for P (correspondingly, an equally bad one for R!), whereby this information is scale-independent and therefore can be used for scaling up such reactors. Second, it is shown that by suppressing back-mixing it is already possible to obtain the same selectivity with P/q_A values, these being two orders lower.

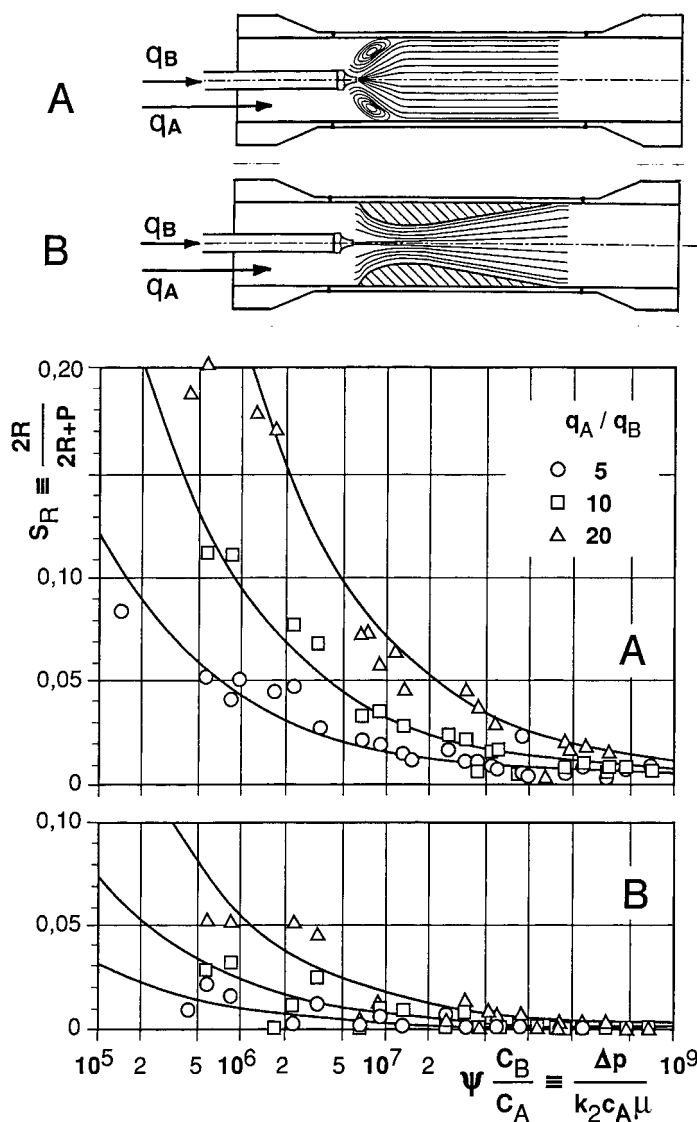


Fig. 94 Selectivity S_R of the undesired by-product R for a fast reaction taking place in a tubular reactor with a propulsion jet nozzle (Fa. Schlick) as mixing device as a function of q_A/q_B and $\Psi \times c_B/c_A$. The upper graph is valid for developed, the lower graph for suppressed back-mixing.

If these results are considered for the selectivity, S_P , with reference to the desired product P (in Fig. 94, the selectivity S_R of the *undesired* by-product is plotted!), it becomes apparent that this increases with increasing Ψ but decreases superproportionally with increasing throughput ratio q_A/q_B . This is because the

propulsion jet power related to the *total* liquid throughput $\Delta p_{q_B}/(q_A+q_B)$ decreases with increasing q_A/q_B .

If the kinetic parameters $k_1/k_2 = 7300/1.63 = 4480$ and the mol ratio $c_A q_A/c_B q_B = 1.05$ are known, it is possible to calculate the selectivities S_R which would result from the ideal plug-flow tubular reactor and in the completely back-mixed vessel. The corresponding values are 0.001 and 0.008 respectively. This is in good agreement with the results obtained for high values of $\Psi \times c_B/c_A$.

Example 48: Mass transfer limitation of the reaction rate of fast chemical reactions in the heterogeneous material gas/liquid system

To allow a chemical reaction to take place between a gaseous and a liquid reaction partner, the gaseous component must first be dissolved (absorbed) in the liquid. In this case, the overall reaction rate will depend on the rates of the mass transfer and the chemical reaction step.

The so-called "Two Film Theory" (Lewis and Whitman, 1923–24) assumes the formation of laminar boundary layers on both sides of the interphase. Mass transfer through these boundary layers can only be effected by means of diffusion, while the phase transition is immeasurably fast, **Fig. 95**. Consequently, an equilibrium predominates in the interphase and the saturation concentration c_G^* of the gas in the interphase (*) obeys Henry's law:

$$p_G = H y c_G^* \quad (14.42)$$

(Hy – Henry coefficient, p_G – partial pressure of the gaseous reaction partner; indices: G – gas, L – liquid)

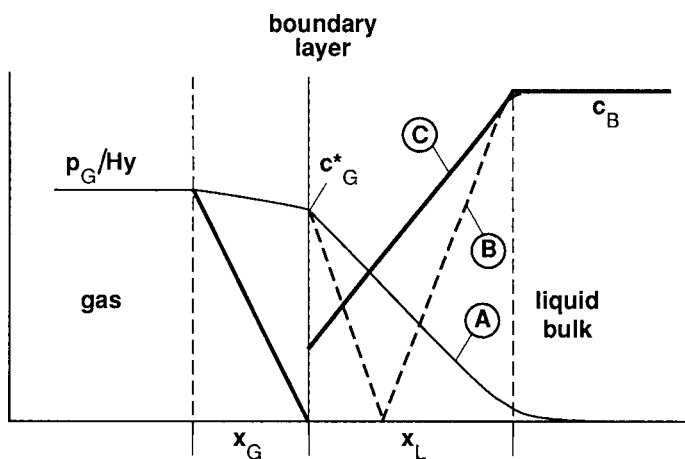


Fig. 95 Graphic depiction of the concentration profiles on both sides of the interphase when the gas absorption is followed by (A) a very slow, (B) a fast and (C) an extremely fast chemical reaction.

The two mass transfer coefficients k_G and k_L give the ratio of the respective diffusion coefficients, D_i , to the respective boundary layer thickness, x_i :

$$k_i \propto D_i/x_i \quad (14.43)$$

Since $k_G \gg k_L$, only the influence of k_L is taken into account in the following.

If the mass transfer of a gaseous reaction partner into the liquid is accompanied by a chemical reaction, the following case can occur depending on the reaction rate and the mobility of the reaction partners: The concentration of A is not only reduced to zero in the solution; in addition, the reaction front shifts from the liquid bulk to the liquid-side boundary layer. As a result, the liquid-side boundary is apparently reduced and finally eliminated in a chemical way ("chemisorption"), see Fig. 95. This process increases the mass transfer coefficient by the "enhancement factor E" as compared to its numerical value for purely physical absorption.

The target quantity E will depend on the parameters of mass transfer $\{k_L, D_A, D_B\}$ and reaction kinetics $\{k_2, c_A^*, c_B\}$, where k_2 stands for the rate constant of a 2nd order reaction:

$$\{E; k_L, D_A, D_B; k_2, c_A^*, c_B\} \quad (14.44)$$

Apart from the trivial, obvious dimensionless numbers $\{E, D_A/D_B, c_A^*/c_B\}$, four parameters remain which form a single additional dimensionless number, namely the *Hatta* number *Hat* (for a 2nd order reaction):

$$\text{Hat}_2 \equiv \sqrt{D_A k_2 c_B} / k_L \quad (14.45)$$

If the numbers D_A/D_B and c_A^*/c_B are formulated as being a ratio of the diffusion currents

$$Z \equiv \frac{D_B c_B}{z D_A c_A^*} \quad (14.46)$$

the resulting functional relationship is:

$$E = f(\text{Hat}_2, Z). \quad (14.47)$$

This pi-relationship can be treated theoretically by assuming that the gradients of the diffusion rates of the two mass flows and the chemical reaction rate are equal (steady-state approximation):

$$D_A \frac{d^2 c_A^*}{dx^2} = D_B \frac{d^2 c_B}{dx^2} = k c_A^* c_B \quad (14.48)$$

This differential equation was numerically solved by *van Krevelen and Hoftijzer* [71] using the simplifying assumption of an idealized concentration profile and the results were graphically presented as a work sheet, **Fig. 96**. The following three ranges can be differentiated:

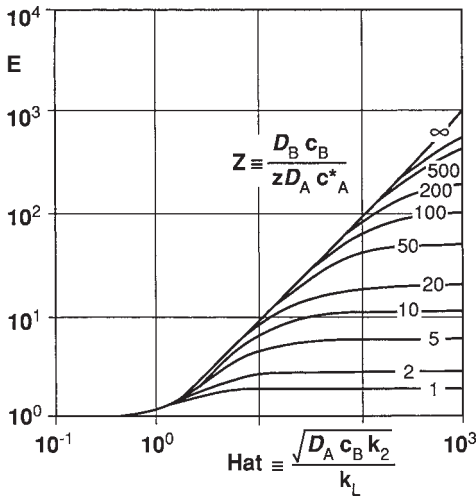


Fig. 96 Enhancement factor E for gas absorption with subsequent chemical conversion as a function of the Hatta number, Hat , and the ratio of the diffusion currents, Z .

1. $\text{Hat} < 0.5$: $E = 1$. In this case, physical absorption governs the rate. This can be increased by stirring more vigorously or with a better dispersion of the gas throughput.
2. $\text{Hat} > 2$; $c_B \gg c_A^* \rightarrow Z \approx \infty$ Because of $c_B \gg c_A^*$ we are dealing with a pseudo-1st order reaction: $k_2 c_B = k_1'$ and the Hatta number therefore reads:

$$\text{Hat}_1 \equiv \sqrt{D_A k_1'} / k_L \quad (14.49)$$

In this range, E increases directly proportionally to Hat_1 . Since E was defined as the *enhancement* or acceleration factor in relation to the respective value in pure physical absorption ($g^o \equiv G/A$ – mass flux density at physical absorption)

$$E \equiv \frac{g}{g^o} = \frac{g}{k_L c_A^*} \quad (14.50)$$

it follows here from the valid relationship $E = \text{Hat}_1$ that

$$\frac{g}{k_L c_A^*} = \frac{\sqrt{D_A k_1'}}{k_L} \rightarrow g = \sqrt{D_A k_1'} c_A^* \quad (14.51)$$

From this relationship it can be deduced that k_L is replaced, in the case of chemisorption, by the kinetic expression $\sqrt{D_A k_1'}$, which in the given reaction system only depends on the temperature.

3. $\text{Hat} > 2$; $Z < \infty$: With increasing $k_2 c_B$ and, subsequently, increasing Hatta number, the molecular mobility of B and, consequently, Z become increasingly important. In this range, E is increasingly dependent on the mixing intensity in the reaction system which reduces diffusion resistances.

It should be pointed out that the relationship $\{E, \text{Hat}_2, Z\}$ in Fig. 96, found by *van Krevelen* and *Hofijzer* on the basis of theoretical reasoning, was convincingly confirmed by laboratory measurements; see works of *Nijssing* et al. [93], *Yoshida* and *Miura* [190], as well *Hofer* and *Mersmann* [53].

15

Selected Examples for the Dimensional-analytical Treatment of Processes within the Living World

Introductory Remark

The results of evolution – the so-called living world: fauna and flora – cannot be lumped together by means of dimensional analysis, because each group, as such, does not consist of geometrically similar species and is not adapted to the same living conditions (animals, birds, fishes). The biological treatises, which refer to the “Size and Shape in Biology” [80] or to the “Size and Scaling in Human Evolution” [112], only mean that in the living world, in spite of strong deviations from the geometric similarity (differently shaped bodies), astounding relationships exist, which exert a “similar” influence of the relevant physical parameters upon the size and shape of the species.

In contrast, correlations between the egg mass of any bird species and the physical parameters influencing it, can be quite easily represented in a dimensional-analytical manner because here we are concerned with practically geometrically similar objects.

On the other hand, it is beyond question that creatures are subjected to the same physical regularities and frame conditions on Earth as inanimate nature. These are therefore describable by the same dimensionless numbers as in inanimate nature and technology.

For instance, the processes of motion in the living world are perfectly describable by dimensional analysis and from these correlations valuable information is obtained about a similar process concerning another, larger or older species. The scale-invariance of dimensionless representation is an advantage for the living world, which should not be underestimated. The relevant dimensions of length here span over the whole of eight decades.

To illustrate this, some examples will be given in this chapter.

Example 49: The consideration of rowing from the viewpoint of dimensional analysis

Why should larger boats containing many oarsman go faster than smaller ones containing fewer oarsmen? *Th. A. McMachon* [79] pursued this question and his approach deserves to be represented here.

Before examining his reasoning, we should remind ourselves of Example 9 in which the drag resistance, F , of a ship's hull was treated. It has been pointed out that its dependence on the geometric, material and process related parameters can be represented by

$$Ne = f(Re, Fr) \quad (15.1)$$

The symbols correspond to

$$Ne \equiv \frac{F}{\rho v^2 l^2}, \quad Re \equiv \frac{vl\rho}{\mu} \quad \text{and} \quad Fr \equiv \frac{v^2 l}{g}.$$

In principle, the relationship (15.1) is, with respect to $f(Re)$, similar to that of the drag coefficient on a sphere. In the range of $Re = 10^3 - 10^5$, the drag coefficient is approximately independent of Re .

The dependence $Ne = f(Fr)$ describes bow wave development and bow wave resistance in ships. Of course, it does not exist in submarines. At moderate speeds and large ratios of boat length to boat width, it is practically negligible. Only the friction loss of the ship's hull has to be overcome. Under these frame conditions (Re and Fr irrelevant),

$$Ne = \text{const} \quad \rightarrow \quad F \propto v^2 l^2 \quad (15.2)$$

This means that at a given speed ($v = \text{const}$) and considering geometrically similar ship bodies, due to the proportionality between the wetted surface of the ship's hull and the displacement volume of the ship's body ($V \propto l^3$), the relationship

$$F/V \propto l^{-1} \quad (v = \text{const}) \quad (15.3)$$

exists. This motivated the construction of larger and larger ships of iron in the 19th century.

McMachon [79] comparatively evaluated the results obtained from four Championships in rowing (Olympics 1964 Tokyo and 1968 Mexico City; World rowing championship 1970 Ontario, International Championship 1970 Lucerne). First, he ascertained that all competitive shells (single scull, pair-oared, four-oared without cox, eight-oared heavy weight) were geometrically similar. The slenderness ratio l/b is reasonably invariant of boat length. In addition, the boat weight per oarsman is also reasonably invariant. Furthermore, full-scale towing tank tests have shown that the resistance due to leeway and wave-making together, constitute only 8% of the total drag at 20 km/h, the Olympic target speed, for an eight-oared shell. The assumption can be summarized as follows:

1. There is geometrical similarity between the boats and the displaced water volumes, V , of the boats are similar to each other.
2. The boat weight per oarsman (number z), W_0 , is a constant.
3. Each oarsman contributes the same power, P , and weight, W .
4. Skin friction drag is the only hindering force.

From the relationship

$$V \propto z W_0 + z W = z (W_0 + W) \propto z \rightarrow z \propto l^3 \quad (15.4)$$

it follows that the displaced volume per oarsman ($V \propto l^3$) remains constant.

The total power, P , required to move the boat at velocity, v , is proportional to the product of the drag resistance and velocity:

$$z P \propto v F$$

If F is taken into consideration according to (15.2), it follows that:

$$z P \propto v^3 l^2$$

Because P has been assumed to be constant (assumption 3), it follows from eq. (15.4) that:

$$z \propto l^3 \rightarrow l^2 \propto z^{2/3}$$

This gives the relationship between z and v to be

$$v^3 \propto \frac{z}{l^2} \propto \frac{z}{z^{2/3}} \propto z^{1/3} \rightarrow v \propto z^{1/9} \quad (15.5)$$

This relationship is shown in **Fig. 97** for four boat types and four Championships and proves an excellent agreement between the prediction given by dimensional analysis and the measured data.

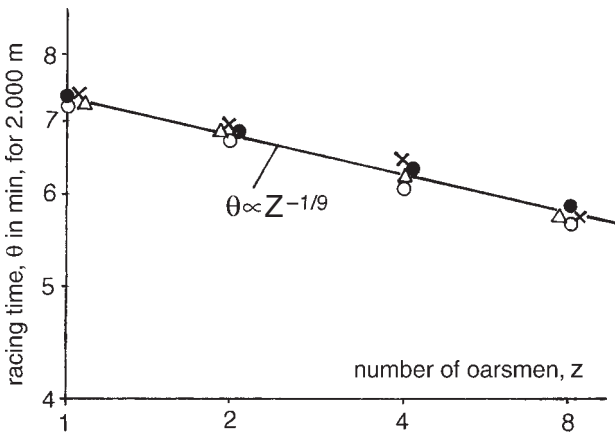


Fig. 97 Correspondence between racing time [min] for 2.000 m in calm or near calm conditions and the number of oarsmen, z . Legend: $\triangle$ Tokyo 1964; $\bullet$ Mexico 1968; $\times$ Ontario 1970; $\circ$ Luzern 1973; from [133].

Example 50: Why most animals swim beneath the water surface

In dealing with this question, we should first look at the flow conditions at the liquid surface. It is well known that longer waves move faster than shorter ones and that large ships can travel faster than small ones. Both things have to do with the wave formation at the liquid surface, for which the gravitational acceleration, g , is also characteristic. Therefore, the motion of a body at the liquid surface is dependent on the Froude number, Fr . From the structure of this pi-number

$$Fr \equiv \frac{v^2}{gl} \quad (15.6)$$

it can be seen that at given flow conditions $v^2 \propto l$ applies.

As the characteristic wave dimension, l , the wave crest or the distance between two crests can be taken. A doubling of this length leads to the four-fold travelling or spreading velocity of the waves.

Due to water displacement, a ship produces waves when travelling. A bow wave, a few waves along the ship's hull and a stern wave are created. At full speed, the so-called "hull speed", it is left with a bow wave and a stern wave, the two separated by the length of the ship's hull. The critical value of the Froude number in this state is $Fr \approx 0.16$. Going faster than this requires that the ship leave its beneficial stern wave astern and try to cut through or climb up its bow wave. Both would result in a dramatic rise in power demand.

The consequence is that a ship with a length of 100 m can easily travel at $v \approx 13$ m/s (25 nautical miles per hour), whereas a ship of 10 m length achieves only $v \approx 4$ m/s (8 nautical miles per hour). This fact also speaks in favor of long ship bodies.

S. Vogel [149] reports on findings in the living world regarding this matter. A duck, with a hull length of about one third of a meter, hits hull speed at 0.7 m/s. Fully submerged, it can swim several times as fast. A mink, towed along the surface above hull speed, had up to ten times as much drag as it had when fully submerged.

The critical value of the Froude number shows why reasonable surface speeds are off-limits for the sizes of most of Nature's "vessels" and why even air breathers mostly swim submerged. An occasional animal porpoises up and down through the interface or planes on the surface, but only a large whale could consider migrating in the same way as a surface ship.

Snorkeling is rare, perhaps because swimming deep enough to keep wave drag low requires breathing against too much hydrostatic pressure.

Example 51: Walking on the Moon

If we assume that the kinematics of motion on the Earth or on the Moon is influenced only by the velocity of motion v , a characteristic length scale L of the body, and the acceleration due to gravity g , then this process of motion is described solely by the kinematic process number, the Froude number Fr :

$$Fr \equiv \frac{v^2}{Lg}$$

If a person of average height ($L = 1.70$ m) feels that a walking speed of $v = 4$ km/h is comfortable, then a child half as tall would feel the same at

$$\left(\frac{v^2}{Lg}\right)_{\text{large}} = \left(\frac{v^2}{Lg}\right)_{\text{small}} \rightarrow v_{\text{small}} = v_{\text{large}} / \sqrt{2} = 0.7 v_{\text{large}} \rightarrow 2.8 \text{ km/h.}$$

On the Moon, the gravitational acceleration g_M , is smaller than on the Earth by a factor of 6.13 ($g_E = 9.81 \text{ m}^2/\text{s}^2$; $g_M = 1.6 \text{ m}^2/\text{s}^2$). On the Moon therefore the same person should walk comfortably at

$$\left(\frac{v}{\sqrt{g}}\right)_M = \left(\frac{v}{\sqrt{g}}\right)_E \rightarrow v_M = v_E / 2.48 = 0.40 v_E$$

That means $0.40 \times 4 \text{ km/h} = 1.60 \text{ km/h}$.

The discovery that a comfortable walking speed on the Moon is smaller than that on Earth is not credible. At a gravitational acceleration, which is smaller by a factor of 6, a comfortable walking speed must be higher than on Earth. This shows that the relevance list $\{v, L, g\}$, which led to the pi-space $Fr = \text{idem}$, is not correct.

The problem of a comfortable walking speed on the Moon has also been investigated by *Thomas Szirtes* [138], who assumed the scaling criterion $Fr = \text{idem}$ as valid. He even went a step further in determining the energy which had to be utilized to cover a certain walking distance. His finding is:

Frame conditions: Mass $m = 85$ kg; distance $L = 1850$ m

Moon $v = 1.86 \text{ km/h}$ $E/L = 87.05 \text{ kJ/m}$

Earth $v = 4.61 \text{ km/h}$ $E/L = 533.75 \text{ kJ/m}$

$$\frac{v_E}{v_M} = 2.48 \quad \frac{E_E}{E_M} = 6.13$$

To put it more clearly: The walking speed at $Fr = \text{idem}$ on the Earth is a factor of ≈ 2.5 larger, but the energy consumption on the Earth is a factor of ≈ 6 larger. Therefore, both walking speeds cannot be felt as equally comfortable.

Pawłowski [111] refers to the fact that locomotion with a target quantity v cannot be achieved by L and g alone, but by the momentum $I = m v [\text{MLT}^{-1}]$ which is given to the body as it pushes off from the ground. Therefore the body mass m has to be added to the relevance list. For further discussion of the problem it is

more suitable to replace the body mass by the momentum velocity $u = I/m$ which leads to the relevance list $\{v, u, L, g\}$ and thus to the process relationship:

$$v/u = f\left\{\frac{u^2}{Lg}\right\} \quad (15.7)$$

Therefore the walking speed on the moon depends on the nature of the function f . This function can be explicitly determined, if – for the sake of simplicity – the locomotion of a jumping frog is considered. This movement is defined as a sequence of equally long jumps with short breaks in between.

In the following calculation, these abbreviations are used:

g	acceleration due to gravity
t	time
T	duration of a jump
T_0	duration of break between two jumps
u	momentum velocity
$v = X/(T + T_0)$	mean locomotion speed
x	abscissa in the direction of motion
X	jump length
y	ordinate in the vertical direction
φ	jump angle $\angle (x, u)$

The dynamics of the jump (free throw in the gravitational field) is given by the differential equations

$$dx/dt = u \cos \varphi \quad \text{as well as} \quad d^2y/dt^2 + g = 0$$

with starting conditions

$$x(0) = 0; y(0) = 0; \quad (dy/dx)_{t=0} = u \sin \varphi$$

and with

$$x(T) = X; y(T) = 0; \quad v = X/(T + T_0).$$

Their solutions deliver the jumping parabola

$$x(t) = u t \cos \varphi$$

$$y(t) = -g t^2/2 + u t \sin \varphi \quad (0 \leq t \leq T)$$

From this it follows that

$$T = 2 u/g \sin \varphi$$

$$X = 2 u^2 / g \sin \varphi \cos \varphi$$

$$v = 2 u^2 / g \sin \varphi \cos \varphi / (T_0 + 2 u / g \sin \varphi)$$

and one obtains (in frog-like movement!) for the pi relationship between

$$A \equiv (v/u), \text{Fr}^* \equiv (v/g T_0) \text{ and } \varphi$$

the expression

$$A = f(\text{Fr}^*, \varphi) = \frac{\text{Fr}^* \sin \varphi \cos \varphi}{1/2 + \text{Fr}^* \sin \varphi} \quad (15.8)$$

The speed of the frog's movement decreases with increasing g ($\partial A / \partial \text{Fr}^* > 0$). The frog moves faster on the Moon than it does on the Earth.

Further, the analysis of eq. (15.8) shows that $A(\varphi)$ passes a maximum at $\varphi = \varphi_{\text{opt}}(\text{Fr}^*)$. Then eq. (15.7) is transferred to

$$A_{\text{opt}}(\text{Fr}^*) = f\{\text{Fr}^*, \varphi_{\text{opt}}(\text{Fr}^*)\} \quad (15.9)$$

The function $\varphi_{\text{opt}}(\text{Fr}^*)$ is fixed by determining eq. (15.10), which follows from $\partial A / \partial \varphi = 0$:

$$F(\text{Fr}^*, \varphi_{\text{opt}}) = (1 + 2 \text{Fr}^* \sin \varphi_{\text{opt}}) \cos 2\varphi_{\text{opt}} - \text{Fr}^* \cos \varphi_{\text{opt}} \sin 2\varphi_{\text{opt}} = 0 \quad (15.10)$$

Pawlowski [111] determined this correlation and calculated eq. (15.9). In the table some values for Fr^{*-1} , φ_{opt} and A_{opt} are summarized.

For frog jumps, therefore, the following equation is valid:

$$\frac{v_{\text{Moon}}}{v_{\text{Earth}}} = \frac{(A_{\text{opt}} \text{Fr}^{*-1})_{\text{M}}}{(A_{\text{opt}} \text{Fr}^{*-1})_{\text{E}}} = 3.04$$

On the Moon the jumps are 'flatter' than on Earth:

$$\varphi_{\text{opt,M}} = 0.577 ; \varphi_{\text{opt,E}} = 0.726$$

Pawlowski points out that these relationships, obtained with $u = \text{const}$, cover only the outer dynamics of movement. A further discussion of the question of whether and how u reacts to the different gravitation will lead into the physiology of locomotion. The momentum velocity acts only as an intermediate quantity, which connects the outer jump dynamics with the inner, physiologically influenced, kinetics. This analogy can obviously be applied to humans, whereby a human will acquire another type of jumping movement.

Table 5 Correlation $\{Fr^{*-1}, \varphi_{opt}, A_{opt}\}$ for frog jumps.

Fr^{*-1}	φ_{opt}	A_{opt}	
0.1	0.345	0.820	
0.2	0.418	0.733	
0.4	0.498	0.619	
0.8	0.577	0.484	Moon
1.0	0.600	0.438	
3.0	0.697	0.230	
4.905	0.726	0.159	Earth
7.0	0.741	0.119	
10.0	0.753	0.088	

Example 52: Walking and jumping on water

The high surface tension of water is an important physical property in the living world. It allows the tiniest creatures (insects, spiders) to walk and even to jump on the water surface. The question arises about which frame conditions have to be fulfilled for this.

We should first ask what could be the maximum size which limits this process. As a characteristic length, l , the perimeter of the wetted surface of the legs is taken. Because here the surface tension, σ , counteracts the gravitation, g , the Bond number will describe the process:

$$Bd \equiv \frac{\rho g l^2}{\sigma} \triangleq \frac{m g}{\sigma l} < 1 \quad (15.11)$$

Its numerical value has to be < 1 , if surface tension has to overcome the gravitational force. The left-hand side expression confirms that only very tiny creatures will manage this, because the length enters this fraction with the square (leg surface).

The equivalent expression on the right-hand side is formed by the mass, $m \propto \rho l^3$. An insect of $m = 0.1$ g will need $l = 1.3$ mm. This perimeter of the wetted leg surface can easily be provided by the four legs displaying a fringe of hydrophobic foot hairs. They are needed, if the insect is to jump vertically from the water surface. In this case, surface support has to be an order of magnitude greater.

In the determination of the lower limit of this process, it is not the support but the locomotion we are interested in. The water's surface tension will pull against an animal whichever way it tries to move. Can it get enough inertial force to offset

the force of surface tension? These frame conditions are determined by the Weber number:

$$\text{We} \equiv \frac{\rho l v^2}{\sigma} \quad (15.12)$$

So the animal has to be sufficiently large and fast, because size and speed act in the same way. Therefore, really tiny creatures have problems. *D'Arcy Thompson* (quoted in [149]) has put it in this way:

"A water beetle finds the surface of a pool a matter of life and death, a perilous entanglement or an indispensable support."

Example 53: What makes sap ascend up a tree?

Could capillary rise account for the ascent of sap? This question can be easily answered by formulating the Bond number as a fraction of the weight force of the lifted water column, to the force exerted by the surface tension:

$$\text{Bd} \equiv \frac{\rho g h r}{2\sigma} \quad (15.13)$$

(h – capillary rise height, r – capillary radius).

For the Bond number not to exceed one with a typical conduit radius of $r = 0.05$ mm, the rise height must remain at $h < 3$ m. This wouldn't be much of a tree; therefore capillary rise cannot be responsible for it [149].

It is a generally accepted opinion, that columns of sap are maintained by the considerable internal cohesion of water, in essence hanging from the tops of trees and drawn up by evaporative water loss from the leaves. How can the water columns remain open to the air at the top? If the water vapor quite clearly leaves the trees, why doesn't air enter? The answer is that the relevant capillary radius of the cell walls is much smaller, $r \approx 10^{-4}$ mm. With this radius, the Bond number will not rise above one and air will not be pulled in by gravity until a tree exceeds 1500 m in height – over an order of magnitude higher than any tree ever known.

16

Brief Historic Survey on Dimensional Analysis and Scale-up

16.1

Historic Development of Dimensional Analysis

Dimensional analysis has been used for the last 125 years, its foundations were published 180 years ago but its roots reach back for centuries. These roots will not be examined in detail here. It should, however, be mentioned that the ancient Greeks had already been following some fundamental dimensional rules when dealing with physical quantities in their calculations. In this context, the names of *Euclid*, *Ptolomeus*, *Pappus*, etc., are cited [77, 37]. (The following statements are based on both of these sources as far as dimensional analysis is concerned.)

With the exception of statics and hydrostatics, the Greeks did not develop other areas of physics into a classic science as they did with geometry and astronomy. One of the problems may have been caused by the fact that they saw inevitable difficulties in executing an unusual calculation in order to determine velocity by dividing the length by the time, e.g., secondary, derived quantities were neither recognized nor accepted as authentic quantities.

We have to go back to medieval times to find writers, like *Johannis de Muris*, who dealt with products and quotients of quantities of different dimensions. In the early Renaissance, a current of research began in the schools of Paris and Oxford which yielded clear fundamental concepts of kinematics and statics; a path can be followed from these scholars to *Galilei* and *Descartes*, and we find that the latter had already used a language that conveyed modern concepts:

“The force to which I refer always has two dimensions, and it is not the force that exists (a weight) which has only one dimension”.

It was, however, left to *Newton* to enunciate in his *Principia* (1687; book 2, Lemma 2) the general idea of derived quantities. *Leibnitz*, too, made the same statement in the year 1707.

Great contributions were made to the analysis of physical phenomena during the 18th century and yet very little attention was paid to matters concerning physical dimensions during most of that period. Only *Euler* occupied himself in his numerous writings with the meaning of physical quantities and with the mathematical expressions of physical relationships as can be seen in his *Mechanica* (1736)

and *Theoria motus corporum solidorum seu rigidorum* (1736). Euler devoted a chapter to questions of units and homogeneity. He was aware that there is nothing absolute concerning systems of units and dimensions of physical quantities.

It is remarkable that Euler's ideas on dimensions and homogeneity did not have repercussions as is demonstrated in the works of *Lagrange* and *Laplace*. It was left to their countryman, J.B. *Fourier*, to continue the work initiated by *Euler*. In the last of his three successive versions of the *Théorie analytique de la chaleur* (1822) [30] *Fourier* established the foundations of dimensional analysis. His approach to the study of heat conduction constitutes a good model of the physical analysis that should always precede mathematical formulation and solution. He formulated the term physical dimension and at the same time stressed that a physical equation must exert that property which today is defined as the dimensional homogeneity with respect to the chosen system of dimensions. In his equations he recognizes the existence of dimensionless groups.

In the works of *Fourier*, consideration is also given to the flow of heat in similar bodies. Because much confusion has existed between similarity and dimensional analysis for almost a century, it is interesting to see that *Fourier* did not establish any connection between the two questions. It also appears in this regard that *Fourier* failed to see one of the applications of the general principles that he had so clearly established.

After *Fourier*, no important development in dimensional analysis took place for half a century. It has to be pointed out, however, that *H.v.Helmholtz* (1873) developed all the dimensionless products (numbers), important for hydromechanics, and among others found the Reynolds number – ten years before *Reynolds* published his findings on transition from laminar to turbulent motion in pipes (1883). Why this publication of *Helmholtz* [46], dealing in particular with steering balloons, has been forgotten, is reported in [1].

In *Lord Rayleigh's* monumental work in two volumes "*The Theory of Sound*" (1877–78) [117] an entry "Method of Dimensions" is found. It marks the beginning of the application of principles enunciated by *Fourier* more than 50 years previously. The fact that *Rayleigh* developed a method, while *Fourier* established a principle, reflects the different attitudes of these two men of science with respect to physical dimensions. *Rayleigh* never gave a logico-didactic structure to his method, but made extensive and good use of it as a research tool; he noted at the end of the book "*The Theory of Sound*":

"In the course of this work we had frequent occasion to notice the importance of the conclusions that may be arrived at by the method of dimensions".

He also confirmed the scale invariance of the pi-space which can also be interpreted in the way that a physical law retains its validity independently of the dimensional system in which it was formulated:

"This expression must retain its form unchanged, whatever may be the fundamental units by which the quantities are numerically

expressed, as is evident, when it is considered that in deriving it no assumption would be made as to the magnitude of those units”.

To achieve this, the relationships have to be formulated in a dimensionally homogeneous manner or better still, should be dimensionless.

While in England the development of dimensional analysis towards the end of the 19th century lay exclusively in the hands of *Lord Rayleigh*, in France a host of engineers and physicists were occupying themselves with this method more fundamentally: *Bertrand*, *Lucas*, *Carvallo*, *Clavenad*, *Vaschy*, *Mereadier* and *Vogt* contributed to the development of the method by the end of the century. *Carvallo* and *Vaschy* were the first to undertake – in a sort of personal contest with each other – to formulate the general theorem for the method of dimensions; *Vaschy* achieved it in 1892 [144]. In his book [145] he formulates it as follows:

“Any homogeneous relationship among p quantities $a_1, a_2, \dots, a_p$, the values of which depend on the choice of the units, may be reduced to a relationship involving $(p - k)$ parameters that are monomial combinations of $a_1, a_2, \dots, a_p$, and that are of zero dimensions ($L^0 M^0 T^0 = 1$), if among the p units of the quantities $a_1, a_2, \dots, a_p$, one can choose k arbitrarily. Obviously this number k cannot be greater than the number of fundamental units, n ”.

Vaschy’s justification of this theorem is based essentially on the invariance of the relationship with respect to changes in the values of the units, and it makes use of the mathematical conditions of homogeneity of functions. Some of the modern detailed proofs of the *Vaschy* theorem are essentially based on this approach.

After *Vaschy*, the theorem was apparently forgotten, even by the French, and no references were found in the literature for many years. Thus there is a good reason to believe that in 1911, when *A. Federmann* [29] and *D. Riabouchinsky* [121] rediscovered the fundamental theorem of dimensional analysis in Russia, they did it independently. In 1911, *A. Federmann* proved the pi-theorem mathematically stringently in the course of his research on partial differential equations; for details see [37]. *Riabouchinsky* proved the pi-theorem by showing that the physical relationship in question can be expressed by a series expansion of power products of the corresponding variables.

E. Buckingham did not know the work of *A. Federmann*; in his opinion *D. Riabouchinsky* was the discoverer of the pi-theorem. The reason why posterity has attributed the pi-theorem to *E. Buckingham* is found in his paper in the year 1914 [16] – in which he introduced the term ‘pi-theorem’ – and with which he provoked strong interest amongst the experts for this great method.

This theorem states that it is always possible to formulate a physical process in a dimensionless way, whereby the number of the dimensionless products, which describe the process completely, is equal to the number of dimensional parameters minus the number of base dimensions contained in them. Later (1922) *P. W. Bridgman* [15] showed that this rule of thumb is not always correct: it is more

accurate to speak of the rank of the dimensional matrix than of the number of base dimensions.

P. W. Bridgman (1922 and 1931) [15] and H. L. Langhaar (1951) [74] popularized dimensional analysis and the Theory of Models which was based on it, by their excellent textbooks. In Russia the textbook of L. I. Sedov [132] is well known. Among the books in German the work of J. Pawłowski [103], which treats a series of dimensional-analytical questions in an exact mathematical way, must be mentioned. It offers an extremely simple method for the formation of dimensionless numbers by matrix transformation. It is the only book which treats variable physical properties (e.g., non-Newtonian liquids) [103, 109] in detail and gives a mathematical guideline for the evaluation of reference-invariant material functions. All this was consequently complied with in this book. Pawłowski presents a proof of the pi-theorem using the matrix transformation, whereas H. Görtler in his textbook [36] also treats dimensional analysis in a concise mathematical way; he compares and comments on several different proofs of the pi-theorem.

16.2

Historic Development of Scale-up

The end result of dimensional analysis is a complete set of dimensionless numbers which describe a physical process and which outline the conditions under which this process behaves “similarly” in the model and its full-sized counterpart; thus dimensional analysis is the basis of scale-up methods. Lord Rayleigh was aware of this when he referred to the studies in which he employed dimensional analysis as the study of *similitude*. Let us take this reference as our starting point in a historical survey of the scale-up which begins with the first attempts at scaling up a model; attempts made at a time when the very concept of dimensions was still unknown.

The desire to obtain information about a full-scale process by first carrying out tests on models has existed for centuries. Leonardo da Vinci [146] wrote:

“Vitruvius says that small models are of no avail for ascertaining the effects of large ones; and I here propose to prove that this conclusion is a false one”.

Galileo Galilei [33] later investigated the strength of mechanical parts of machines and Isaac Newton [92] clearly defined the concept of “mechanical similarity”. Bertrand [9] formulated these rules unequivocally in 1847 and expressed Newton’s general law of similarity as a constraint which has to be satisfied by the four ratios of length, time, force and mass. However, the technical breakthrough in the field of similarity did not come – unrecognized by most – until 1869/70, when William Froude [84, 32] determined the drag on a ship by using model experiments, and in 1883, when Osborne Reynolds [120] published the results of his model-based experiments on the flow of liquids through pipes. The “Science of Models” was

born. The significance of this method is well expressed by the famous words of L. H. Baekeland [3]:

“Commit your blunders on a small scale and make your profits on a large scale”.

It should be added that Baekeland (inventor of the first plastic material, bakelite) had in mind only experiments on the laboratory and bench scale, but the foundations of dimensional analysis were surely unknown to him.

But how was it done? Engineering science in Germany demanded the “general physical similarity” of basic quantities (one spoke of geometrical, temporal, thermal and other forms of similarity) and used this similarity to formulate “scale-up rules” for secondary quantities such as velocities, accelerations, etc. The comparison of forces, energies, etc., led to the definition of the first dimensionless groups. Moritz Weber comprehensively described these techniques of deriving dimensionless numbers in his two fundamental papers (1919 and 1930) [152, 153].

Physicists studied hydrodynamics as well as heat and mass transfer by applying the *Navier-Stokes* differential equations of hydrodynamics [40]. By making those equations dimensionless, power products (dimensionless numbers) were achieved. According to a proposal of H. Gröber [39] these numbers were named after renowned scientists, whereby the first two letters were chosen as the symbol. Thus one spoke of *Newtonian*, *Reynolds's*, *Froude's*, etc., “model law”.

It has to be seen that this quite successful development in engineering science made dimensional analysis unnecessary. In addition, the books which should have popularized it, were written by mathematicians and (technical) physicists, whose language is strange to engineers (not to speak of industrial chemists!).

On the other hand, it can be seen that modern technology (process technology, industrial chemistry, etc) cannot survive without dimensional analysis. Its application is indispensable for the evaluation of experiments which are necessary to scale up industrial plants reliably. From this point of view, a victorious career for the dimensional analyst still lies ahead. In this context some textbooks, which popularize it, should be mentioned [134, 138, 162].

17

Exercises on Scale-up and Solutions

17.1

Exercises

1 The model of a ship on a scale of $\mu = 1 : 15$ is examined in a water channel. What is the towing speed v_M in the water channel, if the ship's speed should be $v_T = 20 \text{ km/h}$ and the scale-up rule is $Fr = \text{idem}$?

2 The so-called "friction factor c_w " of a body submerged in a liquid (which corresponds to the Newton number Ne based on force F per area A instead of that based on the power P), in separation-free flow, takes a constant value:

$$c_w \equiv \frac{F/A}{\rho v^2} = \text{const} \quad Re \gg 10^3$$

This fact induced ship builders of the 19th century to build bigger and bigger ships. Why?

3 The flow resistance of water piping of $2'' \varnothing$ with curvatures and fittings is to be determined by measurements with air. The water speed will be 2.2 m/s . What must be the air speed, which should be blown through the piping, in order to obtain similar conditions? ($v_{\text{air}}/v_{\text{water}} = 15$)

4 What is the pressure loss Δp in an oil pipeline of 2 m diameter and 100 km length have, if it transports $5.000 \text{ m}^3/\text{h}$ of crude oil ($\rho = 900 \text{ kg/m}^3$; $\mu = 0.5 \text{ Pa s}$)? What is the pumping power that is necessary if the effectiveness of the compressors is $\mu = 0.7$?

5 Tin melt ($\rho = 7.230 \text{ kg/m}^3$; $\mu = 1 \text{ mPa s}$) must be pressed through complex moulds for the production of tin solders. We wish to investigate the flow pattern for models of transparent plastics with water at 20°C and to determine the pressure loss. At which throughput ratio q_M/q_T is the hydrodynamic similarity reached and what is the $\Delta p_M/\Delta p_T$ value if the experiments are performed on models of equal size ($\mu = 1$) or with $\mu = 0.5$?

6 Which pi-space describes the pouring of a liquid from a vessel under gravity in (a) friction-free conditions (Torricelli)? By which dimensionless numbers is the pi-space widened if, instead of an opening in the vessel wall, a pipe is used through which the liquid can no longer pour out free from friction (b)?

7 What power is needed by a leaf stirrer of diameter $d = 2$ m in a vessel of $D = 4$ m, if the stirrer speed is $n = 10 \text{ min}^{-1}$ and the kinematic viscosity of the liquid is $\nu = 1 \times 10^{-4} \text{ m}^2/\text{s}$?

Measurements can be obtained on a geometrically similar apparatus of $D = 0.2$ m and $d = 0.1$ m, of which the stirrer speed n can be varied. The measurements should be performed with water ($\nu = 1 \times 10^{-6}$).

- (a) What stirrer speed do we need to have in the model?
- (b) What is the power of a full-scale stirrer, if in the model measurement we find that $Ne = 8.0$?

8 What power does a turbine stirrer of $d = 0.8$ m in a baffled vessel of $D = 4$ m need, if it rotates in water with a speed of $n = 200 \text{ min}^{-1}$ and the air flow supplied from below amounts to $q = 500 \text{ m}^3/\text{h}$? The corresponding pi-space is given by $Ne(Q)$, but the functional correlation is unknown.

A geometrically similar model appliance of $D = 0.4$ m and $d = 0.08$ m, with a stirrer speed of $n = 750 \text{ min}^{-1}$ is available.

- (a) What should the air supply be in order to obtain $Q = \text{idem}$?
- (b) What stirrer power does the industrial stirrer have under the process conditions mentioned above, if in the model measurement $Ne = 1.75$ is found?

9 Which dimensionless number characterizes the creeping flow of a highly viscous liquid, which is flowing under the influence of gravity?

10 How will the material similarity in Newtonian fluids be determined with regard to the temperature dependency of a physical property (e.g., viscosity, density, etc.)?

11 How will the material similarity in non-Newtonian fluids be determined with regard to the shear viscosity?

12 In which way will the material function and the process equation be defined if *pseudoplastic* fluids are treated? (What are the determining parameters?)

13 In which way will the material function and the process equation be defined if *viscoelastic* fluids are treated? (Which are the determining parameters?)

14 By which intermediate quantity can the relevance list for the *sedimentation* process be constricted? Which process and material parameters does it replace?

15 By which intermediate quantity can the relevance list for the *flotation* process be constricted? Which process and material parameters does it replace?

16 The state of suspension during fluidization in the systems G/S or L/S is described by the dimensionless numbers

$$\text{Fr}_p' \equiv \frac{w_{ss}^2 \rho}{d_p g \Delta \rho} \quad \text{and} \quad \text{Re}_p \equiv \frac{w_{ss} d_p \rho}{\mu}$$

where w_{ss} represents the sinking velocity of the particle swarm and d_p is the particle diameter. Construct two dimensionless numbers of which one primarily represents d_p and the other w_{ss} .

17 Which pi-space describes the mass transfer for *surface aeration* and which for *volume aeration*? What is the difference between the two?

18 Which pi-space describes the mass transfer in the material system S/L (e.g., dissolution of salt particles in liquids)? What is the difference compared with systems having two fluid phases (G/L, L/L)?

19 Which pi-space describes non-steady-state *heat conduction* in solids?

20 Which pi-space presents the heat transfer characteristics in the case of *natural* and *forced convection*, respectively?

21 What is expressed by the definition of the *kinematic viscosity* ν ? What is its physical meaning? Why is it preferable to the *dynamic viscosity* μ in dimensional analysis?

22 What is expressed by the definition of the *thermal diffusivity* a ? What is its physical meaning?

23 The material parameters a , D , ν influence the molecular exchange in the transfer processes (momentum, mass and heat transfer). All three have the same dimension. What is it and which three pi-numbers are formed using these material parameters?

24 Measurements have been conducted in two geometrically similar vessels of different scale. $V_1 = 10 \text{ l}$ and $V_2 = 10 \text{ m}^3$. What is the scale factor μ between both?

25 Which pi-space describes the mass transfer from gas to falling water drops in a spray column? Tests should enable us to dimension a spray column as a spray dryer. (In this way we can determine spray drying from the analogy between mass and heat transfer.)

17.2

Solutions

1 $Fr = \text{idem} \rightarrow 5.16 \text{ km/h}$

2 With ρ and $v = \text{const}$ and $m = \rho V \propto L^3$ for the cargo-related force F/m the relationship follows

$$\frac{F}{A} \sim \frac{F}{L^2} = \text{const} \rightarrow \frac{F}{m} \triangleq \frac{F}{L^3} = \text{const } L^{-1}$$

The cargo-related force diminishes proportionally to the ship's scale.

3 $Re = \text{idem} \rightarrow 33 \text{ m/s}$

4 $v = 0.45 \text{ m/s}$; $Re = 1600$; The *Hagen-Poiseuille* equation for the ζ (Re) characteristics in creeping flow ($Re < 2.300$) results in:

$$\zeta Re \equiv \frac{\Delta p}{v^2 \rho / 2} \frac{d}{L} \frac{v d \rho}{\mu} = 64 \rightarrow \frac{\Delta p d^2}{v \mu L} = 32 \rightarrow \Delta p = 1.8 \cdot 10^5 \text{ Pa} \triangleq 1.8 \text{ bar}$$

$$P = q \Delta p / 0.7 = 357 \text{ kW}$$

5 a) $\mu = 1$: $Re = \text{idem}$; $d, \mu = \text{idem}$: $q_M/q_T = 7$ $\Delta p_M/\Delta p_T = 7$

b) $\mu = 0.5$: $Re = \text{idem}$: $q_M/q_T = 3.5$ $\Delta p_M/\Delta p_T = 28$

6 (a) $Eu = \frac{\text{potential energy}}{\text{kinetic energy}} = \frac{\rho g h}{\rho v^2 / 2} = 1 \rightarrow v = \sqrt{2 g H}$

(b) $Eu = f(Re, L/d)$ (relevance list is enlarged by μ, d, L)

7 $Re \equiv n d / v = 6.7 \times 10^3$

(a) $n d^2 / v = 6.7 \times 10^3 \rightarrow n = 40 \text{ min}^{-1}$

(b) $Ne = 8 \rightarrow P = Ne \rho n^3 d^5 = 8.0 \times 1 \times 10^3 \times (10/60)^3 \times 2^5 = 1185 \text{ W} \equiv 1.2 \text{ kW}$

8 $q/nd^3 = 8.14 \times 10^{-2} \rightarrow q \approx 1.88 \text{ m}^3/\text{h}$

$P = Ne \rho n^3 d^5 = 1.75 \times 1 \times 10^3 \times (200/60)^3 \times 0.8^5 = 21\,200 \text{ W} \equiv 21 \text{ kW}$

9 Dimensional matrix $\{\mu, d, v, g\rho\} \rightarrow \frac{g \rho d^2}{\mu v} \equiv Re / Fr$

10 The dependency $x(T)$ must be standardized:

$$\{x/x_0, T/T_0, (1/x_0)(dx/dT)_{T_0} \Delta T\}; x = \mu, \rho, \text{ etc.}$$

- 11 The dependency $\mu(\dot{\gamma})$ must be standardized:
 $\{\mu/\mu_0, \dot{\gamma}/\dot{\gamma}_0\}$. At $m \neq \text{idem}$ the material similarity is not given!
- 12 Material function: $f\{\mu/\mu_0, \dot{\gamma}/\dot{\gamma}_0, m\} = 0$.
 Process equation: pi-numbers which contain μ must be formed with μ_0 , there is an additional pi-number which contains $\dot{\gamma}_0$, e.g., $v \dot{\gamma}_0/L$ or $\dot{\gamma}_0/n$, etc.
- 13 Material function: $f\{Wi/Wi_0, \dot{\gamma}/\dot{\gamma}_0\} = 0$.
 Process equation: pi-numbers which contain μ , must be formed with μ_0 , additional pi-numbers are Wi and one which contains $\dot{\gamma}_0$, e.g., $v \dot{\gamma}_0/L$ or $\dot{\gamma}_0/n$, etc.
- 14 The settling velocity w_s replaces d_p , φ_p , $g\Delta\rho$ and μ .
- 15 The floating velocity w replaces d_p , φ_p , $g\Delta\rho$, q_{Gas} and all physical parameters, which govern the process: pH , c_{Tensid} , etc.
- 16 $(Re_p^2 / Fr_p')^{1/3} \equiv d_p (\rho g\Delta\rho / \mu^2)^{1/3} = d_p (g\Delta\rho / v^2 \rho)^{1/3}$
 $(Fr_p' Re_p)^{1/3} \equiv w_{ss} (\rho^2 / g\Delta\rho \mu)^{1/3} = w_{ss} (\rho / v g\Delta\rho)^{1/3}$
- 17 $k_L a$ is an extensive target quantity. It must depend on *extensively* formulated process numbers: (d, n) or (d, q) or (d, v) .
 This leads to kinematic dimensionless numbers like Re and Fr .
 $k_L a$ is an *intensive* target quantity. It must depend on *intensively* formulated process numbers: P/V and q/S . This leads to $(P/V)^*$ and $v^* \equiv (q/S)^*$.
- 18 The material system L/S has a solid phase with a solid surface, therefore $A \propto d^2$. Thus a corresponding dimensionless number $Sh \equiv k_L d_p / D$ can be produced.
- 19 The only material parameter here is thermal diffusivity $a \equiv \lambda / \rho c_p$. From this it follows: $\{d, a, T_0, \Delta T, t\} \rightarrow \{a t / d^2, \Delta T / T_0\} = f\{Fo, \Delta T / T_0\} = 0$
- 20 Natural convection: $\{a; d; \rho, v, \lambda, c_p; \beta \Delta T, g\} \rightarrow Nu = f(Gr, Pr, \text{Geometry})$
 Forced convection: $\{a; d; \rho, v, \lambda, c_p; v\} \rightarrow Nu = f(Re, Pr, \text{Geometry})$
- 21 a) Physical: The reverse influence of ρ and μ on the momentum exchange.
 b) Dimensional-analytical: The dimension of v has one base dimension less than μ .
- 22 a) Physical: The reverse influence of λ and ρc_p on the heat convection.
 b) Dimensional-analytical: The dimension of a does not contain the base dimension of the temperature.

23 All three material parameters have the *same* dimension [$L^2 T^{-1}$]

Schmidt number $Sc \equiv \nu/D$

Prandtl number $Pr \equiv \nu/a$

Lewis number $Le \equiv Sc/Pr \equiv a/D$

24 The scale factor μ always refers to the linear scale:

$$\mu = l_T/l_M = (V_T/V_M)^{1/3} = (1000/1)^{1/3} = 10$$

25 Water is atomized by a nozzle in a vertical column. A droplet distribution is produced whose median value (d_{50}) is denoted by the particle diameter d_p . The falling water drops absorb oxygen from the air. The O_2 uptake is measured at different column heights l and thus at different falling times (mean residence times) $\tau = l/v$ (v – falling velocity). From the slope of the straight line in the graph $\ln c_{O_2} = f(\tau)$, $k_L a$ is determined.

Because of the surface absorption $G = k_L A \Delta c$, the measured $k_L a$ must be multiplied by the sample volume: $k_L a V = k_L A$. $k_L A \equiv G/\Delta c$ is the target quantity of this process.

The relevance list: $\{k_L A, d_p, \rho_L, \mu_L, D, \tau\}$
 by means of the dimensional matrix $\{\rho_L, d_p, \tau \mid k_L A, D, \mu_L\}$
 results in the following three dimensionless numbers:

$$\Pi_1 \equiv \frac{k_L A \tau}{d_p^3}; \quad \Pi_2 \equiv \frac{D \tau}{d_p^2}; \quad \Pi_3 \equiv \frac{\mu_L \tau}{\rho_L d_p^2}$$

All three contain τ as well as d_p . By combining them correspondingly we obtain a target number, a process number and a pure material number:

$$\Pi_1 \Pi_2^{-1} \equiv \frac{k_L A}{d_p D} \triangleq \frac{k_L d_p}{D} \equiv Sh - \text{Sherwood number (with } A \propto d_p^2)$$

$$\Pi_2 \equiv \frac{D \tau}{d_p^2} \equiv Fo' - \text{Fourier number for a non steady-state mass transfer}$$

$$\Pi_2^{-1} \Pi_3 \equiv \frac{\nu}{D} \equiv Sc - \text{Schmidt number}$$

The process thus takes place in the following pi-space:

$$\frac{k_L d_p}{D} = f\left(\frac{D \tau}{d_p^2}, \frac{\nu}{D}\right) \rightarrow Sh = f(Fo', Sc)$$

18

List of important, named pi-numbers

Name	Symbol	Pi-number	Remarks
A Mechanical process engineering			
Archimedes	Ar	$g \Delta \rho l^3 / \rho v^2$	$\equiv (\Delta \rho / \rho) Ga$
Bond	Bd	$\rho g l^2 / \sigma$	$\equiv We / Fr$
Brinkman	Br	$v^2 \mu / (J k \Delta T)$	J – Joule' heat equivalent
Deborah	De	λn	λ – relaxation time
Euler	Eu	$\Delta \rho / (\rho v^2)$	
Froude	Fr	$v^2 / (l g)$	
	Fr*	$v^2 \rho / (l g \Delta \rho)$	$\equiv Fr (\rho / \Delta \rho)$
Galilei	Ga	$g l^3 / v^2$	$\equiv Re^2 / Fr$
Hedström	He	$d^2 \rho \tau_0 / \mu_0^2$	$\equiv Re_0 \dot{\gamma}_0 / n$
Laplace	La	$\Delta p d / \sigma$	$\equiv Eu We$
Mach	Ma	v / v_s	v_s – velocity of sound
Newton	Ne	$F / (\rho v^2 l^2)$	F – force
		$P / (\rho v^3 l^2)$	P – power
Ohnesorge	Oh	$\mu / (\rho \sigma l)^{1/2}$	$\equiv We^{1/2} / Re$
Reynolds	Re	$v l / \nu$	$\nu \equiv \mu / \rho$
Stokes	Sto	$\rho_p d_p^2 v / (l \mu)$	$\equiv Re_p (d_p / l)$
Strouhal	Sr	$l f / v$	f – frequency
Viscosity ratio	Vis	μ_w / μ	w – wall
Weber	We	$\rho v^2 l / \sigma$	
Weissenberg	Wi	N_1 / τ	N_1 – 1. normal stress

Name	Symbol	Pi-number	Remarks
B Thermal process engineering (heat transfer)			
Fourier	Fo	$a t/l^2$	a – thermal diffusivity
Grashof	Gr	$\beta \Delta T g l^3 / \nu^2$	$\equiv \beta \Delta T Ga$
Nusselt	Nu	$h l/k$	h – heat transfer coefficient k – heat conductivity
Péclet	Pe	$v l/a$	$\equiv Re Pr$
Prandtl	Pr	ν/a	$a \equiv k/(\rho c_p)$
Rayleigh	Ra	$\beta \Delta T g l^3 / (a \nu)$	$\equiv Gr Pr$
Stanton	St	$a / (v \rho c_p)$	$\equiv Nu/(Re Pr)$
C Thermal process engineering (mass transfer)			
Bodenstein	Bo	$v l/D_{ax}$	D_{ax} – dispersion coefficient
Lewis	Le	a/D	$\equiv Sc/Pr$
Schmidt	Sc	ν/D	D – mass diffusivity
Sherwood	Sh	$k l/D$	k – mass transfer coefficient
Stanton	St	k/v	$\equiv Sh/(Re Sc)$
D Chemical reaction engineering			
Arrhenius	Arr	$E/(RT)$	E – energy of activation
Damköhler	Da	$c \Delta H_R / (C_p \rho T_0)$	eq. (14.7)
	Da_I	$k_1 \tau$	k – reaction rate constant τ – mean residence time
	Da_{II}	$k_1 L^2/D$	$\equiv Da_I Bo$
	Da_{III}	$k_1 \tau \left(\frac{c \Delta H_R}{\rho C_p T_0} \right)$	$\equiv Da_I Da$
	Da_{IV}	$\frac{k_1 c \Delta H_R l^2}{k T_0}$	$\equiv Da_I Re Da$
Hatta	Ha_{t_1}	$(k_1 D)^{1/2}/k_L$	1st order reaction
	Ha_{t_2}	$(k_2 c_2 D)^{1/2}/k_L$	2nd order reaction
Prater	β	eq. (14.29)	$\equiv Da Le^{-(1+n)}$
Thiele modulus	Φ	$L \sqrt{k_1/D}$	$\equiv Da_{II}^{1/2}$
Weisz modulus	Ψ'	eq. (14.44)	$\equiv \eta_K \Phi^2 = \eta_K Da_{II}$

19

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Index

a

- Acceleration due to gravity 31
- Atomization of liquids 140

b

- Boussinesq* 95
- Bridgman*, P.W. 9
- Bubble columns
 - dimensioning by rules of thumb 48
 - gas hold-up 170
 - mixing-time 49
 - heat transfer 185
 - mass transfer 196

c

- Catalytical reactions *see* Solid-catalyzed gas reactions
- Centrifugal filters, spin drying process in – 163
- Coalescence in gas/liquid contacting 203
- Cold-flow-model 226

d

- Damköhler*, G. 212
- Density, standard representation of $\rho(T)$ 63
- Dimension 3
- Denotation of diagrams, wrong – 23
- Density, temperature dependency
 - of – 63, 72
- Diffusivity of water in plastics 66
- Dimensional analysis 3
 - fundamental principle 3
 - of variable physical properties 57
 - advantage of using – 51
 - scope of applicability of – 52
 - typical problems and mistakes in the use of – 101
- Dimensional constants 4
- Dimensional homogeneity 7

Dimensional systems 5

Dryers 205

Drying

- of films 206
- of wet polymeric mass 66

e

- Emulsifiers 146
 - colloid mills 146
 - high pressure emulsifiers 148
 - jet emulsifiers 149
- Exercises on Scale-up and Solutions 253

f

- Flotation
 - dissolved Air –, DAF 34
 - induced Air –, IAF 156
- Foam breakers, centrifugal 39
- Free Fall, Law of – 9
- Froude*, W. 45

g

- Galilei* 9
- Görtler*, H. 14
- Gravitational constant 31
- Grinding of solids in stirred media mills 150

h

- Heat transfer
 - in bubble columns 185
 - in stirring vessels 121, 182
 - in pipelines 184
 - from a heated wire to air stream 27
- Hanging film phenomenon 143
- History of Dimensional Analysis and Scale-up 247
- Homogeneous reactions of 1st order 212
- Homogenization of liquid mixtures 33

i

- Injectors for gassing liquids 196
- Intermediate Physical Quantities 37

l

- Living World, Processes in the Realm of – 237
- Liquid atomization 140

m

- Mass transfer
 - surface aeration 191
 - volume aeration (in mixing vessels) 193
 - volume aeration (in bubble columns) 196
 - limitations in fast chemical reactions 233
- Material function
 - dimensionless representation of – 59
 - reference-invariant representation of – 68
 - in non-Newtonian fluids 72
 - in *Ostwald-de Waele* fluids 73
 - concept of *Metzner and Otto* 86
- Matrix transformation 17
- Measuring units
 - base 5
 - derived, named 5
- Micro-reactors 105
- Mini-plants 105
- Mixing of liquids *see* Stirring
- Mixing of solids 131
- Mock-up plants 226
- Model measurements
 - change of scale in – 54
 - preparation of the – 111
 - execution of the – 111
 - evaluation of the – 111
- Model scale
 - state of flow and – 101
 - accuracy of measurement and – 107
- Moon, walking on the — 241

n

- Newton, Isaak* 4, 9
 - 2nd law of motion 4
 - law of gravitation 4
- non-Newtonian fluids
 - pseudoplastic flow behavior 73
 - viscoelastic flow behavior 76
- Normal stress coefficients 77
- Nozzles *see* Injectors

o

- Optimization of process conditions 113

p

- Partial similarity 42
- Particle strenght, dependig on particle diameter 64
- Particle separation by inertial forces 166
- Pawlowski, J.* 14, 62, 74, 86, 241
- Pendulum, period of oscilation 7
- Petrochemical plants 226
- Physical quantity 3
 - base and derived 4
- Physical constants, universal – 31
- Physical properties, variable – 57
- pi-space at – 59
- temperature dependence of viscosity 59
- temperature dependence of density 63
- Pi-numbers, important, named – 259
- Pi-space
 - for variable physical properties 69
 - in processes with non-Newtonian fluids 72
 - reduction of the – 93
- Pi-set
 - generation 17
 - complete recording 109
- Pi-theorem 14
- Pipe flow, pressure drop of – 17
- Pseudoplastic fluids – flow behaviour 73

q

- Quantity, physical 3
 - Base – 4
 - Derived – 4

r

- Rank of the matrix 17
 - reduction of the – 28
- Rayleigh, Lord* 14, 31
 - Controversy *Rayleigh* – *Riabouchinsky* 93
- Reference-invariant material functions 68
- Relevance list of the problem 17
- Rheology *see* non-Newtonian liquids
- Rheological Standardization Functions 72
- Riabouchinsky* paradox 95
- Roasting time in cooking 12
- Rowing, dimensional analysis of – 238
- Rules of thumb *see* Partial siminarity

s

- Scale
 - scale factor 27
 - scale invariance 25

- Scale-up 39
 - at unavailability of the model material system 39
 - at partial similarity 42
 - in processes with non-Newtonian liquids 91
 - experimental techniques for – 53
 - under change of scales 54
 - and Mini plants 101
 - Screw machines 135
 - Sensitivity of target quantity 106
 - Ship's hull, drag resistance 43
 - Similarity
 - complete 26
 - partial 42
 - rules of thumb 47
 - Solid-catalyzed gas reactions, mass and heat transfer
 - 1st order reactions 215
 - outer transfer processes 219
 - inner transfer processes 222
 - petrochemical plants 226
 - Standardization of material functions 59
 - Standardization of rheological functions 73, 80
 - Stirring
 - dimensioning by rules of thumb 47
 - power characteristics in Newtonian liquids 113
 - power characteristic in gassed liquids 125
 - homogenization characteristic in Newtonian liquids 115
 - homogenization characteristic in liquids with $\Delta\rho$ and $\Delta\mu$ 33
 - homogenization characteristic in viscoelastic liquids 90
 - homogenization characteristic at minimum mixing work 116
 - hollow stirrers, power and gas throughput 118
 - mass transfer in surface aeration 191
 - mass transfer in volume aeration 193
 - mass transfer limitation of fast chemical reactions 233
 - heat transfer characteristic 97
 - heat transfer, optimization of stirring conditions 121
 - complete suspension of solids 106
 - Surface-bound processes, scale-up of – 49
 - Système International d'Unités (SI) 5
- t**
- Tableting process 174
 - Torricelli's formula 11
 - Tubular reactor
 - for a homogeneous reaction 212
 - for a heterogeneous catalytic reaction 215
 - for complex homogeneous reactions 229
- u**
- Universal Physical Constants 37
- v**
- Viscosity
 - temperature dependency of – 59, 70
 - effective acc. to Metzner–Otto 86
 - Viscoelastic fluids
 - flow behaviour 76
 - dimensional analysis of – 78
- w**
- Weissenberg's phenomenon, dimensional analysis 81